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Propagation and smoothing of nonuniform thermal fronts

Louis Baker
Sandia National Laboratories, Albuquerque, New Mexico 87185
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Calculated Rayleigh-Taylor growth for perturbations on ablation fronts may be reduced by an order of magnitude due to thermal smoothing. Because studies of the development of perturbations on ablation layers have generally derived growth rates by assuming that conditions are quasisteady in a reference frame moving with the front, they neglect this stabilizing effect. If confirmed experimentally, this will allow substantial improvements over present designs for inertial confinement fusion implosions. Using a combined Galerkin perturbation technique, we develop analytic models for the propagation of a rippled thermal front into a uniform medium. We find that the mean front advances slightly more rapidly than a uniform front, and the perturbation on the front rapidly dies off.

I. INTRODUCTION

The stability of thermal fronts is of great importance in the design of targets for inertial confinement fusion. Such fronts produce ablation in the outer shell of such targets, producing a rocket effect which accelerates the shell inward. Instabilities can adversely affect the target, requiring thicker ablators and consequent larger power and energy fluxes to achieve a given implosion velocity.

Despite a great deal of theoretical effort, the question of how stable these fronts are has not been satisfactorily resolved. Theoretical work generally takes one of three approaches. Full two-dimensional numerical simulations[1-5] cannot cover a broad region of parameter space due to cost, have complicated physics which obscures the physical mechanism, and have problems obtaining sufficient resolution. To date they have given conflicting pictures of front stability. One-dimensional simulations of implosions with subsequent two-dimensional perturbation evolution calculated about the basic state produced in the one-dimensional runs[6-8] are less costly but have the other problems. Simple theoretical models[9-11] must make simplifying assumptions and approximations, first linearizing and then neglecting one or more terms. The experimental work to date[12-14] has not detected any instability growing as fast as the classical Taylor instability; this is not surprising, as there have been many stabilizing mechanisms proposed that would reduce the growth rate of perturbations on an ablation layer relative to the growth rate of perturbations on an interface between two distinct, immiscible, liquids.

Because of the contradictory results of the various theoretical treatments of the problem, along with the lack of experimental guidance, the simple theoretical models should be of great value in clarifying the important physical processes in determining the stability of ablation layers and gauging their stability. One approximation these models all seem to contain is that, in a frame moving with the ablation front, the conditions are assumed to be independent of time. This simplification generally permits the assumption of a simple time dependence for the perturbation of the form $\exp(\gamma t)$ (but see Refs. 15–17 for studies of more classical forms of instability which do not make such an assumption). In this paper it is shown that the assumption of constant properties in the moving frame neglects an important stabilizing effect; namely, that as a perturbed or rippled ablation front advances into a uniform material, it tends to smooth itself due to the diffusive nature of its penetration. Thus the perturbation acting as a source to the hydrodynamic instability is itself decreasing with time. The evolution of the perturbation with the hydrodynamic Rayleigh–Taylor instability of the ablation layer included is not treated, but only the self-smoothing of the front. A simple model will be quite adequate for demonstrating this self-smoothing. In the regime of quasisteady conditions near the ablation front and small enough perturbations to justify the simple linear theories, the effects of the self-smoothing and hydrodynamic growth of the perturbation are separable. Self-smoothing can easily reduce perturbation amplitudes by an order of magnitude or more for parameters of interest, hence greatly reducing the growth that would otherwise be expected.

II. MODEL

The equation to be solved is

$$\partial_t \Theta = \nabla \cdot \kappa(\Theta) \nabla \Theta \ , \qquad (1)$$

where $\Theta(x,y,t)$ is the temperature field, a function of time t. The coordinate x is normal to and y is along the front, and the thermal conductivity κ is in general a function of Θ. Two special cases are considered: constant κ, and power law dependence $\kappa = \Theta^n$, where $n = 2.5$, the value characteristic of the Spitzer conductivity of a plasma. The constant κ or $n = 0$ case leads to linear equations, the general case giving a nonlinear problem in Θ. Θ is separated into a mean temperature field $\Theta_0(x,t)$ and a "ripple" or perturbation field $\Theta_1(x,y,t)$. On the plane $x = 0$, we impose the boundary condition: either specified temperature, $\Theta = \Theta_0 + \Theta_1 f(y)$ or specified flux:

$$\partial_x \Theta = \partial_x \Theta_0 + \partial_x \Theta_1 f(y) \ ,$$

where in the former case Θ_0 and Θ_1 and in the latter case $\partial_x \Theta_0$ and $\partial_x \Theta_1$ are specified constants. The former case is of interest with regard to perturbations induced by nonuniformities in the shells of inertial confinement fusion (ICF) targets, and the latter is of concern with regard to asymmetries in the illumination providing a seed perturbation for instabilities. We will therefore consider both cases.

