Charged-particle scattering in a magnetic and a laser field and nonlinear bremsstrahlung

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A nonrelativistic theory of charged-particle scattering by a background potential $V(r)$ in the simultaneous presence of a magnetic and a laser field is developed. The particle motion in the magnetic and laser field in dipole approximation is treated exactly. Transitions between exact quantum-mechanical states of the charged particle are considered to lowest-order Born approximation in $V(r)$. Conservation laws and selection rules for the scattering are derived for the case of laser light circularly polarized in the plane perpendicular to the magnetic field direction. It is shown that nonlinear bremsstrahlung may occur accompanied by a corresponding change of the electron's angular momentum component along the magnetic field axis. Transition amplitudes for these cases are presented in closed form and their electric and magnetic field dependences are discussed in the limiting cases of weak and strong magnetic and laser fields. The connection of this approach to previous work is established.

I. INTRODUCTION

In recent years considerable efforts have been devoted to the experimental and theoretical investigations of the interaction of intense radiation fields with matter. One of the most extensively studied phenomena is the potential scattering of charged particles in such fields.[1] Moreover, much interest has been devoted to the study of charged-particle scattering in strong magnetic fields, mainly in connection with magnetically confined plasmas.[2,3] Apparently, both phenomena also have considerable relevance to astrophysics, where extremely strong electromagnetic fields may occur.[4] We think, therefore, that the study of the joint action of a strong magnetic and a radiation field is of practical importance as well as an interesting problem in itself. Recently, Ferrante *et al.*[5(a),5(b)] have developed a formalism to treat the potential scattering of charged particles in the simultaneous presence of a magnetic and a laser field. They found that during scattering the particle may emit or absorb integer multiples of light quanta $\hbar\omega$ and make transitions between different Landau levels at the same time. Their treatment, however, was restricted to the case in which the polarization of the laser light was parallel to the direction of the magnetic field. This allows a separation of the equation of motion, i.e., the laser field in dipole approximation only influences the motion of the particle in the direction of its polarization whereas the magnetic field confines the motion to a plane perpendicular to its direction and gives rise to the appearance of the Landau levels. Altogether, the problem has cylindrical symmetry. Classically, we have helical motion and the axis of the helix is determined by the direction of the magnetic field. Since the scattering potential usually has inversion symmetry, the cylindrical symmetry is maintained even after inclusion of the scattering potential. This symmetry property permits separation in cylindrical coordinates and the whole scattering problem can be treated analytically.

The case of a laser-light polarization not directed along the magnetic field causes more difficulties, for then the cylindrical symmetry is lost and the motion induced by the laser field gets coupled to the motion in the magnetic field. Thus the problem becomes intrinsically time dependent. Furthermore, the symmetry of the scattering potential cannot be exploited to its full extent which makes the derivation of an analytic expression more difficult.

In Sec. II we briefly outline the solution of the

Schrödinger equation for a particle in a magnetic and in a laser field. Recently the exact states were found for this problem by two of the present authors.[6] In Sec. III we develop a formalism for the calculation of transition amplitudes, based on these exact quantum-mechanical states. The formalism is similar to the one used previously by the present authors to describe scattering phenomena in intense radiation fields.[7] We derive conservation laws for energy and angular momentum. These yield simple interpretations. The transition amplitude splits into an infinite sum of incoherent amplitudes each of which corresponds to the emission or absorption of n light quanta. The same integer n appears in the conservation of angular momentum. In Sec. IV we summarize our main results, their connection with previous calculations and with other possible approximation methods. Special attention is paid to the case of a weak magnetic field. Moreover, we present a simple physical interpretation of our results. Finally, we discuss another implication of our results according to which stimulated emission may be favored as compared with absorption and thus net amplification may occur. For the free-electron laser as a device very popular in present investigations,[8] this possibility suggests the use of nonconventional arrangements, i.e., an axial guiding magnetic field. This aspect of the problem, however, still needs more elaboration.

II. QUANTUM-MECHANICAL STATES OF A CHARGED PARTICLE IN A MAGNETIC AND A LASER FIELD

The Schrödinger equation of a particle of mass m and charge e in an external field reads

$$i\hbar\left[\frac{\partial\psi}{\partial t}\right]=(1/2m)\left[\hat{\vec{p}}-\left[\frac{e}{c}\right]\vec{A}\right]^2\psi\,. \qquad (2.1)$$

