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Coulomb scattering in the presence of a low-frequency laser field

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The cross section for scattering by a Coulomb potential, cut off at large distance, in the presence of a low-frequency laser field, is obtained as a power series in the laser frequency. The long-range nature of the potential introduces a change in the leading term (ω^0) as well as a new term proportional to $\omega \ln\omega$. It is also found that the Coulomb cutoff parameter can, under some circumstances, become an observable.

The original work by Kroll and Watson[1] and almost all subsequent papers[2] on scattering in the presence of a low-frequency laser field ignored the long-range effects of the Coulomb potential. They all arrive at a cross section which can be written

$$\frac{d\sigma_l(\vec{p}_f,\vec{p}_i)}{d\Omega}=J_l^2(\vec{\alpha}\cdot(\vec{p}_f-\vec{p}_i))\times\left.\frac{d\sigma(\vec{p}_f,\vec{p}_i)}{d\Omega}\right|_0+O(\omega)\ , \quad (1)$$

where the cross section on the left describes scattering with the transfer of l photons so that[3]

$$\frac{p_f^2}{2m}=\frac{p_i^2}{2m}-l\omega\ . \quad (2)$$

The cross section on the right side of (1) is that in the absence of the field. The field strength parameter $\vec{\alpha}=e\vec{E}/m\omega^2$ is held fixed as the limit $\omega\to 0$ is taken. A result, very similar to (1), applies even when the potential is replaced by an atom[4] and the result is only slightly changed[5] when we deal with an ionizing cross section. However, the long-range effects of the Coulomb potential have been ignored, and this is particularly important in ionizing collisions.

In this Communication we deal with only Coulomb *potential* scattering and take the point of view that the Coulomb potential is an idealization of a real potential which is cut off at some large radius R.[6] That is, we set $V(r)=-e^2/r$ for $r<R$ and $V(r)=0$ for $r>R$. At the end of the calculation we shall then let $R\to\infty$. More precisely, we shall find that the crucial way that R enters is as the product $(mn\omega R/p)$, where n is a (not large) integer and p is some characteristic momentum. For a CO_2 laser and an electron with a few eV of kinetic energy, a reasonable value of R makes this parameter large and so we shall set $R=\infty$ where possible. There are also situations in which $n=0$ so the $R\to\infty$ limit will give very different results for these terms.

Previous results for finite-range potential scattering in a low-frequency laser field may then be used. The T matrix for the transfer of l photons can be expanded in powers of the laser-projectile interaction in intermediate states which is also a power series in ω. The first three terms are[7]

$$T^{(0)}+T^{(1)}=\sum_{\lambda=-\infty}^{\infty}J_{l-\lambda}(\vec{\alpha}\cdot\vec{p}_f)J_\lambda(-\vec{\alpha}\cdot\vec{p}_i)(\vec{p}_f|T_R(\epsilon_{p_i}-\lambda\omega)|\vec{p}_i)\ , \quad (3)$$

$$T^{(2)}=\frac{\omega}{2}\sum_{\lambda=-\infty}^{\infty}\sum_{x=\pm 1}J_{l-\lambda+x}(\vec{\alpha}\cdot\vec{p}_f)J_\lambda(-\vec{\alpha}\cdot\vec{p}_i)\times(\vec{p}_f|V_RG_R^{(+)}[\epsilon_{p_f}-(l-\lambda+x)\omega]\vec{\alpha}\cdot\vec{p}G_R^{(+)}(\epsilon_{p_i}-\lambda\omega)V_R|\vec{p}_i)\ , \quad (4)$$

where $\epsilon_p = p^2/2m$ and the subscript R indicates that the quantity is evaluated for the cutoff potential. The $R \to \infty$ limit for the T matrix is tricky when the matrices are either on-shell or half-on-shell, but there is no difficulty for off-shell T matrices.[6,8] We therefore will not discuss the $l=0$ case here; that is, we exclude this one experimental situation in this paper.[8] We single out the $\lambda=0,l$ terms in (3) since these give half-on-shell T matrices. The remaining terms are off shell and the $R\to\infty$ limit may be taken immediately. After some algebra[9] this becomes