By introducing a "modal expansion"[18–20] for Θ,

$$\Theta = T_0(x,t) + T_1(x,t) f(y) \ , \qquad (2)$$

a form of Galerkin approximation, the problem is reduced from one partial differential equation in three independent variables to two equations in two independent variables, while retaining the important physics of the problem. Equations for the mean field T_0 and the perturbation field T_1 are obtained by successively multiplying Eq. (1) by 1 and $f(y)$ and averaging over the y coordinate using the operator:

$$\langle f \rangle = \frac{k}{2\pi} \int_0^{2\pi/k} f \, dy \ . \qquad (3)$$

The following properties of the $\langle \ \rangle$ operator are used, where $f = 2^{1/2} \sin(ky)$: $\langle f \rangle = 0$, $\langle 1 \rangle = 1$, $\langle f^2 \rangle = 1$, $\langle f f_{yy} \rangle = -k^2 \langle f^2 \rangle = -k^2$, $\langle f^3 \rangle = 0$, and $\langle f_y^2 \rangle = -\langle f f_{yy} \rangle = k^2$. Expanding,

$$\kappa = T^n = (T_0 + f T_1)^n$$
$$= T_0^n + n T_0^{n-1} f T_1 + n(n-1)/2 f^2 T_0^{n-2} T_1^2 + \cdots \ ,$$

and it is assumed that $|T_1| \ll |T_0|$ permitting the ripple to be treated as a perturbation on the mean field. This assumption is physically reasonable and rigorously justifies the severely truncated expansion employed; from experience[18–20] it is expected that the modal expansion will be qualitatively valid over a much broader range, however. Positivity of the temperature field requires $|T_1| < |T_0|$. In general, an expansion of Θ as $T_0(x,t) + \sum f_i(y) T_i(x,t)$ may be used if it is considered desirable to do so. We then find the equation for the evolution of the mean field is $(D = \partial_x, \kappa_T = \partial\kappa/\partial T)$

$$\kappa_{\text{eff}} = \kappa + n(n-1)/2 T_1^2 T_0^{n-2} \ , \qquad (4)$$

$$\partial_t T_0 = D(\kappa_{\text{eff}} D T_0) + D T_0 T_0^{n-2} n(n-1) T_1 D T_1 + \kappa_T [T_1 D^2 T_1 + (D T_1)^2] \ . \qquad (5)$$

The "fluctuation field" T_1 acts through the factor κ_{eff} to enhance the penetration of the mean field in that it contributes (for $n > 1$) positive semidefinite terms to an effective diffusion coefficient κ_{eff}. The other terms modify $T_0(x)$ near the front. These terms have little effect on the solution, as verified by numerical solution of Eqs. (5) and (6) (see below).

The equation for the fluctuation field T_1 is

$$\partial_t T_1 = T_0^n (D^2 T_1 - k^2 T_1) + n(T_0^{n-1} D^2 T_0 T_1 + 2 T_0^{n-1} D T_0 D T_1) + n(n-1) T_0^{n-2} (D T_0)^2 T_1 \ . \qquad (6)$$

