# Energy loss and straggling of ions with any velocity in dense plasmas at any temperature

Gilles Maynard and Claude Deutsch
*Laboratoire de Physique des Plasmas, Université Paris XI, 91405 Orsay, France*
(Received 19 February 1982)

The exact random-phase-approximation dielectric function is used to compute the energy loss and straggling of nonrelativistic charged particles in very dense electron fluids ($r_s \leq 1$) at any degeneracy and for any velocity ratio $V/V_F$. Relevance to ion-driven inertial fusion is stressed.

With the growing attractiveness of ion beams as an inertial confinement fusion (ICP) driver, we are currently witnessing a new and enlarged interest in energy losses and straggling of nonrelativistic charges in dense and hot matter. In contradistinction to the highly nonlinear[1] coupling encountered in the laser–dense-plasma interaction, at the critical density, the ion-beam target is expected to display a mostly "classical" behavior[2] monitored by weak but numerous Coulomb collisions between a projectile ion and the electrons, free or bound in the dense medium.

This rather pedestrian approach to the beam-pellet coupling brings in the possibility of accurate calculations for the ion ranges and energy deposition profile[3] in a given target. Moreover, integrating these elementary events on a pellet radius during a compression time of the order of a few nsec ($10^{-9}$ sec) allows, through appropriate hydrodynamical codes,[4] to optimize the beam characteristics; emittance, density, energy, pulse shape, etc., in order to achieve a given compression.

In this area, the present emphasis lies mostly on stopping characteristics of a dense and hot plasma with an electron temperature comparable or smaller than the Fermi one. This new situation raises the obvious question of how to extrapolate the usual low-temperature ($k_B T \ll E_F$) estimates.

To fulfill these goals, we give a complete and numerically exact solution for the energy loss and straggling of swift nonrelativistic ions in a very dense electron fluid of arbitrary degeneracy and for any ion-velocity–Fermi-velocity ratio $V/V_F$. This problem has already recently received considerable attention.[2,5,6]

Usually, the free-electron fluid provides the overwhelming contribution to the stopping processes[3] as soon as

$$r_s = \left(\tfrac{4}{3}\pi n\right)^{-1/3} a_0^{-1} \leq 1 \ .$$

As a consequence, the previous work[2,5,6] was mostly devoted to a small $V/V_F$ approximation where the partial degeneracy effect may be worked out through a simplified[4] low-frequency form of the random-phase-approximation (RPA) dynamic dielectric function $\epsilon(k,\omega)$.

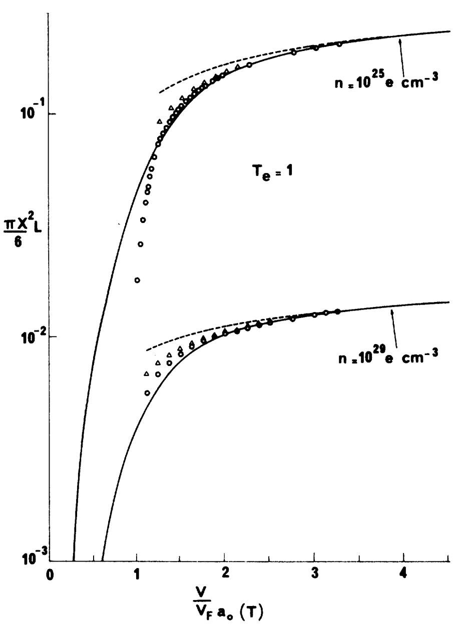


FIG. 1. $(\pi X^2/6)L$ [Eq. (2)] and its large $V_F/V$ approximants (factorized with $\pi X^2/6$); ---represents $\ln(2mV^2/\hbar\omega_p)$; $\triangle$ represents $\ln(2mV^2/\hbar\omega_p) - T_e[F_{3/2}(\alpha)/F_{1/2}(\alpha)]V_F^2/V^2$; $\circ$ represents Eq. (8) at $T_e = 1$. $V$ is measured by the mean thermal-electron velocity[9] $V_F a_0(T)$.

 

The purpose of this paper is to take advantage of an exact[8] RPA $\epsilon(k,\omega)$ to remove this velocity constraint by considering any $V/V_F$ ratios, which allows for a much more realistic modeling of the ion-beam-target coupling. Using the notations of Ref. 5, the stopping power reads

$$\frac{dE}{dx}=\frac{4\pi Z^2e^4}{mV^2}nL \ , \tag{1}$$

where $Z$ is the projectile charge and $n$ and $m$ denote, respectively, the free-electron density and the electron mass. The stopping number $L$ is given by[9,10]

$$L=\frac{6}{\pi X^2}\int_0^{V/V_F}du\,u\int_0^{\infty}dz\frac{z^3X^2f_2(u,z)}{[z^2+X^2f_1(u,z)]^2+[X^2f_2(u,z)]^2} \tag{2}$$

in terms of the standard dimensionless units

$$z=\frac{k}{2k_F},\quad u=\frac{\omega}{kV_F},\quad X^2=\frac{\alpha r_s}{\pi},\quad \alpha=0.5211\ .$$

It should be noticed that Eq. (2) was considered previously[10] by Lindhard and Winther for the case $T=0$. The purpose of the present work is to generalize these RPA calculations to any temperature (degeneracy).