In our case the vector potential $\vec{A}$ is composed of two parts

$$\vec{A}(\vec{r},t)=\vec{A}_L(t)+\vec{A}_B(\vec{r})\,, \qquad (2.2a)$$

where $\vec{A}_L(t)$ corresponds to the laser field in dipole approximation

$$\vec{A}_L(t)=[A_x(t),A_y(t),A_z(t)]\,, \qquad (2.2b)$$

and $\vec{A}_B(\vec{r})$ is the vector potential of the magnetic field which we assume to be oriented along the z axis

$$\vec{A}_B(\vec{r})=[-(\tfrac{1}{2})By,\ (\tfrac{1}{2})Bx,0]\,. \qquad (2.2c)$$

In solving (2.1) we first note that we can easily eliminate the z dependence and the irrelevant $\vec{A}_L^2$ term by the ansatz

$$\psi(x,y,z,t)=\psi_K(z,t)\chi(x,y,t)\,, \qquad (2.3a)$$

where $\psi_K(z,t)$ is the one-dimensional nonrelativistic Volkov state

$$\psi_K(z,t)=\left[\frac{1}{L_z}\right]^{-1/2}\exp\left[\left[\frac{i}{\hbar}\right]\left[\hbar Kz-(1/2m)\times\int_0^t\{[\hbar K-(e/c)A_z(t)]^2+(e^2A_x^2/2mc^2)+(e^2A_y^2/2mc^2)\}d\tau\right]\right]\,. \qquad (2.3b)$$

The structure of the equation for $\chi(x,y,t)$ is more easily recognized if we introduce absorption and emission operators $(a,a^\dagger)$, $(b,b^\dagger)$ instead of $(x,\hat{p}_x)$, $(y,\hat{p}_y)$ with the usual definitions for harmonic oscillators[9] of frequency $(\omega_c/2)$, where $\omega_c=(eB/mc)$ is the cyclotron frequency. Then $\chi(x,y,t)$ obeys the equation

$$i\hbar\left[\frac{\partial\chi}{\partial t}\right]=[\hbar(\omega_c/2)(a^\dagger a+\tfrac{1}{2})+\hbar(\omega_c/2)(b^\dagger b+\tfrac{1}{2})+(a+ib)\alpha+(a^\dagger-ib^\dagger)\alpha^*-i\hbar(\omega_c/2)(ab^\dagger-a^\dagger b)]\chi\,, \qquad (2.4a)$$

where

$$\alpha=[e\hbar\omega_c/2c(m\hbar\omega_c)^{1/2}](A_y+iA_x)\,. \qquad (2.4b)$$

Equation (2.4a) describes two resonantly coupled harmonic oscillators both exhibiting a complex, time-dependent displacement. These displacements differ by $\pi/2$ in their relative phase but have equal magnitude.

If we define displacement operators D_υ, as introduced by Glauber,[10] by

$$D_{\sigma_a}=e^{\sigma_a a^\dagger-\sigma_a^* a} \qquad (2.5)$$

and similarly for D_{σ_b}, then we can define the following new wave function:

$$\phi = D_{\sigma_a}^{-1} D_{\sigma_b}^{-1} \exp\left[\left[\frac{i}{\hbar}\right]\int_0^t 2\,\mathrm{Re}(\sigma_a \alpha)d\tau\right]\chi(x,y,t) . \quad (2.6)$$

We subject the parameters σ_a and σ_b of the displacement operators to the conditions

$$\dot{\sigma}_a = -i\omega_c \sigma_a - (i/\hbar)\alpha^* , \quad \sigma_b = -i\sigma_a . \quad (2.7)$$

Substituting (2.6) into (2.4a) and using (2.7) we find the Schrödinger equation for ϕ to satisfy

$$i\hbar\left[\frac{\partial\phi}{\partial t}\right] = [\hbar(\omega_c/2)(a^\dagger a + \tfrac{1}{2}) + \hbar(\omega_c/2)(b^\dagger b + \tfrac{1}{2}) - i\hbar(\omega_c/2)(ab^\dagger - a^\dagger b)]\phi . \quad (2.8)$$

This equation describes a charged particle in a magnetic field alone. If we return to the coordinate representation (x,y) and introduce cylindrical coordinates (ρ,ϕ), the stationary solutions of (2.8) can be written as[3]

$$\phi_{n,s}(\xi,\phi,t) = \Phi_{n,s}(\xi,\phi)\exp\left[-\left[\frac{i}{\hbar}\right][\hbar\omega_c(n+\tfrac{1}{2})t]\right] , \quad (2.9a)$$

with

$$\Phi_{n,s}(\xi,\phi) = \exp[i(n-s)\phi]I_{n,s}(\xi) ,$$
$$I_{n,s}(\xi) = \begin{cases} (\gamma/\pi)^{1/2}(n!s!)^{-1/2}\exp^{-\xi/2}\xi^{(s-n)/2}L_n^{s-n}(\xi), & s \geq n \\ (\gamma/\pi)^{1/2}(n!s!)^{-1/2}\exp^{-\xi/2}\xi^{(n-s)/2}L_s^{n-s}(\xi), & s < n . \end{cases} \quad (2.9b)$$

Here the L_α^β denote the associated Laguerre polynomials $\xi = \gamma\rho^2$ and $\gamma = (eB/2ch)$. Equations (2.9a) and (2.9b) are the stationary solutions of the two-dimensional harmonic oscillator, where $n = 0,1,2,\ldots$ is the principal quantum number of the nth Landau level with energy $\hbar\omega_c(n+\frac{1}{2})$. The energy levels are degenerate with respect to the quantum number s $(s = 0,1,2,\ldots)$. The number $m = n - s$ determines the value of the angular momentum component along the direction of the magnetic field. On account of (2.9a), (2.6), and (2.3a) the complete solution of the Schrödinger equation (2.1) can be written as

$$\psi(r,t) = \psi_K(z,t)D_{\sigma_a}D_{\sigma_b}\Phi_{n,s}(\xi,\phi)\exp\left[-\left[\frac{i}{\hbar}\right]\int_0^t [\hbar\omega_0(n+\tfrac{1}{2}) + 2\,\mathrm{Re}(\sigma_a\alpha)]d\tau\right] , \quad (2.10)$$

which we shall employ in the evaluation of the scattering amplitude.

