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Possible mechanism for transitions in wavy Taylor-vortex flow

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We studied a sequence of transitions between wavy-vortex-flow states with $m = 3$, in which a vortex pair was gained or lost. We found that the local-vortex pair width λ became nonuniform as the Reynolds number was increased. Transitions occurred when the *local* λ was larger (smaller) near the center (ends) than the stability limit for the underlying Taylor vortex flow, extrapolated into the wavy-vortex-flow regime.

The motion of a fluid confined between two concentric cylinders with the inner one rotating at angular velocity Ω undergoes a series of transitions with increasing Reynolds number $R(\Omega)$.[1] The initial transition is from azimuthal flow to Taylor vortex flow (TVF) of axial wavelength $\lambda = \lambda_c$ at $R = R_c$. It is followed by a transition to wavy vortex flow (WVF) at $R = R_w > R_c$, with R_w slightly dependent on λ. For a given $R > R_c$, many stable states are possible.[2,3] They differ in the number of vortices p and/or the number of azimuthal waves m. Upon increasing R, various transitions from one state to another occur, often with considerable hysteresis.

We measured the stability boundary for TVF in the range $R/R_c - 1 \lesssim 0.1$ and $-0.1 \lesssim \lambda/\lambda_c - 1 \lesssim 0.2$. We found good agreement with theory[4,5] for $\lambda \geq \lambda_c$, but for $\lambda < \lambda_c$ TVF became unstable for values of λ somewhat larger than the calculated values.

Upon increasing R very slowly beyond R_w, we always found WVF with $m = 3$. We observed a sequence of transitions $p \rightarrow p \pm 2 \rightarrow p \pm 4 \ldots$, which occurred with increasing R. We also found that λ became nonuniform as R increased. It grew beyond its mean value $\bar{\lambda}$ near the center of the column and decreased below $\bar{\lambda}$ near the ends. A transition in which a vortex pair was gained ($p \rightarrow p + 2$) always occurred near the center and only when the local wavelength exceeded the stability boundary calculated for TVF at that R, *in the absence of waves*. When a pair was lost ($p \rightarrow p - 2$), this occurred near the ends where the local λ was small. It thus appears that the stability boundary for the underlying vortices, extrapolated into the WVF regime, controls the observed transition sequence.

Our inner cylinder radius was 3.118 cm, and the radius ratio was $\eta = 0.893$. The end boundaries were rigid and nonrotating. The aspect ratio $L \equiv H/d$ was 53.9 except as noted (H is the height and d the gap). The inner cylinder speed was controlled to $\pm 0.05\%$ and temperature to ± 5 mK. The fluid was 30% glycerol in water by volume, with 0.6% by volume of a "Kalliroscope" flake suspension added for visualization. A signal was derived from a movable probe which detected the scattering of laser light by the flakes. Digital records of the signal were used to obtain power spectra.

The WVF transitions were observed visually and by measuring as a function of R/R_c the dimensionless wave speed $s \equiv \omega_1/m\Omega$. Here ω_1 is the fundamental angular frequency in the power spectrum. States with p ranging from 44 to 56 (p even) were prepared in the TVF state by manipulating the top boundary, and R was then increased quasistatically through R_w, resulting in $m = 3$ in each case. The results for s are shown in Fig. 1.[6] The arrows indicate the transitions which were always of the form $p \rightarrow p \pm 2n$ with $n = 1$ or 2 and always toward $p = 46$. The state $p = 46$ persisted to larger values of R than any other and underwent period doubling near $R = 1.5R_c$ immediately before becoming quasiperiod-

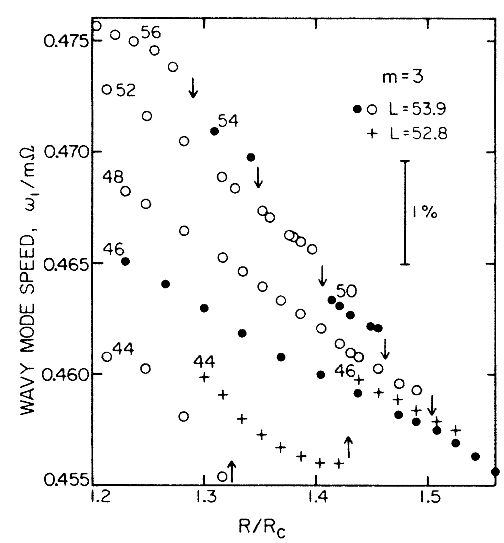

FIG. 1. Dimensionless wave speed vs R/R_c for different states. The numbers near the data sets are the values of p. Vertical arrows indicate transitions of the form $p \rightarrow p \pm 2$. The period-doubling and quasiperiodic transitions occur for the $p = 46$ state near $R/R_c = 1.5$.