The imaginary and real parts[8] of $\epsilon(k,\omega)$ read

$$f_2(u,z)=-\frac{\pi T_e}{8z}\ln\left[\frac{1+\exp\left[\dfrac{\gamma-p_+^2}{T_e}\right]}{1+\exp\left[\dfrac{\gamma-p_-^2}{T_e}\right]}\right], \tag{3}$$

$$f_1(u,z)=\int_0^{\infty}dk\,n^0(k)$$
$$+\pi T_e\sum_{n=0}^{\infty}\left[\frac{b_n}{r_n}-\frac{1}{4z}\left[\tan^{-1}\frac{P_+-a_n}{b_n}+\tan^{-1}\frac{p_+-a_n}{b_n}-\tan^{-1}\frac{p_-+a_n}{b_n}-\tan^{-1}\frac{p_--a_n}{b_n}\right]\right], \tag{4}$$

where ($\epsilon_F=1.84r_s^{-2}$),

$$T_e=\frac{k_BT}{\epsilon_F},\quad n^0(k)=\left[\exp\left[\frac{k^2-\gamma}{T_e}\right]+1\right]^{-1},\quad P_\pm=u\pm z\ ,$$

$$\gamma=\frac{\mu}{\epsilon_F}\ ,$$

with the coefficients

$$a_n=\pm\frac{1}{\sqrt{2}}\{\gamma+[\gamma^2+(2n+1)^2\pi^2T_e^2]^{1/2}\}^{1/2}\ .$$

and

$$b_n=\pm\frac{1}{\sqrt{2}}\{-\gamma+[\gamma^2+(2n+1)^2\pi^2T_e^2]^{1/2}\}^{1/2}\ .$$

Similarly, the straggling

$$\Omega^2=\frac{4\pi Z^2e^4}{mV^2}nL' \tag{5}$$

is expressed in terms of a straggling number

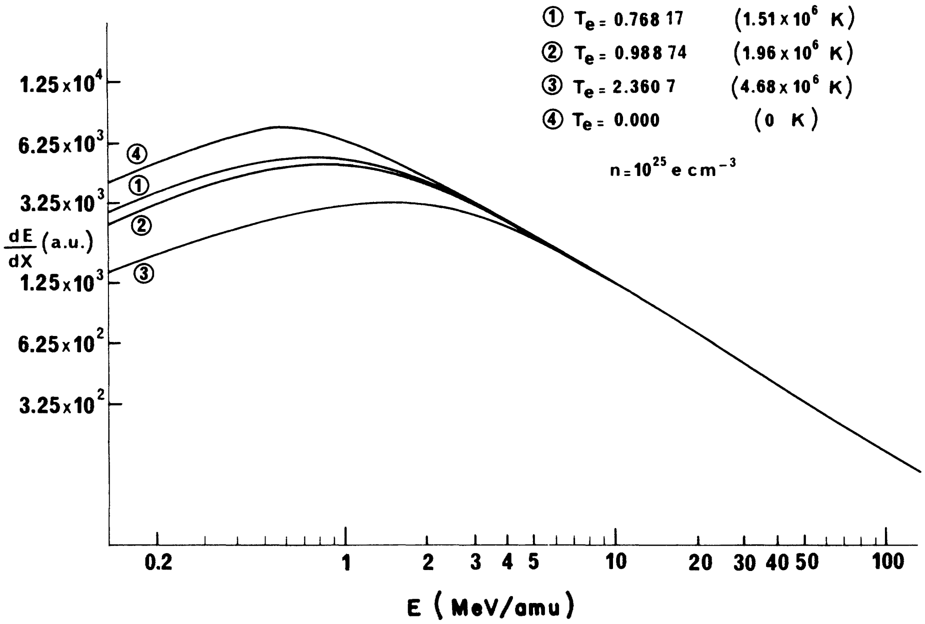


FIG. 2. Stopping power $dE/dx$ (a.u.) at $n=10^{25}e$ cm$^{-3}$ and various temperatures.

$$L'=\frac{24\epsilon_F}{\pi X^2}\int_0^{V/V_F} du\, u^2\int_0^{\infty}\frac{dz\, z^4 f_2(u,z)}{[z^2+X^2f_1(u,z)]^2+[X^2f_2(u,z)]^2}\left[\frac{2}{\exp\left[\dfrac{4zu}{T_e}\right]-1}+1\right] . \tag{6}$$

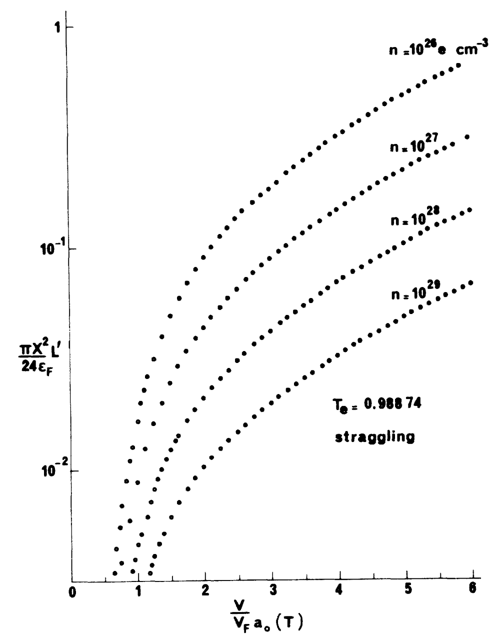


FIG. 3. $(\pi X^2/24\epsilon_F)L'$ [Eq. (6)] at $T_e=0.9887$, where $\alpha=\gamma=0$.

