Bifurcation gap in a hybrid optically bistable system

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The truncation of a period-doubling sequence in a hybrid optically bistable device with delayed feedback is observed as a function of the noise present in the system. Increasing the noise level decreases the number of period doublings observed.

Many dynamical systems have been studied that follow a period-doubling route to chaos.[1–10] Using a hybrid optical bistable device with delayed feedback, we investigate experimentally the effect of varying levels of intensity-dependent noise on the period-doubling sequence that lies on the lower branch of our device.[11] We find the period-doubling sequence is truncated, but in a way different from that predicted by the addition of intensity-independent Gaussian noise.[5,12]

Many nonlinear systems make a transition to chaos as some parameter, which we denote by μ, is increased monotonically. We denote a periodic output waveform by P_p where pt_R is the period of the waveform and t_R is some characteristic time of the system (in our case the delay time of the feedback). For period doubling, Feigenbaum has shown that the domain of μ in which any particular waveform is observed becomes geometrically smaller as the period increases. The period of the waveform thus becomes infinite at some finite value, $\mu=\mu_\infty$. If μ is increased beyond μ_∞, the waveforms become chaotic. These waveforms appear as superpositions of periodic waveforms with chaotic ones.[13] We denote these waveforms as N_p, where p indicates the period of the periodic part. Lorenz has shown that, as μ is increased, the period of the waveform decreases in steps of one-half.[13] He called this a "reverse bifurcation" sequence. The total bifurcation sequence in the case of period doubling is[14]

$$P_2 \to P_4 \to P_8 \to \cdots \to P_\infty \to N_\infty \to \cdots \to N_8 \to N_4 \to N_2 \to C \ ,$$

where $C \equiv N_1$ denotes a "fully developed" chaos with no periodic part.

By studying a forced anharmonic oscillator and Feigenbaum's quadratic map,[1] Crutchfield and Huberman predicted that the addition of Gaussian noise to a system would cause the number of period doublings to be finite.[5] They called this a "bifurcation gap" since, instead of period doubling *ad infinitum,* some waveform of finite period bifurcates to the chaotic waveform of the same periodicity, leaving a gap in the sequence of observed waveforms. In a previous experiment we observed such a gap after two period doublings.[6,7] Others have also observed a gap in the period-doubling sequence.[8,9]

In contrast, it was found in a hydrodynamic experiment that noise has no significant effect on the transition to chaos.[15] Moreover, noise is not the only potential cause of a bifurcation gap. Chow[16] has predicted that a gap should be observed in our system in the absence of noise, if the ratio of the response time τ to the delay time t_R is small enough. With the timescales of our present system we would not expect to see this type of truncation. We demonstrate that noise is the cause of the bifurcation gap by varying its amplitude. We observe that the location of the truncation in the bifurcation sequence moves monatonically toward fewer period doublings with increasing noise. If the truncation was deterministic, we would expect the bifurcation sequence to be independent of the noise level.

Figure 1 shows our experimental setup. A helium-neon laser beam passes through a Glan prism polarizer, then through a potassium dihydrogen phosphate

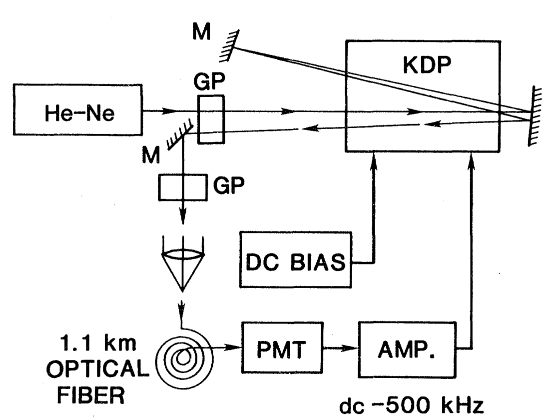

FIG. 1. Experimental layout: He-Ne laser; Glans prisms (GP); KDP, crystal; mirrors (M); photomultiplier (PMT).

