Heating of and force acting on the electrons of the magnetic plasma by the electromagnetic wave

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(Received 4 February 1982)

The heating of and the force acting on the electrons of the magnetic plasma by the externally launched electromagnetic wave are investigated from first principles. The quantum-mechanically extended Vlasov equation is first derived to treat properly the inverse bremsstrahlung, and then a quasilinear equation is derived from this Vlasov equation to consider properly the electron velocity-space diffusion by the turbulence induced by the electromagnetic wave. The heating rate of and the force acting on the electrons are explicitly expressed in the forms of hydrodynamic energy and momentum equations derived from the quasilinear equation. The heating is shown to be due to the collisional and collisionless inverse bremsstrahlen of the electromagnetic wave in the magnetic field under the influence of the plasma field, and the Landau and cyclotron resonance damping of the plasma waves induced by the electromagnetic wave while the force is due to only the former. It is shown that the force is negligible in the vicinity of the electron cyclotron frequency except for the ordinary wave propagating in the direction perpendicular to the magnetic field.

I. INTRODUCTION

The heating and current driving by the electromagnetic wave in the vicinity of the electron cyclotron frequency are of great potential importance in tokamaks for a supplementary heating and the steady-state operation, respectively.

If the electromagnetic wave is externally launched into the plasma, the wave becomes distorted as it propagates since the wave gains or loses its wave energy via interactions with charged particles and plasma waves. The plasma waves include the ripples, which are the differences between the primary wave and the distorted waves. These ripples are produced by the random collective dynamics of the electrons driven by the primary wave itself in a self-consistent manner.[1] Hence, these ripples are random (turbulent) in nature and consist of an infinite number of different modes. Since the electrons are accelerated by these random fields of the ripples, the electron velocities diffuse in the velocity space; consequently, the temperature increases or decreases.

The velocity-space diffusion by the turbulence induced by the electromagnetic wave appears to have received attention in recent years,[2] while numerous papers have been devoted to the study of the velocity-space diffusion by the spontaneously generated turbulence ("thermal fluctuation") by means of quasilinear theory in both magnetized[3] and unmagnetized[4] plasmas.

Past works on propagation and absorption of the electromagnetic wave in the magnetized plasma deal only with the over-all ohmic dissipation (including scattering) based on the computations of the high-frequency conductivity,[5] or the ray tracing studies.[6-10] The former work[5] can not give the detailed features as to how the energy of the electromagnetic wave is transferred to the plasma electrons. The latter works[6-10] modeled the distorted wave (the primary wave plus the ripples) as being of the single mode. This modeling is equivalent to the unreasonable assumption that the heating electromagnetic wave propagates in the plasma without heating or cooling the plasma since the velocity-space diffusion can not be produced by a few definite waves; the electron velocity distribution periodically oscillates with the least-common-multiple period of these wave periods.

Besides the neglect of this thermodynamic principle, in these works,[6-10] the primary wave itself is viewed as a (small) perturber to the electron distribution.[11] For example, Antonsen and Manheimer[6] calculated the attenuation of the heating wave via Landau and cyclotron resonance damping by using the usual first-order perturbation theory in which the heating primary wave is viewed as the perturber. The result of the first-order perturbation theory is valid only when the perturbed quantity is small compared with the unperturbed quantity. Hence, their calculation must be unreasonable. For example, the absorption coefficient of the ordinary wave propagating in the direction perpendicular to the magnetic field in their theory is several orders of magnitude greater than that of the extraordinary wave propagating in the same direction, although the former can be attenuated only by the Landau damping whereas the latter can be attenuated by both the Landau damping and the cyclotron resonance damping.[6]

Further, in these works[6-10] the collisionless inverse bremsstrahlung can never be included, since they preclude the ripples which act as the catalyzer field for the collisionless inverse bremsstrahlung.[2]

In terms of energy transfer, the velocity-space diffusion by the plasma waves can be viewed as a chain of processes in the following manner. The energy of electromagnetic wave is transferred to the plasma waves through interactions with charged particles and the already existing plasma waves. Then, energy flows from plasma waves to the electrons by Landau and cyclotron resonance damping, which diffuses the electrons in the velocity space.

Since this mechanism is in play in the turbulent plasma waves, it has been generally misunderstood as the heating mechanism proper to only turbulent plasmas. Moreover, this mechanism is called the anomalous heating mechanism by authors of other papers.[2,12] However, I call it the "classical heating mechanism" since it is not only the universal heating mechanism by the electromagnetic wave irrespective of whether the plasma irradiated by the electromagnetic wave is turbulent or quiescent but also a fully understandable mechanism using the classical theory.

Besides the classical heating, the energy of the electromagnetic wave can be transferred directly to the individual electrons by either collisional or collisionless inverse bremsstrahlung.[2,13-16] Both inverse bremsstrahlen become resonantly enhanced as the frequency of the electromagnetic wave approaches the electron cyclotron frequency, except for the ordinary wave propagating in the direction perpendicular to the magnetic field.[16] Especially the collisionless inverse bremsstrahlung can be anomalously enhanced because not only the absorption mechanism of photons of the electromagnetic wave resonantly increases but also the plasma waves, which act as the catalyzer-field, can be resonantly produced as the frequency of the electromagnetic wave approaches the electron cyclotron frequency. The inverse bremsstrahlung is essentially a quantum-mechanical process so that even the identification of this mechanism is not fully possible by the classical theory based on the ordinary Vlasov equation.[2] Hence, in order to properly include inverse bremsstrahlung, the quantum-mechanically extended Vlasov equation is used here.