We see that the terms in the equation for the mean field T_0 involving the fluctuation field T_1 are all of second order in that field; hence to an excellent approximation in T_1 we may neglect these terms, obtaining for the T_0 field the equation

$$\partial_t T_0 = D[\kappa D T_0] \ . \qquad (7)$$

Approximate solutions to (7) are at hand in Anderson and Lisak[21] for the nonlinear case, and exact solutions[22] exist for the linear, constant κ $(n = 0)$ case. For the linear case, T_1 satisfies

$$\partial_t T_1 = \kappa (D^2 - k^2) T_1 \ , \qquad (8)$$

which is solved for the fixed-temperature case[22] and which may be solved following Ref. 23 for the fixed perturbation flux case. Note that in the linear case, Eq. (8) reduces to Eq. (7) if $k=0$. For fixed temperature,

$$T_0=1-\mathrm{erf}(Z)=\mathrm{erfc}(Z),\quad Z=x/2(\kappa t)^{1/2}\;, \tag{9}$$

$$T_1=T_1(x=0)/2[\exp(kx)\mathrm{erfc}(Z+b/Z)+\exp(-kx)\,\mathrm{erfc}(Z-b/Z)] \tag{10}$$

with $b=kx/2$, normalization $T_1(x=0)=1$, while for the case of an imposed flux

$$T_0=2[\exp(-Z^2)[\kappa t/\pi]^{1/2}-x/2\,\mathrm{erfc}(Z)]\;, \tag{11}$$

$$T_1=DT_1(x=0)/(2k)[\exp(-kx)\mathrm{erfc}(Z-b/Z)-\exp(kx)\mathrm{erfc}\;.(Z+b/Z)]\;. \tag{12}$$

Anderson and Lisak[21] give the following results for nonzero n, with the fixed-temperature solution:

$$T_0=\begin{cases}[1-Z/Z_0+8Z^3/(12Z_0)]^p, & p=1/(1+n),\ 0<Z<1\\ 0, & Z>1\end{cases} \tag{13}$$

and for fixed flux

$$T_0=\begin{cases}Ft^q(1-Z/Z_1)^p, & 0<Z<1\\ 0, & Z>1\end{cases} \tag{14}$$

with

$$q=1/(n+2)\;,$$

$$F=\{(n+2)[DT_0(0)]^2\}^q\;,$$

$$Z_1=(n+2)/(n+1)^2\{(n+2)[DT_0(0)]^2\}^{nq}\;,$$

$$Z_0=2(n+2)/(n+1)^2\;.$$

Equation (6) will be solved numerically, but the following simple approximation may be justified *a posteriori*. The approximate[21,24] and exact[25] solutions for T_0 all show that T_0 is almost constant out to a temperature "front," beyond which T_0 is zero. Hence DT_0 and D^2T_0 are substantial only near the front. Furthermore, as will be shown, the T_2 field propagates more slowly than the T_0 field; consequently, it can be substantial only where T_0 is appreciably constant. Thus T_1 for nonzero n is approximately the same as that for the $n=0$ cases. The numerical results below show this approximation is good throughout and excellent away from the front; there, the neglected terms serve to increase T_1. The approximation is worst at very early times which are not of interest.

III. FRONT VELOCITY

To interpret the solutions obtained in the previous section, let us define a "front" velocity as the rate of propagation of an isotherm. For the nonlinear conduction case, which have well-defined fronts, this is a weak function of the isotherm chosen, since the variation in temperature is confined to a small region. If the front temperature is T_f, then the front velocity is

$$V_f=\left.\frac{dx}{dt}\right|_{T_f}=\left.\frac{-\partial T}{\partial t}\right|_{T_f}\Big/\left.\frac{\partial T}{\partial x}\right|_{T_f}\;. \tag{15}$$

For the linear $k=0$ case, i.e., $T_0=\mathrm{erfc}(Z), V_f=(\kappa/t)^{1/2}[\mathrm{inverf}(1-T_f)]$, where $y=\mathrm{inverf}(x)$ is defined by $x=\mathrm{erf}(y)$, and the front position is $x_f=2(\kappa t)^{1/2}[\mathrm{inverf}(1-T_f)]$ (κ is constant); note $V_f=x_f/(2t)$; the general $\kappa=T^n$ case has the same dependence upon t and κ.[24] For the linear case with general k, we find, from Eq. (10),

$$V_f=(x_f/2t)/\{\cosh kx_f+0.25k(\pi\kappa t)^{1/2}\exp(Z_f^2+k^2T^2)[\exp(-kx_f)\mathrm{erfc}(Z_f-b_f/Z_f)-\exp(kx_f)\mathrm{erfc}(Z_f+b_f/Z_f)]\}\;, \tag{16}$$

with $T=(\kappa t)^{1/2}$, $b_f=0.5kx_f$, and $Z_f=x_f/[2(\kappa t)^{1/2}]$. Note that for $k=0$, (16) becomes $V_f=x_f/2t$, in agreement with the $k=0$ result above. The denominator is composed of two positive definite terms [compare Eq. (12) with the second term] and greater than 1 unless $k=0$. It is therefore clear from (16) that the perturbation temperature field T_1 with nonzero k will have a smaller