ic.[7] The values of R/R_c at the transitions are shown by crosses in Fig. 2 as functions of $\bar{\lambda}$.[8] The range of stable WVF states shrinks to a unique state near $R/R_c \simeq 1.55$ and $\bar{\lambda} \simeq 2.33$ ($\bar{\lambda}$ is given in units of d). When R was increased beyond about $1.6R_c$, defects developed in the Taylor vortex lattice,[9] and all spectral features broadened (at higher R, periodic states were again encountered but with $m > 3$).

The experimental results for the stability boundaries of TVF are also shown in Fig. 2. The upper boundary is given by the solid circles which show the onset of WVF at R_w. We also explored the lower stability boundary of the TVF states for various values of p by decreasing R in small steps until a transition occurred to a different p. The results are given in Fig. 2 as open circles. At large $\bar{\lambda}$, they are in excellent agreement with the theoretical boundary[4,5] for TVF. For small $\bar{\lambda}$, we were never quite able to reach the theoretical boundary but, nonetheless, it appears that at least the qualitative features of the TVF regime are given correctly by the theory. A simple extrapolation of the theory *or* the data beyond R_w would, therefore, suggest a wide range of $\bar{\lambda}$ for WVF, contrary to the measurements shown by the crosses in Fig. 2.

In order to search for the mechanism which could determine the upper stability boundaries of WVF, we measured the local axial wavelength of the underlying vortices while in WVF states. We did this by determining the spectral power contained in the line at the fundamental frequency ω_1 versus vertical position. This power had a spatial periodicity[7] of λ. Typical

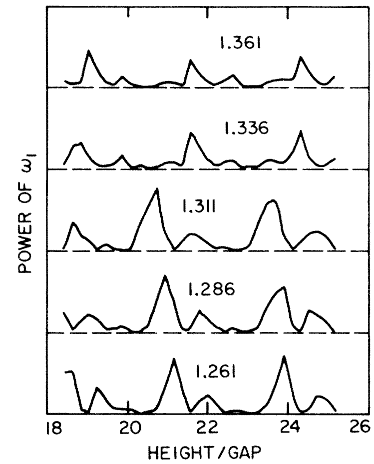

FIG. 3. Power (arbitrary units) contained in the fundamental frequency of WVF, with $m = 3$, as a function of axial displacement, near the center of the column. The numbers in the figure are the values of R/R_c, and each segment is displaced vertically relative to the others.

results for several values of R are shown in Fig. 3. The lower three sets of data are for $p = 44$. The increase in the separation between the main peaks with increasing R is evident. Between $R/R_c = 1.311$ and 1.336, an abrupt transition to $p = 48$ occurred. Quantitative evaluation of data such as those in Fig. 3, taken near the center of the column, resulted in the values of the maximum vortex widths λ_{max} shown in Fig. 4. Near R_w, λ_{max} equaled the mean wavelength,

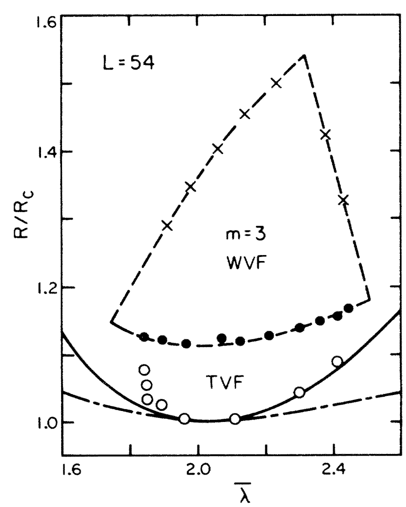

FIG. 2. Phase diagram showing the theoretical stability boundary for TVF (solid line) and Couette flow (dashed-dotted line) as well as the experimentally determined region of stability of TVF and WVF. Open circles: lower limit of stability of TVF. Solid circles: onset of WVF with $m = 3$. Crosses: upper limit of WVF with $m = 3$. Dashed lines: drawn through the data.