III. TRANSITION AMPLITUDE OF POTENTIAL SCATTERING

We assume the interaction of the charged particle with the magnetic and laser field and/or its kinetic energy to be larger than the interaction with the ionic background described by the potential $V(r)$. We therefore treat $V(r)$ as a perturbation. Accordingly, we shall describe the initial and final states of the electron in a magnetic and a laser field by the wave functions (2.10). Details of such a perturbation theory in which the basis is formed by the exact quantum-mechanical states of the charged particle in the intense field(s) and where all the other interactions are treated as perturbations can be found, e.g., in Ref. 11. To lowest order in $V(r)$ the transition amplitudes between such states are given by

$$T_{fi} = -\left[\frac{i}{\hbar}\right]\int dt \langle \psi_{K_f n_f s_f} | V(r) | \psi_{K_i n_i s_i} \rangle . \quad (3.1)$$

To carry out all the integrations contained in (3.1) in the general case of arbitrary geometrical configurations of the two fields is very clumsy though expressions in closed form may be derived even in that case using the exact solutions (2.10). We therefore restrict our further considerations to a few particular configurations. First of all we find in the case $\vec{E}_L \| \vec{B}$ that $\sigma_a = \sigma_b = 0$ and so (2.10) reduces to the wave function employed by Ferrante *et al.*[5] and consequently T_{fi} coincides with their expression. This treatment shows that the laser-field component parallel to the direction of the magnetic field can always be included in the description since it only appears in that part of the equation of motion which is unaffected by the magnetic field. We therefore assume $E_z=0$ and we shall only investigate in detail the case for which the radiation field has components perpendicular to the direction of the magnetic field, i.e., polarization in the xy plane.

Let us start by writing the explicit expressions of σ_a and σ_b for some particular cases as follows:

(a) Linearly polarized laser light along the x axis. The vector potential reads

$$\vec{A}_L(t)=[-(cE/\omega)\sin\omega t,0,0] , \tag{3.2}$$

where E is the field amplitude and ω the angular frequency. Introducing this expression into (2.4b) we can immediately calculate σ_a and σ_b from (2.7). This yields

$$\sigma_{aL}=(\tfrac{1}{2})\sigma^{+}e^{i\omega t}+(\tfrac{1}{2})\sigma^{-}e^{-i\omega t}, \quad \sigma_{bL}=-i\sigma_{aL} , \tag{3.3a}$$

where

$$\sigma^{+}=-[eE\omega_c/2\omega(\omega+\omega_c)(m\hbar\omega_c)^{1/2}] , \quad \sigma^{-}=-[eE\omega_c/2\omega(\omega-\omega_c)(m\hbar\omega_c)^{1/2}] . \tag{3.3b}$$

In (3.3a) the subscript L refers to linear polarization.

*(b) Right-handed circularly polarized (*rcp*) laser light in the x-y plane.* The vector potential is

$$A_L(t)=[-(cE/\omega)\sin\omega t,+(cE/\omega)\cos\omega t,0] , \tag{3.4}$$

and for σ_a and σ_b we obtain

$$\sigma_{ar}=\sigma^{+}e^{i\omega t}, \quad \sigma_{br}=-i\sigma_{ar} , \tag{3.5}$$

where the subscript r refers to rcp.

*(c) Left-handed circularly polarized (*lcp*) laser light in the x-y plane.* The vector potential now reads

$$A_L(t)=[-(cE/\omega)\sin\omega t,-(cE/\omega)\cos\omega t,0] , \tag{3.6}$$

and for σ_a and σ_b we obtain

$$\sigma_{al}=\sigma^{-}e^{-i\omega t}, \quad \sigma_{bl}=-i\sigma_{al} , \tag{3.7}$$

where the subscript l refers to lcp.