The heating rate derived from the quantum-mechanically extended Vlasov equation can give the detailed features as to how and what amount of energy of the electromagnetic wave is transferred to the electrons. Further, since the change of the electron distribution by the plasma waves is completely included in the quantum-mechanically extended Vlasov equation in the same manner as Dawson and Oberman's theory,[1] the hydrodynamic results derived from this equation must be consistent with the thermodynamic principles concerning the turbulence induced by the electromagnetic wave.

II. QUANTUM-MECHANICALLY EXTENDED VLASOV EQUATION

We begin by adopting the Vlasov equation to describe the change of the electron distribution in the $(\vec{r},\vec{v})$ space, and the time-dependent Schrödinger equation to describe the dynamics of the individual electrons in the $\vec{r}$ space.

We let the static magnetic field be in the z direction. The spatial dependence of the primary heating electromagnetic wave is neglected since the wavelength of the electromagnetic wave is much longer than those of the plasma waves. This neglect in turn gives rise to another neglect of the magnetic field of the electromagnetic wave because of Faraday's law. The latter neglect is also appropriate for magnetically confined plasmas since the plasmas are permeated by far stronger magnetic fields. By the same reasoning, the plasma field is assumed to be electrostatic.[3]

Under these assumptions, we can write the following equations (in Gaussian units):

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial \vec{r}}-\frac{e}{m}\left[\vec{E}_{em}-\nabla\phi+\frac{\vec{v}\times\vec{B}}{c}\right]\cdot\frac{\partial f}{\partial \vec{v}}=0 , \tag{1}$$

$$i\hbar\frac{\partial\psi}{\partial t}=\left[\frac{1}{2m}\left[\frac{\hbar}{i}\nabla+\frac{e}{c}\vec{A}_t\right]^2-e\phi\right]\psi , \tag{2}$$

where

$$\vec{E}_{em}(t)=-\frac{1}{c}\frac{\partial\vec{A}(y,t)}{\partial t} , \tag{3}$$

$$\vec{A}_t(y,t)=\vec{A}(t)-By\hat{x} . \tag{4}$$

Here, $\vec{E}_{em}$ is the electric field of the electromagnetic wave, $\vec{A}_t$ is the vector potential of the total electric and magnetic field, $\vec{A}$ is the vector potential of the electromagnetic wave only, ϕ is the scalar potential of the plasma field, and the other notations have the usual meanings.

The potential of the plasma field can be expanded in a double Fourier series,[2]

$$\phi(\vec{r},t)=\sum_k\sum_\Omega\phi(\vec{k},\Omega)\exp[i(\vec{k}\cdot\vec{r}-\Omega t)] . \tag{5}$$

Since $\phi(\vec{r},t)$ is real, we have $\phi(-\vec{k},-\Omega)=\phi(\vec{k},\Omega)^*$.

Here k is quantized as follows:

$$\vec{k}=2\pi(l_x/L_x,l_y/L_y,l_z/L_z); \quad l_x,l_y,l_z=0,\pm1,\pm2,\ldots$$

where L_x, L_y, and L_z are the x, y, and z dimensions of the plasma system, respectively.

After substituting Eqs. (3)–(5) into Eq. (2), we solve for the transition probability from the resulting equation by means of first-order perturbation theory where ϕ is the perturbing potential.[14,15] The unperturbed state can be designated by three quantum numbers, n_x, n_z, and n_L which are associated with three constants of motion of the unperturbed state as follows:

$$p_x=\frac{2\pi\hbar}{L_x}n_x,\quad p_z=\frac{2\pi\hbar}{L_z}n_z ,$$

$$E_{n_L}=\hbar\omega_c(n_L+1) ,$$

where E_{n_L} is the Landau energy of the unperturbed state (n_L is called the Landau quantum number), and $\omega_c=|e|B/mc$ is the electron cyclotron frequency.

The transition probability per unit time from a state 1 with quantum numbers n_{1x}, n_{1z}, and n_{1L} to a state 2 with quantum numbers n_{2x}, n_{2z}, and n_{2L} by absorbing or emitting the photons of the electromagnetic wave under the influence of the plasma field is[14,16]

$$T(1\to2)=T(2\to1)=\frac{2\pi e^2}{\hbar}\sum_{\vec{k}}\sum_\Omega|\phi(\vec{k},\Omega)|^2\sum_{\substack{s=-\infty\\ s\neq0}}^{\infty}|F_s(\omega,\omega_c,\vec{k})|^2|U(n_{1L},n_{2L},p_{1x},p_{2x},\hbar)|^2 \times\delta\left[E_{n_{2L}}+\frac{p_{2z}^2}{2m}-E_{n_{1L}}-\frac{p_{1z}^2}{2m}-\hbar\Omega-s\hbar\omega\right] \times\delta_{n_{2x}-n_{1x},l_x}\delta_{n_{2z}-n_{1z},l_z} , \tag{6}$$

where $F_s(\omega,\omega_c,\vec{k})$ is given by[14]