$$(\vec{p}_f|T_\infty(\epsilon_{p_i}-\lambda\omega)|\vec{p}_i) = -\frac{4\pi e^2}{\Delta p^2}\left(\frac{\pi\nu}{\sinh\pi\nu}\right)\xi(\lambda,l)|\lambda(l-\lambda)|^{-i\nu}\left(\frac{2\Delta p^2\epsilon}{m\omega^2}\right)^{i\nu}\left[1+i\left(\lambda-\frac{l}{2}\right)\frac{\omega\nu}{2\epsilon}\ln\frac{2\Delta p^2\epsilon}{m\omega^2}+O(\omega)\right] , \quad (5)$$

where $\Delta\vec{p}=\vec{p}_f-\vec{p}_i$ and all energies have been expanded about a reference energy, which is the average of the initial and final energies,

$$\epsilon = \tfrac{1}{2}(\epsilon_{p_i}+\epsilon_{p_f}) = p^2/2m \quad . \quad (6)$$

The Coulomb parameter is then taken to be $\nu = 1/pa_0$ and, for $l>0$,

$$\xi(\lambda,l) = \begin{cases} e^{-\pi\nu}, & \lambda<0 \\ 1, & 0<\lambda<l \\ e^{\pi\nu}, & \lambda>l \end{cases} , \quad (7)$$

and, for $l<0$,

$$\xi(\lambda,l) = \begin{cases} e^{\pi\nu}, & \lambda>0 \\ 1, & l<\lambda<0 \\ e^{-\pi\nu}, & \lambda<l \end{cases} . \quad (8)$$

The half-on-shell T matrices that result from the $\lambda=0,l$ terms on (3) have been obtained by Ford.[10] The $\lambda=0$ term is given by

$$(\vec{p}_f|T_R(\epsilon_{p_i})|\vec{p}_i) = -\frac{4\pi e^2}{\Delta p^2}\Gamma(1-i\nu)\exp\left(i\eta_l(R)-\frac{\pi\nu}{2}\operatorname{sgn}l\right)\left(1-\frac{il\omega}{4\epsilon}\eta_l(R)+O(\omega)\right) , \quad (9)$$

where

$$\eta_l(R) = \nu\ln\left(\frac{pR\,\Delta p^2}{m\omega|l|}\right) . \quad (10)$$

The $\lambda=l$ term may be obtained from this by using the symmetry[6]

$$(\vec{q}|T(E)|\vec{q}\,') = (\vec{q}\,'|T(E)|\vec{q}) . \quad (11)$$

It implies that we need only reverse the sign of l in (9) to obtain $(p_f|T_R(\epsilon_{p_f})|\vec{p}_i)$. We may assemble these results to obtain

$$\begin{aligned} T^{(0)}+T^{(1)} = -\frac{4\pi e^2}{\Delta p^2}\Bigg\{&\left(\frac{m\omega^2}{2\epsilon\Delta p^2}\right)^{-i\nu}\frac{\pi\nu}{\sinh\pi\nu}\left[K_1+\frac{i\omega\nu}{2\epsilon}\left(K_2-\frac{l}{2}K_1\right)\ln\left(\frac{2\Delta p^2\epsilon}{m\omega^2}\right)\right] \\ &+J_l(\vec{\alpha}\cdot\vec{p}_f)J_0(-\vec{\alpha}\cdot\vec{p}_i)\Gamma(1-i\nu)\exp\left(i\eta_l(R)-\frac{\pi\nu}{2}\operatorname{sgn}l\right)\left[1+\frac{i\omega l\nu}{4\epsilon}\ln\left(\frac{m|l|\omega}{pR\,\Delta p^2}\right)\right] \\ &+J_0(\vec{\alpha}\cdot\vec{p}_f)J_l(-\vec{\alpha}\cdot\vec{p}_i)\Gamma(1-i\nu)\exp\left(i\eta_l(R)+\frac{\pi\nu}{2}\operatorname{sgn}l\right)\left[1-\frac{i\omega l\nu}{4\epsilon}\ln\left(\frac{m|l|\omega}{pR\,\Delta p^2}\right)\right]+O(\omega)\Bigg\} , \end{aligned} \quad (12)$$

where

$$\begin{pmatrix}K_1\\K_2\end{pmatrix} = \sum_{\substack{\lambda=-\infty\\ \lambda\neq 0,l}}^{\infty} J_{l-\lambda}(\vec{\alpha}\cdot\vec{p}_f)J_\lambda(-\vec{\alpha}\cdot\vec{p}_i)|\lambda(l-\lambda)|^{-i\nu}\xi(\lambda,l)\begin{pmatrix}1\\ \lambda\end{pmatrix} . \quad (13)$$