(KDP) crystal four times before coming back through the polarizer. It then passes through another polarizer and is coupled into a 1.1-km optical fiber. The light emerging from the fiber is detected with a photomultiplier, whose output is amplified and impressed on the KDP crystal.

The major source of noise in the experiment is the shot noise from the photomultiplier. We increase the noise level in our system in the following manner: First we attenuate the light reaching the photomultiplier tube using the polarizer placed in front of the fiber; then we increase the voltage on the dynodes of the tube to return the signal to its previous level. Note that the spectral width of the noise at the photomultiplier tube is much larger than the bandwidth of the rest of the device. Other noise sources (the amplifiers, dark current, etc.) contribute about half of the noise signal at the lowest noise levels used in the experiment. We investigate noise levels from 0.3% to 10% rms (measured with the modulator at maximum transmission).

Our device is described by the equations[6,7]

$$T=\tfrac{1}{2}\{1-\xi\cos[X(t-t_R)+X_b]\}\ ,$$
$$\tau\dot{X}(t)+X(t)=2\pi\mu T+\eta(T)\ , \tag{1}$$

where $X=\pi V/V_h$, V is the voltage on the modulator, V_h is the half-wave voltage of the modulator, $X_b=\pi V_b/V_h$ is a variable bias, and $\eta(T)$ is the noise. The transmission function T is proportional to the transmitted intensity. The ability of the system to achieve extinction is measured by $\xi=0.96\pm0.01$, and μ is proportional to the product of the input laser intensity and the amplifier gain.[7] The fiber delay t_R was 6 μs. For all of the data presented, $X_b=-\pi$ and $\tau=1$ μs, but the results are qualitatively the same for $-\pi<X_b<-\pi/4$ and $0.25<\tau<1$ μs. Although for some values of X_b the upper branches of the device are accessible, we confine our observations to the lowest branch.

To determine the bifurcation points in the presence of noise the power spectra of the waveforms were observed in real time on a spectrum analyzer. The spectrum of the P_2 waveform is seen as a set of instrumentally narrow peaks at $f=k/2t_R$, $k=1,3,5,\ldots$, where f is the frequency. When the system bifurcates to P_4 the period-2 peaks remain and peaks at $f=1/4t_R$ and its harmonics appear. Likewise, peaks at $f=1/8t_R$ and its harmonics rise when the system bifurcates to P_8. It has been shown, both formally[17,18] and numerically[4,10] that, for $\mu>\mu_\infty$, chaos appears as a continuous spectral background. We measure the spectral power of the background to locate the bifurcation to chaos.

Our experimental spectra are summarized in Fig. 2 which is a plot of the power of the P_2, P_4, and P_8 frequency peaks and the chaotic background as a

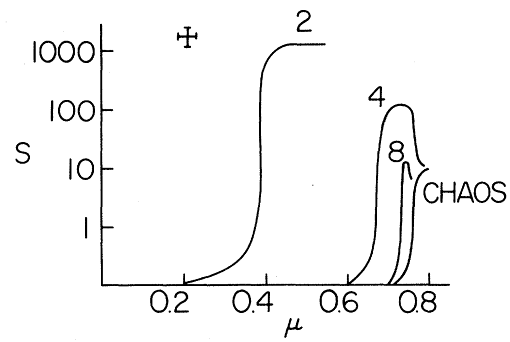

FIG. 2. The log of the power density (on an arbitrary scale of power/Hz) of the most powerful spectral components of $P_2(f=1/2t_R)$, $P_4(f=1/4t_R)$, $P_8(f=3/8t_R)$, and chaos $(f=15/32t_R)$ as a function of μ. The bifurcation to chaos is between period 8 and period-8 chaos. This case has a noise level of 0.3%. Due to the drift in the system (denoted by the horizontal error bar) and the rapid rise of the spectral peaks, no rigorous comparison of the shape of the different curves is significant. The height of the peaks was measured at increments of 0.01 of μ during the rise, and increments of 0.05 before and after it.