$$\sum_{s=-\infty}^{\infty}F_s(\omega,\omega_c,\vec{k})\exp(-is\omega t)=\exp\left[\frac{ik_x}{m}\int^t dt'\left[G-\frac{e}{c}A_x\right]-\frac{ik_yG}{m\omega_c}-\frac{iek_z}{mc}\int^t dt'A_z\right] \tag{7}$$

with

$$G(t)=\mathrm{Re}\frac{e\omega_c}{c}\int^t dt'[A_y(t')-iA_x(t')]\exp[i\omega_c(t-t')] \tag{8}$$

and

$$U(n_{1L},n_{2L},p_{1x},p_{2x},\hbar)=\left[\frac{m\omega_c}{\pi\hbar}\right]^{1/2}(2^{n_1+n_2}n_{1L}!n_{2L}!)^{-1/2}$$
$$\times\int_{-\infty}^{+\infty}dy\exp[-(q_1^2+q_2^2)/2]H_{n_{1L}}(q_1)H_{n_{2L}}(q_2)\exp(ik_yy) \tag{9}$$

with

$$q_1=\left[\frac{m\omega_c}{\hbar}\right]^{1/2}y-(m\omega_c\hbar)^{-1/2}[p_{1x}-G(t)]\ ,$$

$$q_2=\left[\frac{m\omega_c}{\hbar}\right]^{1/2}y-(m\omega_c\hbar)^{-1/2}[p_{2x}-G(t)]\ .$$

In the derivation of Eq. (6), we assumed that there are no phase relations between different components of the plasma fields.

The change of the electron distribution due to the absorption (or emission) of the photons of the electromagnetic wave under the influence of the plasma field [inverse bremsstrahlung (or bremsstrahlung)] is[14,16]

$$\left[\frac{\partial f(\vec{v}_2)}{\partial t}\right]_{\text{ib}}=\lim_{\hbar\to 0}\sum_{n_{1x}}\sum_{n_{1z}}\sum_{n_{1L}}T(1\to 2)[f(\vec{v}_1)-f(\vec{v}_2)]$$
$$=\lim_{\hbar\to 0}\frac{2\pi e^2}{\hbar}\sum_{\vec{k}}\sum_{\Omega}|\phi(\vec{k},\Omega)|^2\sum_{n_{1L}}|U(n_{1L},n_{2L},p_{2z}-\hbar k_z,p_{2z},\hbar)|^2$$
$$\times\sum_{\substack{s=-\infty\\ s\neq 0}}^{\infty}|F_s(\omega,\omega_c,\vec{k})|^2\delta\left[E_{n_{2L}}-E_{n_{1L}}+\hbar k_zv_z-\frac{\hbar^2k_z^2}{2m}-\hbar\Omega-s\hbar\omega\right]$$
$$\times\left[f\left[v_{2\perp},v_{2z}-\frac{\hbar k_z}{m}\right]-f(v_{2\perp},v_{2z})\right]\ . \tag{10}$$

In the Appendix we prove that

$$\lim_{\hbar\to 0}|U(n_{1L},n_{2L},p_{2z}-\hbar k_z,p_{2z},\hbar)|^2=\delta_{n_{1L},n_{2L}}\ . \tag{11}$$

Substituting Eq. (11) into Eq. (10) and eliminating the terms which change their signs as $(\vec{k},\Omega,s)$ changes to $(-\vec{k},-\Omega,-s)$ in the summation over $\vec{k}$, Ω, and s, we further reduce Eq. (10) to

$$\left[\frac{\partial f}{\partial t}\right]_{\text{ib}}=\pi\left[\frac{e}{m}\right]^2\sum_{\vec{k}}\sum_{\Omega}|\phi(\vec{k},\Omega)|^2|k_z|\sum_{\substack{s=-\infty\\ s\neq 0}}^{\infty}|F_s(\omega,\omega_c,\vec{k})|^2$$
$$\times\left[\frac{\partial^2 f(\vec{v})}{\partial v_z^2}\delta\left[v_z-\frac{\Omega+s\omega}{k_z}\right]+\frac{\partial f(\vec{v})}{\partial v_z}\frac{\partial}{\partial v_z}\delta\left[v_z-\frac{\Omega+s\omega}{k_z}\right]\right]\ . \tag{12}$$

In Eq. (1), $(e/m)\vec{E}_{\text{em}}\cdot\partial f/\partial\vec{v}$ is the rate of the change in the number of electrons per unit volume in the $(\vec{r},\vec{v})$ space due to acceleration (or deceleration) of electrons by the electromagnetic wave. From the quantum-mechanical viewpoint, the acceleration (or deceleration) of an electron by the electromagnetic wave means the absorption (or emission) of the photons of the electromagnetic wave. This absorption (or emission) occurs under the influence of the plasma field. Hence,

$(e/m)\vec{E}_{em}\cdot\partial f/\partial\vec{v}$ can be extended to the rate of the change in the electron distribution by inverse bremsstrahlung (or bremsstrahlung) under the plasma field.[2] Accordingly, we have the quantum-mechanically extended Vlasov equation, i.e., quantum-mechanically extended Vlasov equation (or Kim equation) as

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial \vec{r}}-\frac{e}{m}\left[-\nabla\phi+\frac{\vec{v}\times\vec{B}}{c}\right]\cdot\frac{\partial f}{\partial \vec{v}}=\left[\frac{\partial f}{\partial t}\right]_{ib}, \tag{13}$$

where the term $(\partial f/\partial t)_{ib}$ considers all interactions of the electrons with the electromagnetic wave.