To proceed further, we recall the following properties of the displacement operator D_σ (Ref. 10):

$$D_\sigma^{-1}aD_\sigma=a+\sigma, \quad D_\sigma^{-1}a^{\dagger}D_\sigma=a^{\dagger}+\sigma^{*} . \tag{3.8}$$

If we insert (2.10) in (3.1) we recognize that the transition amplitudes can be written in the form of the transition matrix elements of an effective potential

$$V_{\rm eff}(r,t)=D_{\sigma_a}^{\dagger}D_{\sigma_b}^{\dagger}V(r)D_{\sigma_b}D_{\sigma_a} \tag{3.9}$$

taken between the states employed in the calculations of Ferrante *et al.*[5] Using the displacement property (3.8) and replacing in $V(r)$ x and y by

$$x=(\hbar/m\omega_c)^{1/2}(a+a^{\dagger}) , \quad y=(\hbar/m\omega_c)^{1/2}(b+b^{\dagger}) , \tag{3.10}$$

we immediately realize that $V_{\rm eff}(\vec{r},t)$ can be put into the form

$$V_{\rm eff}(\vec{r},t)=V(\vec{r}-\vec{r}(t)) . \tag{3.11}$$

In the cases enumerated above, $\vec{r}(t)$ is given by

$$\vec{r}(t)=[r_L\cos\omega t,(\omega_c/\omega)r_L\sin\omega t,0] , \quad r_L=eE/m(\omega^2-\omega_c^2) , \tag{3.12a}$$

for linear polarization;

$$\vec{r}(t)=(r_{+}\cos\omega t,r_{+}\sin\omega t,0) , \quad r_{+}=eE/m\omega(\omega+\omega_c) , \tag{3.12b}$$

for rcp; and

$$\vec{r}(t)=(r_{-}\cos\omega t,-r_{-}\sin\omega t,0) , \quad r_{-}=eE/m\omega(\omega-\omega_c) , \tag{3.12c}$$

for lcp. From (3.12b) and (3.12c) we infer that in cylindrical coordinates the argument of $V_{\rm eff}$ for the rcp and lcp cases can be simply represented by

$$|\vec{r}-\vec{r}(t)|=[z^2+\rho^2+r_{\pm}^2-2\rho r_{\pm}\cos(\phi\mp\omega t)]^{1/2} . \tag{3.13}$$

We now concentrate on these two cases and insert (3.13) into (3.11), using (3.1) in the form

$$T_{fi}=-\left[\frac{i}{\hbar}\right]\int dt\,\langle\psi^{(0)}_{n_fs_fk_f}|\,V_{\rm eff}\,|\psi^{(0)}_{n_is_ik_i}\rangle . \tag{3.14}$$

The superscript 0 refers to the unperturbed wave functions (2.9a) of the two dimensional harmonic oscillator as used by Ferrante *et al.*[5] We moreover specify $V(r)$ to be a screened Coulomb potential

$$V(r)=(A_0/r)e^{-\alpha r}$$
$$=[A_0/(\bar{\xi}+z^2)^{1/2}]e^{-\alpha(\bar{\xi}+z^2)^{1/2}} ,$$
$$\bar{\xi}=\gamma^{-1}\xi(=\rho^2) . \tag{3.15}$$

Then the effective scattering potential can be obtained by replacing r by $|r-r(t)|$ from (3.13). Taking into account the explicit expression (2.9a) for $\psi_{nsk}^{(0)}$ we can carry out in (3.14) the integration over z, yielding[12]

$$\int_{-\infty}^{\infty} dz \exp(-iq_{fi}z)\{\exp[-\alpha(\bar{\xi}_{\rm eff}+z^2)^{1/2}]/(\bar{\xi}_{\rm eff}+z^2)^{1/2}\}=2K_0[\bar{\xi}_{\rm eff}^{1/2}(q_{fi}^2+\alpha^2)^{1/2}] ; \tag{3.16}$$

$q_{fi}=K_f-K_i$ is the momentum transfer and K_0 denotes the zeroth-order Bessel function of imaginary argument. Furthermore,

$$\bar{\xi}_{\rm eff}=\rho^2+r_\pm^2-2\rho r_\pm \cos(\phi\mp\omega t) \tag{3.17}$$

with rcp (upper sign) and lcp (lower sign), respectively. We are now left with the evaluation of the integral

$$T_{fi}=-(iA_0/\hbar\gamma)\int_{-\infty}^{\infty} dt\, d\xi\, d\phi\, I_{n_f s_f}(\xi)I_{n_i s_i}(\xi)K_0[\bar{\xi}_{\rm eff}^{1/2}(\alpha^2+q_{fi}^2)^{1/2}]e^{-i(m_f-m_i)\phi}$$
$$\times\exp\left[\left[\frac{i}{\hbar}\right]\{[\hbar^2K_f^2/2m-\hbar^2K_i^2/2m+(n_f-n_i)\hbar\omega_c]t\}\right] , \tag{3.18}$$

where $m=n-s$.

Here we used explicit expressions for the unperturbed wave functions of the Landau levels as well as for the phase factors of the exact solutions (2.9a) and (2.10). Application of the summation theorem for Bessel functions [see Gradshteyn and Ryzhik,[12] Eq. (8.530.2)] yields

$$K_0[\bar{\xi}_{\rm eff}^{1/2}(q_{fi}+\alpha^2)^{1/2}]=\begin{cases}\sum_{n=-\infty}^{\infty} J_n[(q_{fi}^2+\alpha^2)^{1/2}\rho]K_n[(q_{fi}^2+\alpha^2)^{1/2}r_\pm]e^{in(\phi\mp\omega t)}, & \rho<r_\pm\\ \sum_{n=-\infty}^{\infty} J_n[(q_{fi}^2+\alpha^2)^{1/2}r_\pm]K_n[(q_{fi}^2+\alpha^2)^{1/2}\rho]e^{in(\phi\mp\omega t)}, & \rho>r_\pm .\end{cases} \tag{3.19}$$

The J_n are the nth-order Bessel functions of the first kind and the K_n are the nth-order modified Bessel functions. If we substitute these expressions into (3.18) two of the remaining three integrations can be carried out.