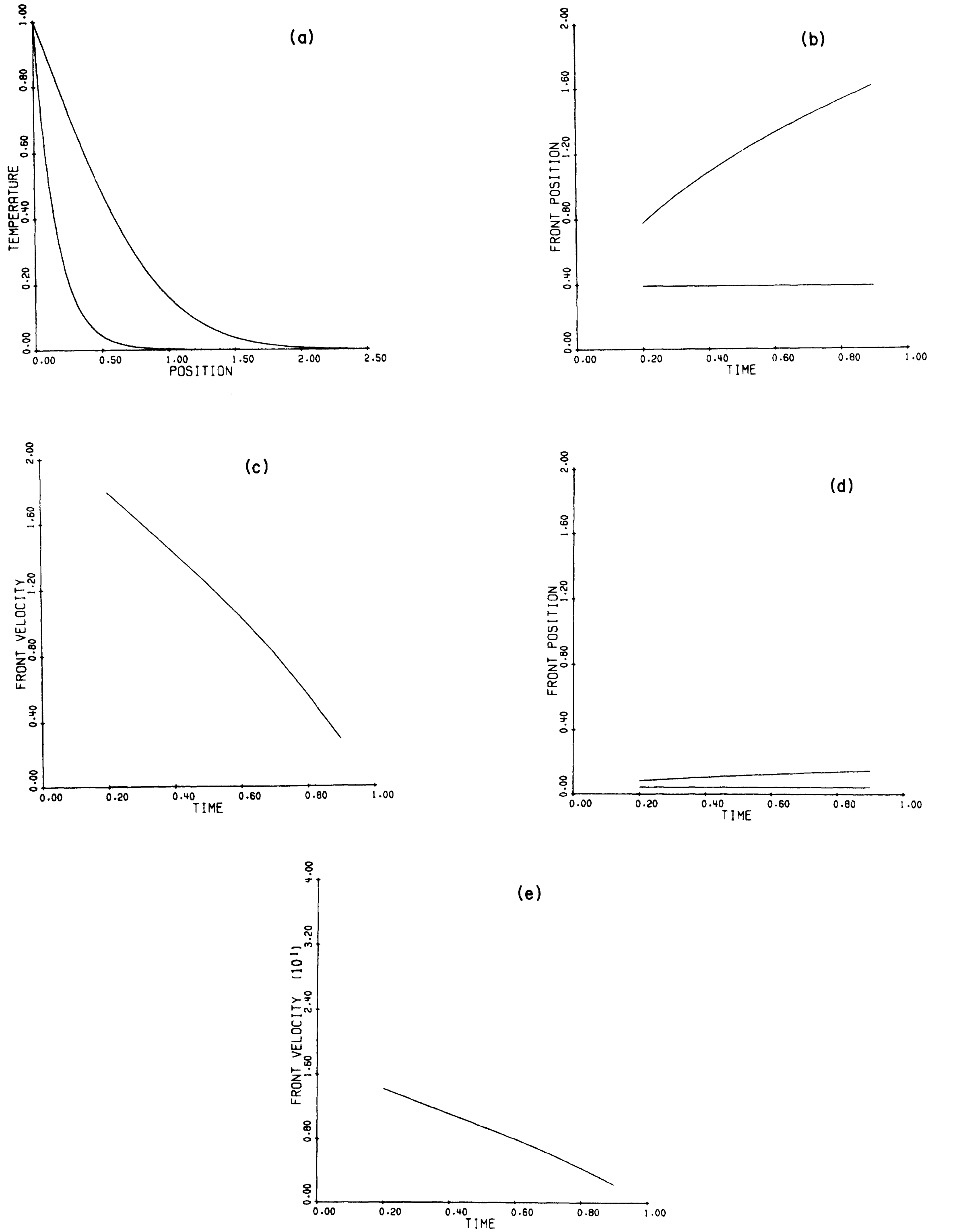

FIG. 1. (a) Temperature fields for the linear heat conduction case (constant κ), at time $t=0.6$. The upper curve is mean temperature T_0 and the lower curve perturbation temperature T_1 for $k=2\pi$. (b) Position of the $t=0.1$ isotherm for the same case, plotted as a function of time, upper curve for T_0 field and lower for T_1. (c) Velocity of the $T=0.1$ isotherm, for the same case. (d) Same as (b) but for $T=0.9$ isotherm. (e) Same as (c) but for $T=0.9$ isotherm.

velocity than the mean field T_0 with $k=0$. For large kx_f the ratio of velocities can clearly be large, so the mean field will leave the perturbation field far behind; thus at the ablation front, which will be x_f for the T_0 field, T_1 will be completely negligible. Figure 1 displays the T_0 and T_1 fields and the "front" positions and velocities, with T_f taken as either 0.1 or 0.9 of $T_i(x=0)=1$ (fixed-temperature boundary condition $i=0,1$), with T_1 having wave number $k=2\pi$ (the reason for this choice will be given in Sec. IV). For the linear case ($n=0$), the fields are uncoupled.

The concept of a front is less arbitrary for a nonlinear case (nonzero n), since the temperature field will then vanish sufficiently far into the material. The leading order approximation for T_0 in the fixed-temperature boundary condition case is of the same form for both Anderson and Lisak[21] and Petschek *et al.*,[25] $T_0=(1.-Z/Z_0)^{1/(1+n)}$, with the former giving $Z_0^2=2(n+2)/(n+1)^2$, the latter $Z_0^2=(n+2)(n+1.5)/(n+1)^2$. Note that for $n=2.5$, there is a 40% difference in Z_0 and hence front position depending upon which approximation is used. We may invert this to find $x_f=[\kappa(x=0)t]^{1/2}Z_0(1-T_f^{n+1})$, and, exactly as in the linear case, $V_f=x_f/2t$. Here κ, evaluated at $x=0$, is in our normalization 1 since $\kappa=T_0^n$ and the boundary condition is $T_0(x=0)=1$; we retain κ in the formula merely as a reminder of the dimensionality. As the laws for V_f are identical in linear and nonlinear cases, the cases should be qualitatively similar in behavior. Figure 2 illustrates this behavior. The amplitude of the perturbation field was taken as $T_1(x=0,t)=0.25$, with the mean field boundary condition $T_0(x=0,t)=1.0$, with $k=2\pi$ (fixed-temperature case). The T_0 and T_1 fields are coupled, and the relative amplitudes can matter, but the effect on T_0 of even such a large T_1 field is negligible. Figure 2 presents the data for this nonlinear case in a manner analagous to Fig. 1 for the linear conduction law case. T_1 has been scaled by dividing by $T_1(0)$ for ease of comparison of its spatial dependence with that of T_0. The remaining graphs are analagous plots for the solutions of (7) and (8), illustrating the effect of the nonlinear coupling terms. Except at early times, the behavior is qualitatively the same in all cases and clearly shows the perturbation being left behind by the mean thermal wave. Figure 3 presents the analagous results for the solutions of the simplified Eqs. (7) and (8). The T_0 field is essentially unchanged as compared to the T_0 field when calculated with the full equations; the T_1 field is more like that of the linear ($n=0$) case, and smaller near the front than the T_1 field as calculated with the full equations.

For the fixed flux boundary condition case, reductions in the ratio T_1/T_0 near the front are even greater. Such numbers will not be tabulated because they may be easily evaluated from formulas (11) and (12) for the linear case and (14) used for comparison in the nonlinear case.

IV. IMPLICATIONS FOR ABLATIVELY DRIVEN ICF TARGETS

How much does an ablation front smooth during the implosion of a typical ICF target shell, due to the mechanism considered in this paper? The most damaging wavelength is generally considered to be approximately equal to the shell thickness, since smaller wavelengths become nonlinear and grow much more slowly when their amplitude is of order of the wavelength, while much longer wavelengths grow more slowly. Hence, scaling all lengths by the shell thickness T, the wave number is $k=2\pi$. The distance traveled by the ablation front is a fraction f times the shell thickness, with f typically about 0.8. Using the approximate expression for x_f presented above, this distance is then $Z_0[\kappa(x=0)t]^{1/2}$, giving us a nondimensional time of $(f/Z_0)^2$.