III. QUASILINEAR EQUATION

Equation (13) is valid irrespective of the cause of the plasma field as long as the plasma field is much weaker than the electromagnetic field. Further, when the strength of the plasma field is so small that the perturbation of the electron distribution due to the plasma field can be adequately treated by means of quasilinear theory, we can reduce Eq. (13) to the quasilinear equation for changes in the electron distribution due to the primary electromagnetic wave itself and the plasma field, which includes the turbulence induced by the electromagnetic wave.

Since Eq. (13) is valid up to the second order in ϕ, we can apply the same methodology used to derive Eq. (2.29) of Ref. 3 as follows.

We separate f into the two terms

$$f(\vec{r},\vec{v},t)=f_0(\vec{v},t)+f_1(\vec{r},\vec{v},t), \tag{14}$$

where

$$f_0(\vec{v},t)=\frac{1}{2\pi}\int_0^{2\pi}d\theta\lim_{V\to\infty}\int_V d^3r\, f(\vec{r},\vec{v},t) \tag{15}$$

with V the volume of the plasma system and θ the azimuthal angle of $\vec{v}$, i.e., $\vec{v}=(v_\perp,\theta,v_z)$. Let $f_1(\vec{r},\vec{v},t)$ be expanded in a double Fourier series as

$$f_1(\vec{r},\vec{v},t)=\sum_{\vec{k}}\sum_{\Omega}f_1(\vec{k},\vec{v},t)\times\exp[i(\vec{k}\cdot\vec{r}-\Omega t)]. \tag{16}$$

The equation for f_0 is obtained by averaging Eq. (13) over the space and the azimuthal angle of velocity, with the result

$$\frac{\partial f_0}{\partial t}=-\frac{ie}{2\pi m}\sum_{\vec{k}}\sum_{\Omega}\sum_{\Omega'}\int_0^{2\pi}d\theta\,\phi(\vec{k},\Omega)\vec{k}\cdot\frac{\partial f_1(-\vec{k},\vec{v},\Omega')}{\partial\vec{v}}\exp[-i(\Omega+\Omega')t]+\left[\frac{\partial f_0}{\partial t}\right]_{ib}. \tag{17}$$

In the derivation of Eq. (17), the third- and higher-order terms in ϕ are neglected since we derive the quasilinear equation.

To find $\partial f_0/\partial t$ to lowest order in ϕ, the linear equation of Eq. (13) is obtained by neglecting all second- and higher-order terms. In addition, $\partial f_0/\partial t$ is neglected because this term is second order in ϕ. The linear result is

$$\left[-i\Omega+i[k_z v_z+k_\perp v_\perp\cos(\theta-\gamma)]+\omega_c\frac{\partial}{\partial\theta}\right]f_1(\vec{k},\vec{v},\Omega)=-\frac{ie}{m}\phi(\vec{k},\Omega)\vec{k}\cdot\frac{\partial f_0(\vec{v},t)}{\partial\vec{v}}, \tag{18}$$

where γ is the azimuthal angle of $\vec{k}$, i.e., $\vec{k}=(k_\perp,\gamma,k_z)$. On the left-hand side of Eq. (18), we replaced $\partial/\partial t$ by $-i\Omega$. This is possible since the time-varying component of $f_0(\vec{r},\vec{v},t)$ on the right-hand side is second order in ϕ as one readily finds it from Eqs. (12) and (17).

Since we are calculating the energy transfer from the plasma waves to the electrons, we should assume the imaginary part of Ω to be negative, i.e.,

$$\Omega=\Omega_r+i\Omega_i,\quad \Omega_i<0 \tag{19}$$

where Ω_r and Ω_i are both real. This assumption should not be confused as we are taking only actually damping modes into consideration. Energy also flows from the actually growing modes to the electrons since any wave also loses its wave energy by Landau and cyclotron resonance damping. However, $\partial f_0/\partial t$ due to the assumed damping waves ($\Omega_i<0$) can be obtained by the proper analytical continuation from $\partial f_0/\partial t$ due to the assumed growing waves ($\Omega_i>0$).[3] In order to ensure the causal solution of Eq. (18), we proceed to $\partial f_0/\partial t$ assuming $\Omega_i > 0$. By the same process as in Ref. 3, we obtain

$$f_1(\vec{k},\vec{v},\Omega) = -\frac{e}{m}\phi(\vec{k},\Omega) \sum_{s=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_s(a) J_n(a) \left[\frac{n\omega_c}{v_\perp}\frac{\partial f_0}{\partial v_\perp} + k_z \frac{\partial f_0}{\partial v_z} \right] \frac{\exp[i(n-s)(\theta-\gamma)]}{n\omega_c - \Omega + k_z v_z} , \tag{20}$$

where $a = k_\perp v_\perp/\omega_c$, and J_n is the Bessel function of order s.