The ϕ integration yields the conservation of angular momentum

$$\int d\phi\, e^{-i[(m_f-m_i)\pm n]\phi}=2\pi\delta_{m_f,m_i\mp n} . \tag{3.20}$$

Similarly the time integration yields the conservation of energy

$$\int_{-\infty}^{\infty} dt \exp\left[\left[\frac{i}{\hbar}\right][\hbar^2K_f^2/2m-\hbar^2K_i^2/2m+(n_f-n_i)\hbar\omega_c+n\hbar\omega]t\right]$$
$$=2\pi\hbar\delta[\hbar^2K_f^2/2m-\hbar^2K_i^2/2m+(n_f-n_i)\hbar\omega_c+n\hbar\omega] . \tag{3.21}$$

Using in (3.18) the results of Eqs. (3.20) and (3.21) we find that the transition amplitude can be represented by a sum over incoherent n-photon scattering processes

$$T_{fi}=\sum_{n=-\infty}^{\infty} T_{fi}^{(n)} ,$$
$$T_{fi}^{(n)}=-2\pi i\delta[\hbar^2K_f^2/2m-\hbar^2K_i^2/2m+(n_f-n_i)\hbar\omega_c+n\hbar\omega]t_{fi}^{(n)} , \tag{3.22}$$

where

$$t_{fi}^{(n)}=(2\pi A_0/\gamma L_z)\int_0^\infty d\xi\, I_{n_f s_f}(\xi) I_{n_i s_i}(\xi)\,[\theta(\xi-\xi_\pm)K_n(\beta\xi^{1/2})J_n(\beta\xi_\pm^{1/2})+\theta(\xi_\pm-\xi)J_n(\beta\xi^{1/2})K_n(\beta\xi_\pm^{1/2})]\,. \tag{3.23a}$$

θ stands for the Heaviside step function and we have introduced the notations

$$\beta=(\alpha^2+g_{fi}^2)^{1/2}\gamma^{-1/2},\quad \xi_\pm=\gamma r_\pm^2\;; \tag{3.23b}$$

$t_{fi}^{(n)}$ contains a single integration over the variable $\xi=\gamma\rho^2$. This integration cannot be carried out analytically, except in the limiting cases of very low and very high field strengths. In the case of very low light intensity, i.e., $\xi_\pm\to 0$, $J_n(\beta\xi_\pm^{1/2})$ can be replaced by $\delta_{n,0}$ and only the first term in (3.23a) survives with $K_n\to K_0(\beta\xi^{1/2})$. This is just the scattering amplitude for a magnetic field alone and for this case the exact expression was found by Ventura[13]:

$$t_{fi}^{(0)}=(2\pi A_0/\gamma L_z)\int_0^\infty d\xi\, I_{n_f s_f}(\xi) I_{n_i s_i}(\xi) K_0(\beta\xi^{1/2})$$

$$=(A_0/L_z)\begin{cases}[(s_i!/n_i!)(s_f!/n_f!)]^{1/2}\psi(s_i+1,s_i-s_f+1,x)L_{n_i}^{n_f-n_i}(x), & n_i<s_i\\ [(n_i!/s_i!)(n_f!/s_f!)]^{1/2}\psi(n_i+1,n_i-n_f+1,x)L_{s_i}^{s_f-s_i}(x), & n_i>s_i\,.\end{cases} \tag{3.24}$$

Here $\psi(a,b,c)$ is the confluent hypergeometric function and $x=\beta^2/4$. We can also find an analytic expression when the electric field is strong and the entire contribution to the transition amplitude practically comes from the second term in (3.23a). In this case we can neglect the first term in (3.23a) and extend in the second term the upper limit of integration to infinity. Then [cf. Eq. (7.422.2) in Ref. 12] for rcp

$$t_{fi}^{(n)}=(A_0/L_z)(-1)^{s_i+n_f}2^{-(n-1)}\beta^n e^{-\beta^2/4}L_{s_i}^{n_i-n_f}(\beta^2/4)L_{n_f}^{s_f-s_i}(\beta^2/4)K_n(\beta\xi_\pm^{1/2})(n_i!s_i!n_f!s_f!)^{1/2}\,. \tag{3.25}$$

[if, e.g., $m_f>0$, $m_f\le 0$ and $n>0$, $n_i>n_f$. Unfortunately, the cases of practical interest lie outside the ranges of applicability of (3.24) and (3.25) and numerical methods have to be used for the evaluation of the general expression (3.23a). Unfortunately, the cases of practical interest lie outside the ranges of applicability of (3.24) and (3.25) and numerical methods have to be used for the evaluation of the general expression (3.23a).

