Validity of nonrelativistic dipole approximation for forward Rayleigh scattering

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The validity of nonrelativistic dipole approximation (NRDPA) in predicting forward Rayleigh scattering amplitudes is examined. Gavrila's analytic Coulomb K-shell amplitudes are compared with the results of a relativistic numerical calculation based on a multipole expansion of the second-order S-matrix element. The real part of the forward amplitude is predicted fairly well in NRDPA at all energies, including the low-energy regime where form-factor approximation fails. The imaginary part of the amplitude is fairly well predicted by NRDPA not too far above threshold, but the prediction fails at higher energies. The dipole contribution to these amplitudes is dominant below and near threshold, but higher multipoles become increasingly important at higher photon energies. The continued usefulness of NRDPA provides another illustration of the cancellation among relativistic, retardation, and higher multipole contributions.

We wish to use the full relativistic numerical partial-wave calculation of Kissel, Pratt, and Roy[1] to illustrate the extended validity of nonrelativistic dipole approximation (NRDPA) in predicting the amplitudes for Rayleigh (R) scattering, the contribution to elastic photon scattering from an atom due to scattering off the bound atomic electrons. We focus our attention here on forward scattering thoughout the x-ray or soft gamma-ray from K-shell electrons, bound in a Coulomb potential, and demonstrate the validity of NRDPA under circumstances where neither photon energy (ω/mc^2) nor $(Z\alpha)^2$ nor (v/c) is small. We have studied the cases of Al, Ag, and Pb. As a result, these R amplitudes can be predicted from a simple analytic form, easy to compute and exhibiting useful scaling features. A similar extended validity of NRDPA has been observed for other processes, including photoeffect,[2] bremsstrahlung,[3] and internal conversion.[4]

Rayleigh scattering may be characterized by two complex amplitudes $A_\perp$ and $A_{||}$, corresponding to scattering of photons with polarizations perpendicular and parallel to the plane of scattering. When other contributions to the elastic amplitudes may be neglected (as is appropriate at these energies) the differential cross section

$$\frac{d\sigma}{d\Omega}=r_0^2(|A_\perp|^2+|A_{||}|^2)\ ,$$

where r_0 is the classical electron radius. In the forward direction $A_\perp=A_{||}\equiv M$ and further, from the optical theorem $4\pi e^2 \mathrm{Im} M=\omega\sigma_{\mathrm{pe}}$ with σ_{pe} the photoeffect total cross section, for photon energies below the pair-production threshold. Gavrila[5] has obtained an analytic expression in hypergeometric

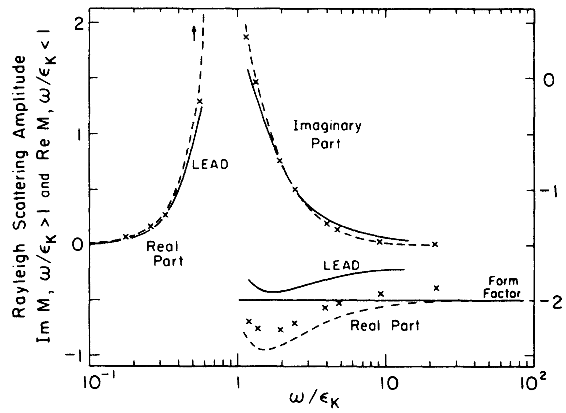

FIG. 1. Real and imaginary parts of the Rayleigh scattering amplitudes M in units of r_0, for K electrons of silver and lead obtained from SNRDPA (Ref. 5) and numerically computed (RMP), are plotted against ω/ϵ_K where ω is the photon energy and ϵ_K is the relativistic K-shell binding energy. The right-hand vertical scale is for ReM for $\omega/\epsilon_K>1$; the left-hand vertical scale is for ImM, $\omega/\epsilon_K>1$ and for ReM, $\omega/\epsilon_K<1$. Here the dashed curve represents the nonrelativistic dipole amplitudes; the solid curve represents the relativistic multipole amplitudes for lead; ($\times$) represents the relativistic multipole amplitudes for silver.

TABLE I. Real and imaginary parts of the Coulomb K-shell Rayleigh scattering amplitudes in units of r_0 for silver and lead, computed from nonrelativistic dipole approximation (Ref. 5) and from our relativistic multipole calculations, NRDPA and RMP, respectively. SNRDPA is obtained from NRDPA by scaling with the shifted relativistic binding energy. RDP is the relativistic dipole amplitude included in our numerical calculation.

Z	Photon energy (keV)	NRDPA Re	NRDPA Im	SNRDPA Re	SNRDPA Im	RDP Re	RDP Im	RMP Re	RMP Im
47	5.41	0.076		0.071		0.067		0.066	
	17.43	1.627		1.414		1.298		1.292	
	22.10	24.77		7.953		7.426		7.422	
	26.0	0.777		−1.798		−1.790		−1.780	
	35.40	−2.359	1.862	−2.330	1.964	−2.117	1.807	−2.162	1.874
	145.4	−2.1555	0.131	−2.161	0.139	−1.249	0.097	−2.036	0.142
	279.2	−2.058	0.033	−2.061	0.036	−0.606	0.019	−1.944	0.041
82	5.41	0.008		0.006		0.0048		0.0046	
	17.43	0.086		0.069		0.054		0.053	
	40.88	0.647		0.483		0.373		0.366	
	74.96	−3.283		28.12		21.07		21.06	
	84.26	−1.383		−2.507		−2.431		−2.423	
	122.5	−2.433	1.489	−2.373	1.788	−1.703	1.344	−1.829	1.531
	661.6	−2.086	0.057	−2.100	0.071	−0.378	0.030	−1.750	0.118
	1332.5	−2.027	0.013	−2.033	0.016	−0.108	0.009	−1.720	0.052

functions for M for hydrogenic atoms in NRDPA; the result, which he tabulated, depends on photon energy ω and nuclear charge Z only in the combination ω/Z^2.

In Table I and Fig. 1 we compare Gavrila'a NRDPA prediction for M with our numerical calculations for the K shell in a Coulomb potential (which, however, are relativistic and include retardation and contributions from all higher multipoles). We show our relativistic multipole results (RMP) for silver ($Z=47$) and for lead ($Z=82$) for photon energies both above and below the K-shell photoeffect threshold. Agreement with NRDPA is generally fairly good, even in these high-Z elements, and is still better in light elements; in aluminum ($Z=13$), (not shown) NRDPA remains correct to 1% from 1 to 22 keV, at 54-keV ReM is still good to 1% and ImM to 5%. ReM varies rapidly near but below the K threshold; since the relativistic threshold shifts significantly in high-Z elements this introduces a large error which can be compensated by scaling NRDPA not by Z^2 (nonrelativistic binding energy) but by the shifted relativistic binding energy (SNRDPA). At high energies ImM, basically the photoeffect cross section, fails as the gamma-ray regime is reached,[2] while for low-Z elements ReM approaches the form-factor prediction -2. A relativistic correction to the form factor of order $(Z\omega)^2$ persists in the high-energy limit[6,7]; this constant correction accounts for much of the deviation from NRDPA at all energies above threshold. We also show in Table I the numerical results for the relativistic dipole term (with retardation) alone (RDP). Below and near threshold RMP and RDP are similar, showing that higher multipole contributions are small, while at high energies RDP becomes small, showing the importance of higher multipoles particularly in maintaining the -2 form factor value of ReM correctly predicted in NRDPA. This once again illustrates the cancellations among relativistic, retardation, and higher multipole contributions.

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