Substitution of Eq. (20) into the first term on the right-hand side of Eq. (17) yields the quasilinear equation as

$$\frac{\partial f}{\partial t} = \left[\frac{\partial f}{\partial t}\right]_{\text{cl}} + \left[\frac{\partial f}{\partial t}\right]_{\text{ib}} , \tag{21}$$

where

$$\left[\frac{\partial f}{\partial t}\right]_{\text{cl}} = -i\left[\frac{e}{m}\right]^2 \sum_{\vec{k}} \sum_{\Omega} |\phi(\vec{k},\Omega)|^2 \sum_{n=-\infty}^{\infty} \left[\frac{n\omega_c}{v_\perp}\frac{\partial}{\partial v_\perp} - k_z \frac{\partial}{\partial v_z}\right] \frac{J_n^2(a)}{n\omega_c + \Omega - k_z v_z} \left[\frac{n\omega_c}{v_\perp}\frac{\partial f}{\partial v_\perp} - k_z \frac{\partial f}{\partial v_z}\right] \tag{22}$$

can readily be identified as the rate of the change in the electron distribution due to the classical heating, i.e., the Landau damping ($n = 0$) and the n-harmonic ($n \neq 0$) cyclotron dampings of the plasma waves. Here, we again used the assumption that there are no phase relations between different modes of the plasma field. The subscript "0" of f_0 is dropped in Eqs. (21) and (22), and the same simplification in notation is adopted in the following, where there is no chance of confusion.

Equation (22) is obtained with the temporary assumption $\Omega_i > 0$. The result of the proper analytical continuation from this temporary hypothetical $(\partial f/\partial t)_{\text{cl}}$ for $\Omega_i > 0$ to the desired $(\partial f/\partial t)_{\text{cl}}$ is simply obtained by applying the Plemelj formula[17] for $\Omega_i < 0$ to $1/(n\omega_c + \Omega - k_z v_z)$ in Eq. (22), i.e.,

$$\frac{1}{n\omega_c + \Omega - k_z v_z} = P\frac{1}{n\omega_c + \Omega - k_z v_z} + i\pi\delta(n\omega_c + \Omega - k_z v_z) , \tag{23}$$

where P means the principal value.

We readily find from Eqs. (22) and (23) that since the first term of Eq. (23) changes its sign as $(\vec{k},\Omega,n)$ changes to $(-\vec{k},-\Omega,-n)$, the contributions to $(\partial f/\partial t)_{\text{cl}}$ from the first term of Eq. (23) when Eq. (23) is substituted into Eq. (22) cancel out by the summation over $\vec{k}$, Ω, and n. Thus, we have

$$\left[\frac{\partial f}{\partial t}\right]_{\text{cl}} = \pi\left[\frac{e}{m}\right]^2 \sum_{\vec{k}} \sum_{\Omega} |\phi(\vec{k},\Omega)|^2 \sum_{n=-\infty}^{\infty} \left[\frac{n\omega_c}{v_\perp}\frac{\partial}{\partial v_\perp} - k_z \frac{\partial}{\partial v_z}\right] J_n^2(a)\delta(n\omega_c + \Omega - k_z v_z) \left[\frac{n\omega_c}{v_\perp}\frac{\partial f}{\partial v_\perp} - k_z \frac{\partial f}{\partial v_z}\right] . \tag{24}$$

IV. HEATING RATE OF AND FORCE ACTING ON PLASMA ELECTRONS BY ELECTROMAGNETIC WAVE

The heating rate per unit volume and the force density acting on the electrons are found by substituting Eqs. (12) and (24) into

$$\frac{\partial\langle\epsilon\rangle}{\partial t} = \int \left[\left[\frac{\partial f}{\partial t}\right]_{\text{cl}} + \left[\frac{\partial f}{\partial t}\right]_{\text{ib}}\right] \tfrac{1}{2} N m v^2 d^3 v = \left[\frac{\partial\langle\epsilon\rangle}{\partial t}\right]_{\text{cl}} + \left[\frac{\partial\langle\epsilon\rangle}{\partial t}\right]_{\text{ib}} \tag{25}$$

and

$$\vec{F} = \int \left[\left[\frac{\partial f}{\partial t}\right]_{\text{cl}} + \left[\frac{\partial f}{\partial t}\right]_{\text{ib}}\right] N m \vec{v}\, d^3 v = \vec{F}_{\text{cl}} + \vec{F}_{\text{ib}} , \tag{26}$$

respectively.

In terms of operators, $(\partial f/\partial t)_{\rm cl}$ and $(\partial f/\partial t)_{\rm ib}$ in Eqs. (25) and (26) are, by means of Eqs. (24) and (12),

$$\left[\frac{\partial f}{\partial t}\right]_{\rm cl}=-\pi\left[\frac{e}{m}\right]^2\sum_{\vec{k}}\sum_{\Omega}|\phi(\vec{k},\Omega)|^2\frac{1}{|k_z|}\sum_{n=-\infty}^{\infty}J_n^2(a)\delta\left[v_z-\frac{\Omega+n\omega_c}{k_z}\right]\times\left[\frac{n\omega_c}{v_\perp}\frac{\partial f}{\partial v_\perp}-k_z\frac{\partial f}{\partial v_z}\right]\left[\frac{n\omega_c}{v_\perp}\frac{\partial}{\partial v_\perp}-k_z\frac{\partial}{\partial v_z}\right] \quad (27)$$

and

$$\left[\frac{\partial f}{\partial t}\right]_{\rm ib}=-\pi\left[\frac{e}{m}\right]^2\sum_{\vec{k}}\sum_{\Omega}|\phi(\vec{k},\Omega)|^2|k_z|\left[\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}|F_n(\omega,\omega_c,\vec{k})|^2\delta\left[v_z-\frac{\Omega+n\omega}{k_z}\right]\right]\frac{\partial f}{\partial v_z}\frac{\partial}{\partial v_z}\,, \quad (28)$$

respectively.