Finally we derive from our result for the transition amplitudes the transition probabilities per unit time P_{fi} and the scattering cross section σ. Taking the square of the modulus of the transition amplitudes and dividing it by the transition time we find

$$P_{fi}=\left[\frac{2\pi}{\hbar}\right]\sum_n |t_{fi}^{(n)}|^2\delta[\epsilon_f-\epsilon_i+(n_f-n_i)\hbar\omega_c+n\hbar\omega]\,. \tag{3.26}$$

Since this expression still contains a delta function, we multiply it with the density of final states in the z direction and integrate over all final states which are allowed by energy conservation

$$P_{\text{tot}}=\left[\frac{2\pi}{\hbar}\right]\sum_n |t_{fi}^{(n)}|^2\rho(\epsilon_f),\quad \epsilon_f=\epsilon_i-(n_f-n_i)\hbar\omega_c-n\hbar\omega\,, \tag{3.27}$$

where

$$\rho(\epsilon_f)=L_z m/2\pi\hbar^2K_f\,.$$

The total cross section is obtained by dividing Eq. (3.27) through the flux component along z of the incident particles, $j_{\text{inc}}=\hbar K_i/L_z m$ yielding

$$\sigma_{\text{tot}}=\sum_n (L_z^2m^2/\hbar^2K_iK_f^{(n)})\,|t_{fi}^{(n)}|^2\,. \tag{3.28}$$

Since $|t_{fi}^{(n)}|^2$ is proportional to L_z^{-2}, the expression (3.28) for the cross section is independent of L_z as it should be. We see that the quantities $t_{fi}^{(n)}$, given by Eq. (3.23a), enter into all the quantities relevant for scattering (transition probabilities per unit time and cross sections). Therefore the description of any scattering process depends on the accuracy to which $t_{fi}^{(n)}$ is known.

IV. DISCUSSION

In previous papers we developed methods to describe scattering processes of charged particles in the presence of an intense radiation field.[1,7,11] The methods were essentially based on the use of the exact quantum-mechanical states for a charged particle in an intense field as the basis set of perturbation theory. Then scattering was treated as a small perturbation and we investigated first-order transitions between these states. A similar approach was employed to treat potential scattering in a magnetic field.[3] With a slight extension of the method the case of a magnetic field and a laser field polarized along the direction of the former has also been included,[5(a),(b)] since in this case the wave function of the particle factorizes into two parts. The first part, describing the motion along the direction of polarization is just the nonrelativistic Volkov solution for a particle in an external time-dependent electric field. The second part corresponds to the motion in a plane perpendicular to the direction of the magnetic field and it is described by the wave functions of a two-dimensional harmonic oscillator. Classically this can be visualized as a confined circular motion and in quantum mechanics the energy belonging to it is quantized in integer multiples of $\hbar\omega_c$ (Landau levels).

The extension to more general field configurations seemed, however, to be a difficult task, for if the laser field has polarization components in a plane perpendicular to the magnetic field, the two types of motions are coupled and the problem becomes intrinsically time dependent. Nevertheless, we recently derived exact solutions for the general field configurations.[6] In Sec. II of the present paper we briefly rederived this solution in a form more suitable for the subsequent calculations of the transition amplitudes, i.e., a form which clearly shows the connection with the Landau level wave functions. We found the following: The motion along the magnetic field direction can still be described by a nonrelativistic Volkov-type wave function. The part corresponding to the motion in the plane perpendicular to the direction of the magnetic field can now be represented (instead of the stationary states of the two-dimensional harmonic oscillator) by displaced harmonic-oscillator states (2.10), where the displacement is a function of time.

In Sec. III we applied our solution to the description of scattering of a charged particle by a static background potential in the simultaneous presence of a magnetic and a laser field. The potential was taken to be a screened-Coulomb potential which is expected to describe most of the phenomena in plasmas. By means of the wave function (2.10) we derived analytic expressions for the transition amplitudes [Eq. (3.23a)] for the cases of left and right circular polarization of the laser field in the xy plane. The problem of linear polarization can also be approximately included in two cases. If $\omega_c \ll \omega$ (which holds for optical frequencies and a wide range of magnetic field strengths) the small displacement in (3.12a) in the direction perpendicular to the polarization of the laser field can be neglected and $\vec{r}(t)$ is one-half of the sum of the lcp and rcp displacements [Eqs. (3.12b) and (3.12a)]. For $\omega_c \ll \omega$ we therefore expect that the transition probabilities and the cross sections for the linearly polarized radiation field can be approximated by the average of the lcp and rcp probabilities and cross sections. If, on the other hand, $\omega_c \approx \omega$ then only the resonant part of r_L gives a contribution and it essentially becomes identical with the lcp case.