The small velocity limit[6] is recovered with

$$L=\frac{2}{\pi}\left[\frac{V}{V_F}\right]^3\int_0^{\infty} dz\, z^3\frac{\left.\dfrac{\partial f_2(u,z)}{\partial u}\right|_{u=0}}{[z^2+x^2g(0)]^2} , \tag{7}$$

while the large $V$ limit $[\omega_p^2=(4\pi ne^2/m)]$,

$$L=\ln\frac{2mV^2}{\hbar\omega_p}-T_e\frac{F_{3/2}(\alpha)}{F_{1/2}(\alpha)}\frac{V_F^2}{V^2}-\frac{T_e^2}{2}\frac{F_{5/2}(\alpha)}{F_{1/2}(\alpha)}\frac{V_F^4}{V^4} , \tag{8}$$

is a novel result, where

$$F_s(\alpha)=\int_0^{\infty}\frac{dx\, x^s}{e^{x-\alpha}+1} .$$

Equation (8) well approximates the complete calculation (see Fig. 1) for $V/a_0V_F\geq 2$.

The effect of temperature is especially noticable at low velocity, which allows us to visualize the discrepancies of the present complete calculations with respect to the $T=0$ calculations.[10] The corresponding $dE/dx$ displayed in Fig. 2 exhibits *important temperature effects* around $T_e \sim 1$ for ions with a few MeV per nucleon, an energy range typical of heavy ions used in ICF. As expected, $dE/dx$ is a decreasing function of $T$.

The straggling number $L'$ is given in Fig. 3. Its large $V$ limit (again a novel result) reads as

$$L' \sim 2\epsilon_F \left[ \frac{V}{V_F} \right]^2 + \frac{4T_e}{3} \frac{F_{3/2}(\alpha)}{F_{1/2}(\alpha)} \ln \frac{V}{V_F} + L'_{\rm res} , \tag{9}$$

where

$$L'_{\rm res} = 4\epsilon_F \left[ \frac{X^2}{3} \right]^{1/2} \times \left[ \frac{2}{\exp \left[ 4 \left[ \frac{X^2}{3} \right]^{1/2} \frac{1}{T_e} \right] - 1} + 1 \right] \frac{L}{2} , \tag{10}$$

reduces to $k_B T L$ when $X/T_e \ll \sqrt{3}/4$. The above computations for the first time allow us to design accurate pseudoanalytic interpolating $L$ and $L'$ formulas valid for any $V/V_F$ ratios.

The present results thus permit accurate predictons for the stopping properties of hot and dense matter in the weak coupling (RPA) approximation when its kinetic-energy density is larger than its potential one. It remains for us to design appropriate models for the strongly coupled case ($r_s \geq 1$), which is of special relevance at the beginning of the compression. However, preliminary calculations show that the required large-$k$ extension amounts only to a few percent of modifications of the present RPA quantities.

## ACKNOWLEDGMENTS

We thank H. Minoo, R. More, S. Skupsky, and Z. Zinamon for stimulating discussions.

---

[1]R. Kidder, in *Laser Interactions and Related Plasma Phenomena,* edited by H. J. Schwartz and H. Hora (Plenum, New York, 1981), p. 303.

[2]Informal Workshop on the Penetration of Charged Particles in Matter Under Extreme Conditions, New York University, January, 1980 (unpublished). See also C. Deutsch, Bull. Soc. Fr. Phys. 40, 5 (1981).

[3]T. A. Mehlhorn, J. Appl. Phys. 52, 6522 (1981).

[4]E. Nardi, E. Peleg, and Z. Zinamon, Phys. Fluids 21, 574 (1978).

[5]N. R. Arista and W. Brandt, Phys. Rev. A 23, 1898 (1981).

[6]S. Skupsky, Phys. Rev. A 16, 727 (1977) and previous references quoted therein.

[7]See, for instance, Eqs. (18)–(20) in Ref. 5.

[8]C. Gouedard and C. Deutsch, J. Math. Phys. (N.Y.) 19, 32 (1978); C. Gouedard, These 3$^e$ Cycle, Orsay, France, 1977 (unpublished).

[9]At $T=0$, $a_0(0)=1$ and $a(T) \underset{T\to\infty}{\sim} (\sqrt{T_e}/\ln T_e)$.

[10]J. Lindhard and A. Winther, K. Dan. Videsk. Selsk.-Mat. Fys. Medd. 34, No. 4 (1964).