We assume the θ average and space average of the electron distribution to be Maxwellian, i.e.,

$$f(\vec{v})=(2\pi v_e^2)^{-3/2}\exp(-v^2/2v_e^2)\,. \quad (29)$$

The calculation of the heating rate and the force by means of Eqs. (25)—(29) is elementary, and gives

$$\left[\frac{\partial\langle\epsilon\rangle}{\partial t}\right]_{\rm cl}=\left[\frac{\pi}{2}\right]^{1/2}\frac{Ne^2}{mv_e^3}\sum_{\vec{k}}\sum_{\Omega}|\phi(\vec{k},\Omega)|^2\frac{\Omega^2}{|k_z|}\sum_{n=-\infty}^{\infty}I_n(b)\exp(-b)\exp[-(\Omega+n\omega_c)^2/2v_e^2k_z^2]\,, \quad (30)$$

$$\left[\frac{\partial\langle\epsilon\rangle}{\partial t}\right]_{\rm ib}=\left[\frac{\pi}{2}\right]^{1/2}\frac{Ne^2}{mv_e^3}\sum_{\vec{k}}\sum_{\Omega}\frac{|\phi(\vec{k},\Omega)|^2}{|k_z|}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}|F_n(\omega,\omega_c,\vec{k})|^2(\Omega+n\omega)^2\exp[-(\Omega+n\omega)^2/2v_e^2k_z^2]\,, \quad (31)$$

$$\vec{F}_{\rm cl}=\hat{z}\left[\frac{\pi}{2}\right]^{1/2}\frac{Ne^2}{mv_e^3}\sum_{\vec{k}}\sum_{\Omega}|\phi(\vec{k},\Omega)|^2\frac{k_z\Omega}{|k_z|}\sum_{n=-\infty}^{\infty}I_n(b)\exp(-b)\exp[-(\Omega+n\omega_c)^2/2v_e^2k_z^2]\,, \quad (32)$$

$$\vec{F}_{\rm ib}=\hat{z}\left[\frac{\pi}{2}\right]^{1/2}\frac{Ne^2}{mv_e^3}\sum_{\vec{k}}\sum_{\Omega}|\phi(\vec{k},\Omega)|^2\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{k_z(\Omega+n\omega)}{|k_z|}|F_n(\omega,\omega_c,\vec{k})|^2\exp[-(\Omega+n\omega)^2/2v_e^2k_z^2]\,, \quad (33)$$

where $b=v_e^2k_\perp^2/\omega_c^2$, and I_n is the modified Bessel function of order n. In the derivation of Eqs. (30) and (32), we used the following formula:

$$\int_0^\infty J_n^2(\beta^{1/2}x)\exp(-x^2/2\alpha)x\,dx=\alpha I_n(\alpha\beta)\exp(-\alpha\beta)\,. \quad (34)$$

For the electrostatic plasma waves, both growth and damping mechanisms are independent in the sense of propagation direction. (For the electromagnetic waves, it is not true. For example, the cyclotron resonance damping depends on whether the polarization is right handed or left handed.) Accordingly, we can conjecture that the spectrum of the plasma waves does not depend on the sense of propagation direction, i.e.,

$$|\phi(\vec{k},\Omega)|=|\phi(\vec{k},-\Omega)|\,. \quad (35)$$

We readily find from Eqs. (7) and (8) that

$$F_n(\omega,\omega_c,-\vec{k})=F_{-n}(\omega,\omega_c,\vec{k})^* \quad (36)$$

Observing the symmetry with respect to Ω and n with use of Eqs. (35) and (36), and $I_{-n}(b)=I_n(b)$ if n is an integer or zero[18] in the summation over Ω and n of Eqs. (32) and (33), we have

$$\vec{F}_{cl}=0\ ,\quad \vec{F}_{ib}\neq 0\ . \tag{37}$$

Equation (37) shows that the externally launched electromagnetic wave induces the force acting on the electrons; consequently, the electron current via the inverse bremsstrahlung whereas it cannot via the classical heating. The force induced by the inverse bremsstrahlung is parallel to the magnetic field regardless of the propagation direction of the electromagnetic wave. This salient feature for the force induced by the externally launched electromagnetic wave can be utilized to drive a current[19] in the toroidal direction in tokamaks for the steady-state operation. The current driven by this force is saturated asymptotically by the drag force due to the difference between the electron drift velocity and the ion drift velocity. As to the other mechanisms driving the current in the magnetized plasmas by the electromagnetic wave, one may refer to Wort,[20] Ohkawa,[21] Fisch,[22] Karney and Fisch,[23] and Fisch and Boozer.[24]

For the case where there is no strong cause for the plasma waves other than the externally launched electromagnetic wave of the intensity level in the present-day electron cyclotron resonance heating (ECRH) using gyrotrons, the frequency of the most dominant mode of the plasma waves is the frequency of the pumping electromagnetic wave. Hence, we can approximately express the potential of the plasma field as

$$\phi(\vec{k},\vec{r})=\sum_{\vec{k}}\exp(i\vec{k}\cdot\vec{r})[\phi(\vec{k},0)+\phi(\vec{k},\omega)\exp(-i\omega t)+\phi(\vec{k},-\omega)\exp(i\omega t)]\ , \tag{38}$$

where the first term in the brackets corresponds to the potential of the static plasma field, and the second and third terms to that of the plasma waves induced by the heating electromagnetic wave.