Furthermore, we have also shown that the expression (3.23a) reduces to the pure magnetic-field results[3,5] if no laser field is present. The integration in (3.23a) can also be carried out analytically at the other extreme of very intense radiation fields. The intermediate cases, which are the most interesting ones from the point of view of practical applications, require a numerical analysis of Eq. (3.23a). This is now in progress. Before doing so it appears to be useful, however, to proceed as far as possible with the analytic calculations and to present the corresponding results here, since they permit a clear physical interpretation and reveal some of the properties of the scattering process.

First we discuss the implications of the conservation laws. From the conservation of energy, determined by the δ function in Eq. (3.22), we conclude that the particle can absorb ($n<0$) or emit ($n>0$) light quanta during the scattering leading to an increase or decrease of its energy (nonlinear direct or inverse bremsstrahlung). The total electron energy is clearly composed of two parts, one belonging to the translational motion in the z direction and the other one stemming from the rotation in the xy plane. During the scattering either of them can be increased or decreased separately, thus giving rise to a large variety of processes. On the whole, this conservation law is similar to the one found for the case of laser-field polarization along the z axis.[5] However, the conservation law for angular momentum gets modified. For polarization in the z direction the angular momentum components along this axis remain conserved ($m_f = m_i$), while in the case of circular polarization in the xy plane the angular momentum conservation law, Eq. (3.20), yields $m_f = m_i \mp n$ where the upper sign holds for rcp and the lower sign for lcp, respectively. This has a very clear interpretation. Since the angular momentum carried by a photon is $+\hbar$ or $-\hbar$ for rcp and lcp, absorption of n rcp photons ($n<0$, upper sign) or emission of n lcp photons ($n>0$, lower sign) will increase the angular momentum of the scattered particle by $\hbar|n|$, etc.

The expressions (2.10) for the wave functions and (3.11)–(3.14) for the transition amplitudes can be also viewed from a different standpoint. In intense field calculations, a frequently used approximation method is based on the so-called space translation transformation originally proposed by Henneberger.[14] The method can be briefly described as follows. Let us introduce the quantity

$$\vec{S}(t) = -(e/mc)\int_0^t \vec{A}_L(\tau)d\tau \tag{4.1}$$

and define the unitary operator

$$\Omega = e^{-(i/\hbar)\vec{S}(t)\hat{\vec{p}}} . \tag{4.2}$$

To lowest order of approximation the space-translation method yields for the perturbed wave function

$$\psi = \Omega\psi^{(0)} , \tag{4.3}$$

where $\psi^{(0)}$ is the solution of the unperturbed problem which in our case is given by (2.9a). To relate the expression (4.3) to our exact solution (2.10), we write Ω in the form of a product of displacement operators of the type (2.5)

$$\Omega = D_{\sigma_a}^{(s)} D_{\sigma_b}^{(s)} . \tag{4.4}$$

The explicit forms of $\sigma_a^{(s)}$ and $\sigma_b^{(s)}$ for the cases discussed in Sec. III are as follows [cf. (3.3)–(3.7)].

(a) Linear polarization:

$$\sigma_a^{(s)} = -[eE\omega_c/2\omega^2(m\hbar\omega_c)^{1/2}]\cos\omega t ,$$
$$\sigma_b^{(s)} = 0 ; \tag{4.5a}$$

(b) rcp:

$$\sigma_a^{(s)} = -[eE\omega_c/2\omega^2(m\hbar\omega_c)^{1/2}]\cos\omega t ,$$
$$\sigma_b^{(s)} = -[eE\omega_c/2\omega^2(m\hbar\omega_c)^{1/2}]\sin\omega t ; \tag{4.5b}$$

(c) lcp:

$$\sigma_a^{(s)} = -[eE\omega_c/2\omega^2(m\hbar\omega_c)^{1/2}]\cos\omega t ,$$
$$\sigma_b^{(s)} = +[eE\omega_c/2\omega^2(m\hbar\omega_c)^{1/2}]\sin\omega t . \tag{4.5c}$$

If we neglect ω_c as compared with ω in the denominators of (3.3b) then we see the connection of the exact σ's with the space translated σ's given above and in all cases we may write

$$\sigma^{(s)} = \text{Re}\sigma(\omega_c \ll \omega) . \tag{4.6}$$

Consequently, our method of solution yields an essential improvement over the space-translation approximation. In the case of the harmonic oscillator the method becomes exact and it also includes resonances. Thus it may serve as a starting point for the description of atomic resonance phenomena as far as atoms in the vicinity of resonances can be approximated by harmonic oscillators.