function of μ, at a noise level of 0.3%. The power of the largest peak in a waveform's spectrum was measured. Specifically, the peaks measured are $f=1/2t_R$ for P_2, $f=1/4t_R$ for P_4, and $f=3/8t_R$ for P_8. (This choice of the P_8 peak also eliminates potential problems with low-frequency filtering within the spectrum analyzer.) The rise of the chaotic background was measured at a frequency of $f=15/32t_R$. This choice of frequency avoids possible complications from any nascent P_{16} peak at $f=7/16t_R$. Note that the bifurcations within the periodic sequence are nonequilibrium second-order phase transitions[1] and the spectral components do not have a discontinuous jump. Critical slowing down produces the rounding off of the rise of the peak, and the rapid rise in the center locates the bifurcation point. Note also in Fig. 2 that the onset of chaos also behaves just like a nonequilibrium second-order phase transition, a result consistent with the work of Huberman and Zisook.[18]

In Fig. 2 the rise of the chaotic background occurs after the period 8 peak is fully developed. This locates the bifurcation to chaos between period 8 and period-8 chaos. Figure 2 describes a bifurcation structure $P_2\to P_4\to P_8\to N_8\to N_4\to N_2$.[14] (If the chaotic background had risen with the period-8 peak the bifurcation structure would have been $P_2\to P_4\to N_8\to N_4\to N_2$ as it is for 1% noise.)

The decay of the period-4 peak is also shown in Fig. 2. The peak decays rapidly when the power in the chaotic part of the waveform approximately equals the power in the spectral components that are associated with P_4. The decays of period-2 and period-8 peaks are analogous and, with the decay of the period-4 peak, they experimentally define the bifurcation structure of the reverse sequence.

In Fig. 3 the solid curves show the bifurcation points as a function of noise. The numbers indicate the waveform present in a domain. (For clarity, only the period of the waveform is shown on the graph. A starred number indicates a chaotic waveform.) Figure 3 shows that more noise is needed to eliminate a chaotic waveform than is needed to eliminate the corresponding periodic waveform. The bifurcation gap presented by Crutchfield and Huberman showed the elimination of a waveform and the corresponding chaotic waveform at the same noise level. One possible explanation for the elimination of the chaotic waveforms at different noise levels in the two systems is the differing characteristics of the noise. The noise in our system is intensity dependent, but Crutchfield and Huberman used intensity-independent Gaussian noise. Another possible explanation lies in the difference of the systems. Our device is a differential delay system while the systems that they studied involve mappings or nonlinear differential equations.

In conclusion, we have demonstrated that noise causes the termination of a period-doubling sequence in our device, although the way that it causes the termination is qualitatively different from that predicted using intensity-independent noise.

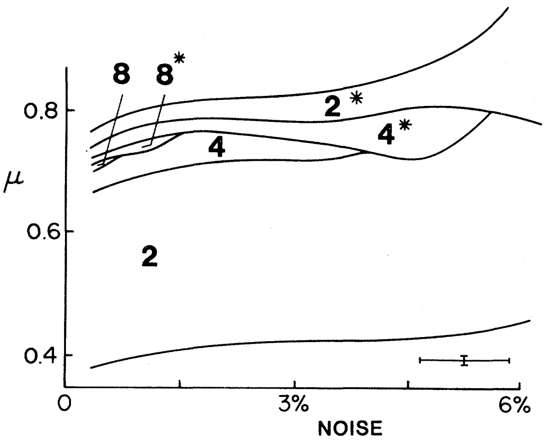

FIG. 3. A diagram of the domains of the waveforms as a function of noise. The lines are the bifurcation points between waveforms. The numbers denote the period of the waveform in units of t_R. The starred numbers denote chaotic waveforms. The noise was increased in increments of 0.3% between noise levels of 0.3% and 2%, and increments of 1% between noise levels of 2% and 6%.

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