Since $b=v_e^2k_\perp^2/\omega_c^2 << v_e^2k_D^2/\omega_c^2=\omega_e^2/\omega_c^2$, where k_D is the Debye wave number and ω_e is the electron plasma frequency, $b<<1$ for $\omega_e \lesssim \omega_c$. The condition $\omega_e\sim\omega_c$ corresponds to $B\sim$ 30 kG for $N\sim 10^{14}/\text{cm}^3$, which is approximately satisfied for tokamak plasmas at the fusion condition of our main interest. Hence, we assume $b<<1$ in the following. Then, $I_n(b)$ is

$$I_n(b)\approx\frac{1}{|n|!}\left[\frac{b}{2}\right]^{|n|}\ . \tag{39}$$

Using Eqs. (36), (38), and (39), Eqs. (30), (31), and (33) can be further reduced to

$$\left[\frac{\partial\langle\epsilon\rangle}{\partial t}\right]_{cl}=(2\pi)^{1/2}\frac{Ne^2\omega^2}{mv_e^3}\sum_{\vec{k}}|\phi(\vec{k},\omega)|^2\left[\left[1-\frac{\omega_e^2k_\perp^2}{\omega_c^2k_D^2}\right]\exp(-\omega^2/2v_e^2k_z^2)+\frac{\omega_e^2k_\perp^2}{2\omega_c^2k_D^2}\{\exp[-(\omega-\omega_c)^2/2v_e^2k_z^2]+\exp[-(\omega+\omega_c)^2/2v_e^2k_z^2]\}\right]\ . \tag{40}$$

$$\left[\frac{\partial\langle\epsilon\rangle}{\partial t}\right]_{ib}=\left[\frac{\pi}{2}\right]^{1/2}\frac{Ne^2\omega^2}{mv_e^3}\times\sum_{\vec{k}}\frac{1}{|k_z|}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}|F_n(\omega,\omega_c,\vec{k})|^2\{|\phi(\vec{k},0)|^2n^2\exp(-n^2\omega^2/2v_e^2k_z^2)+2|\phi(\vec{k},\omega)|^2(n+1)^2\exp[-(n+1)^2\omega^2/2v_e^2k_z^2]\}\ , \tag{41}$$

$$\vec{F}_{ib}=\hat{z}\left[\frac{\pi}{2}\right]^{1/2}\frac{Ne^2}{mv_e^3}\times\sum_{\vec{k}}\frac{k_z}{|k_z|}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}|F_n(\omega,\omega_c,\vec{k})|^2\{\phi(\vec{k},0)^2n\omega\exp(-n^2\omega^2/2v_e^2k_z^2)+2|\phi(\vec{k},\omega)|^2(n+1)\omega\exp[-(n+1)^2\omega^2/2v_e^2k_z^2]\}\ , \tag{42}$$

where the first terms of Eqs. (41) and (42) are due to the collisional inverse bremsstrahlung, and the second terms to the collisionless inverse bremsstrahlung.

As we can find from Eqs. (7) and (8), $F_n(\omega,\omega_c,\vec{k})$ is sharply dependent on $\omega-\omega_c$ except for the ordinary wave propagating in the direction perpendicular to the magnetic field. To investigate the electron cyclotron resonance effect, we consider here only the extraordinary wave propagating in the direction perpendicular to the magnetic field with a frequency in the vicinity of the electron cyclotron frequency. Then, $\vec{A}(t)$ is

$$\vec{A}(t)=\hat{y}(cE_0/\omega)\cos\omega t \;, \tag{43}$$

where E_0 is the electric field amplitude of the electromagnetic wave. Accordingly, we obtain from Eqs. (7) and (8)

$$|F_n(\omega,\omega_c,\vec{k})|^2=J_n^2\left[\frac{\mu}{\omega}\right] , \tag{44}$$

where

$$\mu=\frac{eE_0k'_\perp\omega}{m(\omega-\omega_c)(\omega+\omega_c)} , \tag{45}$$

with $k'_\perp=[(\omega_c k_x/\omega)^2+k_y^2]$.

Since $\omega\approx\omega_c$, we can use the following equation[2,13-16]

$$\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty} |F_n(\omega,\omega_c,\vec{k})|^2\delta\left[v_z-\frac{\Omega+n\omega}{k_z}\right]\approx\frac{1}{2}\left[\delta\left[v_z-\frac{\Omega+\mu}{k_z}\right]+\delta\left[v_z-\frac{\Omega+\mu}{k_z}\right]\right] . \tag{46}$$

Substituting Eq. (46) into Eq. (28) and repeating the same processes used to derive Eqs. (41) and (42), we have

$$\left[\frac{\partial\langle\epsilon\rangle}{\partial t}\right]_{\text{ib}}\approx\left[\frac{\pi}{2}\right]^{1/2}\frac{Ne^2}{mv_e^3}\sum_{\vec{k}}\frac{\mu^2}{|k_z|}\exp(-\mu^2/2v_e^2k_z^2)[\,|\phi(\vec{k},0)|^2+2|\phi(\vec{k},\omega)|^2] , \tag{47}$$

$$\vec{F}\approx\vec{F}_{\text{ib}}\approx\vec{0} . \tag{48}$$

The detailed calculation for the heating rate and the force via the calculation of the plasma field will be given elsewhere.