Figure 3 shows typical temperature fields for a case with $k=2\pi$ and dimensionless time = 0.6. The boundary conditions $T_0(0)=1, T_1(0)=0.25$ were used, with $4T_1$ being plotted to simplify comparison of the behavior of the two fields as a function of x. Near the T_0 front, at the $T_0=0.5$ isotherm, the ratio $[T_1/T_1(x=0)]/[T_0/T_0(x=0)]$ is 0.1. Closer to the front isotherm temperature, the ratio is smaller. Hence we have an order of magnitude of smoothing. The smoothing is approximately the same when the analytic approximation to T_1 is used. The smoothing is similar but somewhat higher for fixed flux boundary conditions, the T_0 field tending to advance somewhat more rapidly in this case than in the fixed-temperature case [compare Eq. (14) vs Eq. (13)], while T_1 moves more slowly [compare Eq. (12) vs Eq. (10)].

Consequently, due to the diffusive nature of the front, there will be smoothing of perturbations by an order of magnitude during the course of an implosion. If the ablation front is unstable, the velocity at which the "bubble" of the Taylor instability penetrates the shell, relative to the mean ablation front, is of concern. This velocity will be approximately $v_p=v_i+v_1-v_0$, where v_i is the velocity due

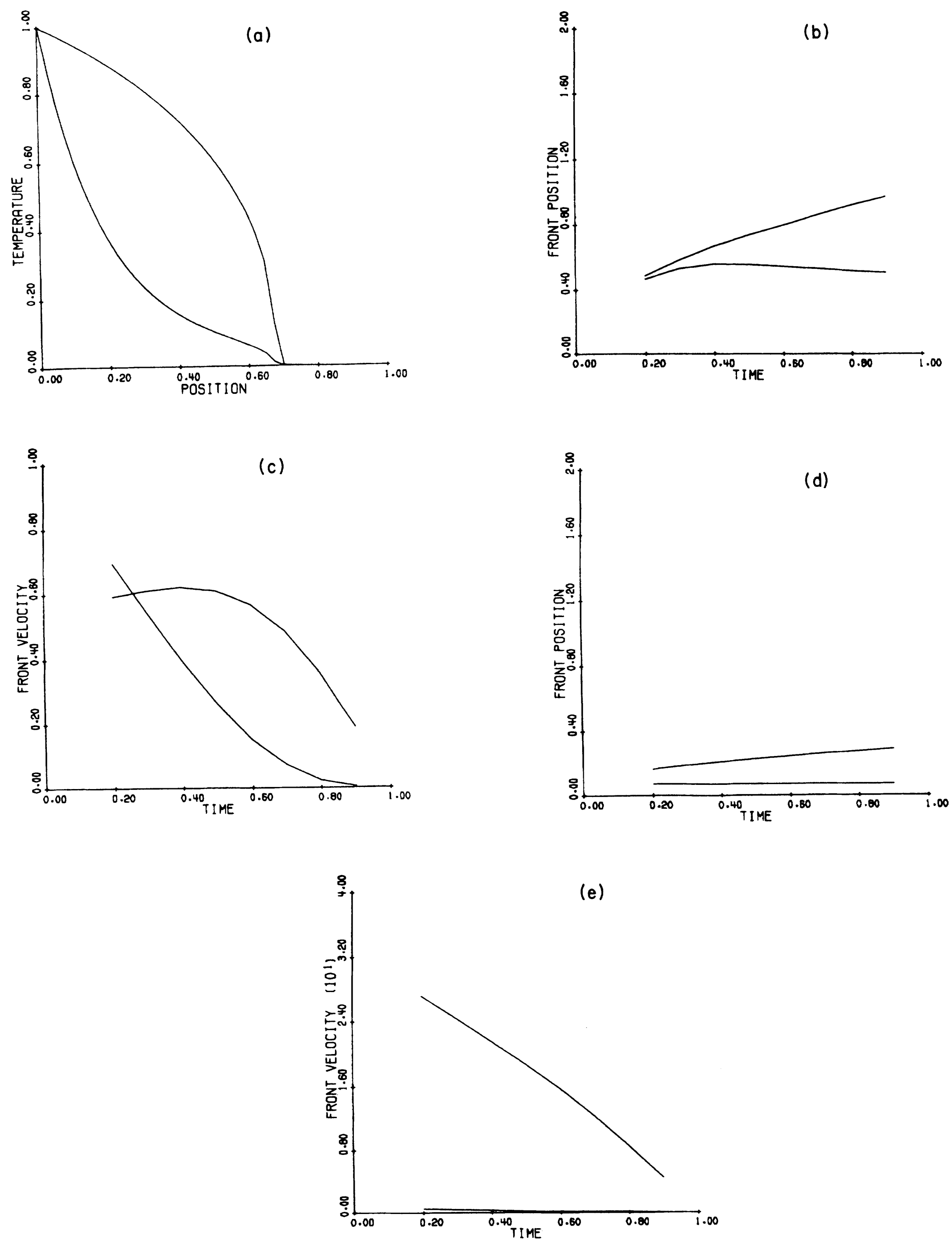

FIG. 2. (a) Temperature fields for the nonlinear conductivity case $\kappa = \Theta^n, n = 2.5$ (Spitzer plasma conductivity), the solution of Eqs. (5) and (6) with fixed-temperature boundary conditions $T_1(x=0)=0.25, T_0(x=0)=1$, wave number for T_1 mode 2π. The mean temperature field T_0 is the upper curve while the lower curve is the perturbation temperature T_1. (b) Position of the $0.1T(x=0)$ isotherm vs time for the solution of (a), upper curve for T_0 and lower for T_1. (c) Velocity of the $0.1T(x=0)$ isotherm vs time. The T_0 field has the large velocity for all but very early times. (d) As in (b) but for the $0.9T(x=0)$ isotherm. (e) As in (c) but for the $0.9T(x=0)$ isotherm.