Finally, we briefly discuss the field dependences of the cross section (3.28) since they shed light on another aspect of the problem. We first consider the dependence on the laser-field intensity. This is determined by the functions $J_n(\beta\xi_\pm^{1/2})$ for $\xi > \xi_\pm$ and $K_n(\beta\xi_\pm^{1/2})$ for $\xi < \xi_\pm$. At low laser intensities ($\xi_\pm$ small) the region $\xi > \xi_\pm$ gives the main contributions to the integral in (3.23a) and the dependence of the cross sections on the laser intensities will be [on account of $J_n(z) \sim (z/2)^n$ for small z and the explicit expression for $\beta\xi_\pm^{1/2}$]

$$\sigma^{(n)} \sim [eE(\alpha^2+q_{fi}^2)^{1/2}/m\omega(\omega\pm\omega_0)]^{2n}$$
$$\approx (eEq_{fi}/m\omega^2)^{2n} . \tag{4.7}$$

The last expression holds for small screening and far from resonances. For high laser intensities ($\xi_\pm$ large) and conventional magnetic fields the region $\xi > \xi_\pm$ still yields the main contributions. Using the asymptotic form of Bessel functions for large arguments, viz.,

$$J_n(z) \approx \left[\frac{2}{\pi z}\right]^{1/2} \cos[z-(2n+1)\pi/4] ,$$

we find

$$\sigma \sim 2m\omega(\omega\pm\omega_0)/\pi eE(\alpha^2+q_{fi}^2)^{1/2}$$
$$\approx 2m\omega^2/\pi e q_{fi} E . \tag{4.8a}$$

For strong magnetic fields the region $\xi < \xi_\pm$ gives the main contributions and with the asymptotic form for the modified Bessel functions for large arguments, $K_n(z) \approx (\pi/2z)^{1/2}\exp(-z)$, we find

$$\sigma \sim [\pi m\omega(\omega\pm\omega_0)/2eE(\alpha^2+q_{fi}^2)^{1/2}]$$
$$\times\exp[-2eE(\alpha^2+q_{fi}^2)^{1/2}/m\omega(\omega\pm\omega_0)]$$
$$\approx (\pi m\omega^2/2eq_{fi}E)\exp(-2eEq_{fi}/m\omega^2) . \tag{4.8b}$$

The field dependences in the cases described by

(4.8a) or (4.8b) are reminiscent to the field dependences of the cross sections for field emission in dc fields. We point out, that in the case of a resonance where the arguments of the Bessel functions tend to infinity the formulas (4.8a) or (4.8b) should also be used.

The simultaneous analysis of the dependence on the magnetic field is slightly more difficult. The dependence is dominated by the functions $K_n(\beta\xi)$ for $\xi > \xi_\pm$ and $J_n(\beta\xi)$ for $\xi < \xi_\pm$ in (3.23a). Using the asymptotic formula for small z

$$K_n(z) \approx [(n-1!)/2](z/2)^{-n} ,$$

and applying Eq. (7.422.2) of Ref. 12 we find for small β (i.e., for large B)

$$\sigma \sim \beta^{-2n} \approx B^n . \tag{4.9}$$

For large β (or equivalently for small B) we can immediately apply Eq. (7.422.2) of Ref. 12 and use the asymptotic form of the Laguerre polynomials, i.e.,

$$L_n^{\sigma}(y) \approx \left[\frac{1}{\sqrt{\pi}}\right] \exp\left[\frac{y}{2}\right] y^{-\sigma/2-1/4} n^{\sigma/2-1/4}$$
$$\times \cos[2\sqrt{ny} - (2\sigma+1)\pi/4] .$$

Thus we find, approximating the product of the two cosine functions by $\frac{1}{2}$, for small B:

$$\sigma \sim \beta^{-2} \sim B . \tag{4.10}$$

Equation (4.9) contains the results of Ferrante *et al.*[5] as a special case, for if we put $n=0$, the strong magnetic-field limit of the cross sections becomes independent of the magnetic field. On the other hand, (4.10) suffers from the same difficulties as the known expressions for scattering in a magnetic field, i.e., it does not reduce to the cross sections without magnetic field in the limit $B \to 0$. We can overcome this difficulty as follows. The stationary eigenstates of the two-dimensional harmonic oscillator (2.9a) *a priori* contain this difficulty since they do not reduce to free-particle states in the limit $\omega_c \to 0$.

Therefore, in the case of a weak magnetic field we have to construct a superposition of states (2.9) such that in the limit $\omega_c \to 0$ the energy of this superposition become $p^2/2m$, i.e., the energy of a free-particle. This can be accomplished by taking a coherent superposition of the states (2.9),

$$\psi_{\alpha,s} = \sum_n (\alpha^n/\sqrt{n!})\psi_{n,s} \exp(-|\alpha|^2/2) , \tag{4.11}$$

with $\alpha = ip/(m\hbar\omega_c)^{1/2}$. These $\psi_{\alpha,s}$ show the correct behavior for $\omega_c \to 0$. To investigate transitions between these states (4.11) is, however, far more complicated than for the states $\psi_{n,s}$. Nevertheless, we can demonstrate that in the limit $B \to 0$ the transition amplitudes tend to the form expected in the field-free case.

Finally we mention one interesting application of our results. In recent theoretical and experimental investigations of free-electron lasers, axial magnetic guiding fields have been introduced in addition to the usual wiggler field.[15] Our results on the nonlinear stimulated bremsstrahlung indicate that the collisions of electrons of the beam with other particles may favorably contribute to the coherent part of the emission processes in a free-electron laser. This, however, would have to be confirmed by relativistic calculations.

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