V. REMARKS

To the author's knowledge, these are the first systematic derivations of the heating rate of and the force acting on the electrons of the magnetic plasma irradiated by the electromagnetic wave from first principles, which consider properly the inverse bramsstrahlung and the thermodynamics of the turbulence induced by the electromagnetic wave.

The author uses in part the theories and calculations devised by Seely[14] to treat the inverse bremsstrahlung in the presence of the static magnetic field, the Vlasov model used by Dawson and Oberman[1] to include the turbulence induced by the heating wave, and the methodology developed by Kennel and Engelmann[3] to consider properly the electron velocity-space diffusion due to the turbulence induced by the heating wave.

Most other calculations[6-10] on the absorption of the electromagnetic wave in the plasma cannot include the inverse bremsstrahlung. In addition, these works[6-10] contradict not only a thermodynamic principle that a primary wave alone can not diffuse the electrons in the velocity space, but also a mathematical principle that the quasilinear theory or the first-order perturbation theory is valid only when the perturbation is small.

ACKNOWLEDGMENT

The author wishes to thank Professor E. G. Harris for supplying relevant materials for the development of this theory.

APPENDIX

We wish first to evaluate the following integral function:

$$U(n_1,n_2,p_{1x},p_{2x},\hbar)=\left[\frac{m\omega_c}{\pi\hbar}\right]^{1/2}(2^{n_1+n_2}n_1!n_2!)^{-1/2}$$
$$\times\int_{-\infty}^{+\infty}\exp[-(q_1^2+q_2^2)]H_{n_1}(q_1)H_{n_2}(q_2)\exp(ik_yy)dy$$
$$=\pi^{-1/2}\alpha(2^{n_1+n_2}n_1!n_2!)^{-1/2}\exp[-(\beta_1^2+\beta_2^2)/2]$$
$$\times\int_{-\infty}^{+\infty}H_{n_1}(\alpha y+\beta_1)H_{n_2}(\alpha y+\beta_2)\exp\{-\alpha^2y^2-[\alpha(\beta_1+\beta_2)-ik_y]y\}dy\ , \tag{A1}$$

where

$$\alpha=(m\omega_c/\hbar)^{1/2}\ ,$$
$$\beta_1=-(m\omega_c\hbar)^{-1/2}[p_{1x}-G(t)]\ ,$$
$$\beta_2=-(m\omega_c\hbar)^{-1/2}[p_{2x}-G(t)]\ .$$

If we use the generating function for the Hermite polynomial

$$\exp(-s^2+2sq)=\sum_{n=0}^{\infty}\frac{H_n(q)}{n!}s^n\ , \tag{A2}$$

then, we obtain

$$\sum_n\sum_j\frac{s^nt^j}{n!j!}\int_{-\infty}^{+\infty}H_n(\alpha y+\beta_1)H_j(\alpha y+\beta_2)\exp\{-\alpha^2y^2-[\alpha(\beta_1+\beta_2)-ik_y]y\}dy$$
$$=\exp(-s^2-t^2+2s\beta_1+2t\beta_2)\int_{-\infty}^{+\infty}\exp\{-\alpha^2y^2-[\alpha(\beta_1+\beta_2-2s-2t)-ik_y]y\}dy$$
$$=\pi^{1/2}\alpha^{-1}\exp[(\beta_1+\beta_2)^2/4-k_y^2/4\alpha^2-ik_y(\beta_1+\beta_2)/2\alpha]$$
$$\times\exp[s(\beta_1-\beta_2+ik_y/\alpha)-t(\beta_1-\beta_2-ik_y/\alpha)+2st]\ . \tag{A3}$$

Equating the coefficients of each power of s and t in Eq. (A3), we obtain, by means of Eq. (A1),

$$U(n_1,n_2,p_{1x},p_{2x},\hbar)=2^{-(n_1+n_2)/2}(n_1!n_2!)^{1/2}\exp[-(\beta_1+\beta_2)^2/2-k_y^2/4\alpha^2-ik_y(\beta_1+\beta_2)/2\alpha]$$
$$\times\sum_{j=0}^{n_1}\frac{(-1)^{n_2-j}2^j(\beta_1-\beta_2+ik_y/\alpha)^{n_1-j}(\beta_1-\beta_2-ik_y/\alpha)^{n_2-j}}{j!(n_1-j)!(n_2-j)!}\ , \tag{A4}$$

where we assumed $n_2\geq n_1$.

If we insert $p_{1x}=p_{2x}-\hbar k_x$, the lowest-order term in $\hbar$ in the summation over j is the term for $j=n_1$. Accordingly, we have

$$\lim_{\hbar\to0}|U(n_1,n_2,p_{2x}-\hbar k_x,p_{2x},\hbar)|^2=\lim_{\hbar\to0}2^{-(n_1+n_2)}(n_1!n_2!)\left[\frac{2^{n_2}(c\hbar)^{(n_2-n_1)/2}}{n_2!(n_2-n_1)!}\right]^2=\delta_{n_1,n_2}\ , \tag{A5}$$

where $c=k_\perp/(m\omega_c)^{1/2}$.

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