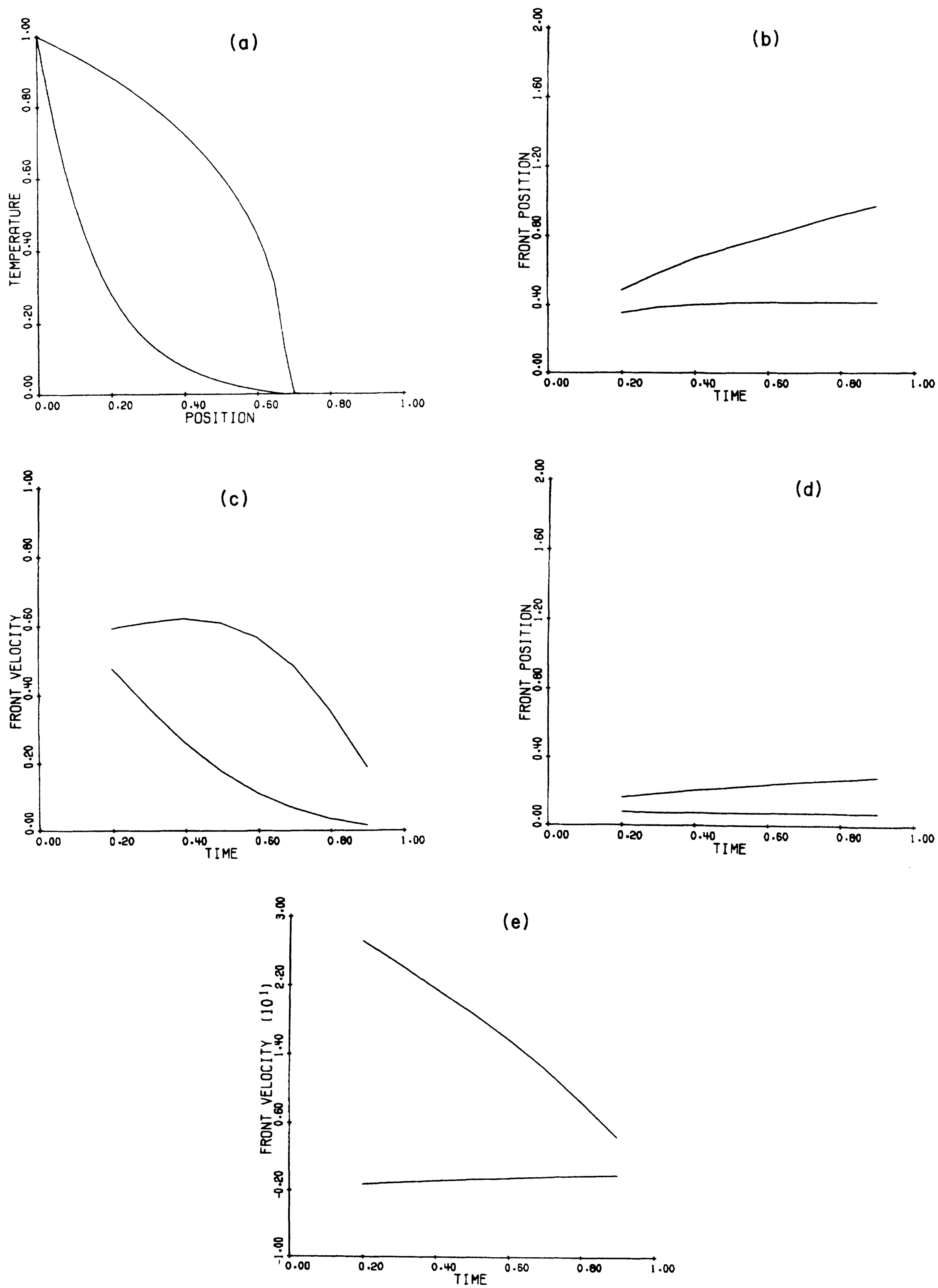

FIG. 3. As in Fig. 2, but the approximate Eqs. (7) and (8) were solved instead of the full Eqs. (5) and (6).

to the hydrodynamic instability, and v_1 and v_0 are the velocities of the respective temperature fields. Let us assume that the instability is still small so that the nonlinear interaction between the instability and the smoothing diffusion discussed here may be neglected and the two effects superimposed. Since v_i is much smaller than v_0, and v_i soon tends to approach a constant value (for classical Taylor instability), v_p can conceivably become zero or negative. This may have been seen in the numerical simulations of laser fronts (private communication, C. Verdon), although other effects such as the finite thickness of the shell may play a role in the calculation.

V. CONCLUSION

It has been shown that ablation fronts smooth themselves as they propagate through a medium. For cases of interest, this results in about an order-of-magnitude reduction in the perturbation, neglecting other effects (i.e., instabilities) that might be present. Consequently, simple theories that assume a quasistatic situation in the frame of the moving ablation front can overestimate perturbation growth by about an order of magntiude. Ablation fronts can therefore be significantly more stable than calculated on the basis of simple theories, even if those theories correctly account for other stabilizing effects.

It has also been shown that the rippled front sees a somewhat larger thermal conduction coefficient in the nonlinear case, resulting in slightly accelerated diffusion of the mean temperature isotherms. This effect is not large, however, and would only be significant, if at all, at very early times.

The physical mechanism responsible for the smoothing is the same as that for the so-called "fire-polishing"[9,26] stabilization of the ablation front, namely, the lateral diffusion of heat which produces increased ablation of any cold "spikes" of material at the ablation front. Whereas fire polishing acts to reduce a temperature fluctuation which is maximal at the front, and which is produced by hydrodynamic effects, the mechanism considered here reduces the "seed" perturbation which could give rise to such an instability, and consequently acts in addition to any smoothing of local front temperature fluctuations due to hydrodynamic instability. It would merely be a question of semantics as to whether the smoothing discussed in this paper should be called fire polishing, since this term has never been precisely defined. I prefer to reserve it for smoothing of perturbations which develop at the front (e.g., due to hydrodynamic motion as might be caused by instabilities); others might instead consider this paper the first quantitative, nonphenomenological treatment of the effect.

Two-dimensional simulations have not been attempted, nor have calculations with more than two modes, or extensive numerical work with flux boundary conditions; however, it is unlikely that these would give rise to any qualitative changes in the results.

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