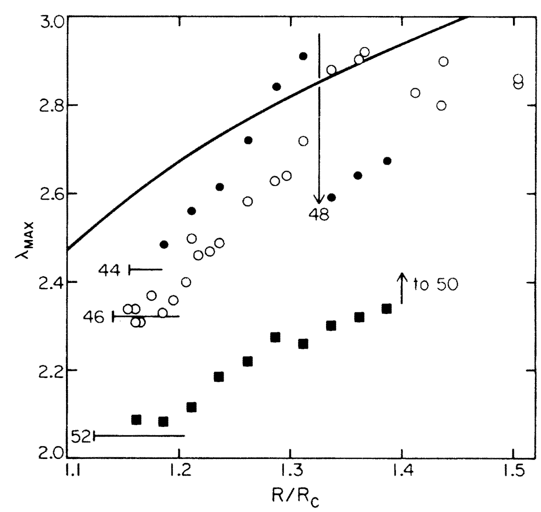

FIG. 4. Maximum local Taylor vortex width λ_{max} measured near the center of the column vs R/R_c. The horizontal lines are the average wavelengths $\bar{\lambda}$, the numbers near the lines are the values of p, and the small vertical bars are the values of R_w/R_c. The solid curve is the extrapolated theoretical boundary of TVF.

but it increased with increasing R/R_c. The stability boundary for TVF[4,5] *extrapolated into the WVF regime* is shown as a solid line in Fig. 4. Shortly after λ_{max} crossed this boundary, with $\lambda_{max} \simeq 1.2\bar{\lambda}$, the $p=44$ state made a transition to $p=48$. The $p=52$ transition at $R/R_c=1.4$ occurred by losing a vortex pair near one end, as was the case for all the transitions in which a pair was lost. Unfortunately, the spectral power near the ends was not sufficient to permit accurate wavelength measurements, but we surmise that the swelling of the pairs near the center, which very noticeably compressed those near the ends, was sufficient to drive an end pair past the stability boundary.

The state with $p=46$ ($\bar{\lambda} \simeq 2.32$) appears to be a special case. It had a λ_{max} which crossed the stability boundary near $R/R_c=1.36$, but Fig. 2 indicates that neither an increase nor a decrease of p would result in a stable state. Instead, a rearrangement in vortex widths occurred near $R=1.4R_c$ which reduced λ_{max}.

Lastly, we mention that the side boundaries of WVF shown in Fig. 2 (crosses) are very aspect-ratio dependent. Reduction of L considerably enhanced the region of stability, whereas an increase in L diminished it. This observation is consistent with the proposed mechanism. A systematic study of the L dependence is now under way.

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[5]We have compared the results of Ref. 4 which strictly holds only for axisymmetric disturbances and for $R \to R_c$, $\lambda \to \lambda_c$, with our data in the region $R/R_c - 1 \leq 0.4$ and $|\lambda/\lambda_c - 1| \lesssim 0.2$.

[6]Figure 1 shows that the wavy-mode speed decreased at constant R/R_c with increasing vortex wavelength $\bar{\lambda}$. The analogy to the volume dependence of the sound velocity in a solid with positive Grüneisen constant suggests itself. We also measured the dependence of s upon m (dispersion) by creating $m=4$ and $m=5$ states through nonmonotonic and/or nonquasistatic variations of Ω. For $\bar{\lambda}=2.20$ and $R/R_c=1.3$, for instance, we found $s=0.4658$ and 0.4559 for $m=3$ and 5, respectively. For $\bar{\lambda}=2.03$ and $R/R_c=1.3$, we obtained $s=0.4697$ and 0.4666 for $m=3$ and 4, respectively. The data show a decrease in s with increasing m in analogy to normal phonon dispersion in solids. The dependence of s upon R in Fig. 1 has an analogy in the usual temperature dependence of sound velocities. Our measured dispersion is consistent with the calculations by Jones [C. A. Jones, J. Fluid Mech. 102, 249 (1981)].

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[8]Stability boundaries for an $m=6$ state have been obtained by G. P. King and H. L. Swinney (private communication), and Phys. Rev. A (in press).

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