The $T^{(2)}$ term (4) can be evaluated in the $R\to\infty$ limit. We show that it is finite and $O(\omega)$, and so can be neglected here. We can do this by allowing both Green's functions in (4) to be on shell. That is, we set $\omega=0$ in the arguments. Then we can use

$$(\vec{p}_f|V_R G_R^{(+)}(\epsilon_{p_f})=(\phi^{(-)}_{p_f,R}|-(\vec{p}_f| \ , \tag{14}$$

$$G_R^{(+)}(\epsilon_{p_i})V_R|\vec{p}_i)=|\phi^{(+)}_{p_i,R})-|\vec{p}_i) \ , \tag{15}$$

where $\phi^{(+)}_{p_i,R}$ and $\phi^{(-)}_{p_f,R}$ are the out and in scattering states, respectively. The plane-wave parts are subtracted off in (14) and (15) so all that remains are the scattered waves. Only the asymptotic forms of the scattered waves are needed to prove that the integral in (4) is finite. This is easily shown so that $T^{(2)}$ can be dropped. A similar argument applies for higher terms. Therefore (12) is the result for the T matrix for the transfer of l photons. Neglecting the overall phase factor it has a term of order ω^0 and a term of order $\omega\ln\omega$.[11] The ω^0 term does not, in general, reproduce the form of the Kroll-Watson result, Eq. (1). However, it can be shown that at high energy, $\nu\to 0$, and the Kroll-Watson form is recovered. In addition, there is a Coulomb distortion phase factor, $e^{i\eta_l(R)}$, in the last two terms of (12). The absolute square of $T^{(0)}+T^{(1)}$ is an observable and this phase factor will affect it.

In the spirit of Ford's[6] suggestion we have treated R as a physical parameter whose value is dictated by the experiment to be described. It could describe the box in which the scattering takes place. It could also arise from a model which attempts to describe the effect of neighboring atoms. In many cases it will not be known for each scattering event and then an ensemble average over R must be performed when a comparison with experiment is made. If the average over R is over a domain which satisfies

$$\frac{R_{\max}}{R_{\min}}>e^{2\pi/|\nu|} \ , \tag{16}$$

then the factor $e^{i\eta_l(R)}$ will average to zero and an effective incoherence will be introduced between the first and the last two terms of (12).

The result (12) and the cross section which comes from it depends upon R, which is the long-range cutoff distance of the Coulomb potential. The fact that it remains in a physical result is unusual but not unheard of since it would also appear in the total Coulomb cross section in the absence of the laser. The appearance of R also leads one to expect that the particular type of cutoff model might also affect the results. This leads us to the suggestion that experiments in this area would first have to specify the details of the cutoff Coulomb potential before other results could be extracted from the data. If this is correct, it would eliminate this field as a profitable experimental area of research. We plan to amplify the $l=0$ problem as well as the others in a future publication.

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[1]N. M. Kroll and K. M. Watson, Phys. Rev. A 8, 804 (1973).

[2]L. Rosenberg, Phys. Rev. A 20, 457 (1979), has considered Coulomb effects but obtains our Eq. (1) as the lowest-order result. The reason for the discrepancy might be the fact that we deal with a cutoff Coulomb potential and then let the cutoff radius become large, whereas Rosenberg deals with an unscreened Coulomb potential but includes wave-packet properties for the projectile.

[3]We use units of $\hbar=1$.

[4]M. H. Mittleman, Phys. Rev. A 21, 79 (1980).

[5]J. Banerji and M. H. Mittleman, J. Phys. B (London) 14, 3717 (1981).

[6]This is the point of view taken by W. F. Ford, J. Math. Phys. (N.Y.) 7, 626 (1966).

[7]See, for example, Marvin H. Mittleman, *Introduction to the Theory of Laser-Atom Interactions* (Plenum, New York, 1982), Chap. 6. The laser is treated as a classical single-mode homogeneous linearly polarized plane wave in the dipole approximation.

[8]A comprehensive review of the results of Ref. 6 and others is given by J. C. Y. Chen and A. C. Chen in *Advances in Atomic and Molecular Physics,* edited by D. R. Bates and I Esterman (Academic, New York, 1972), Vol. 8, p. 71. It is pointed out here and in Ref. 6 that the on-shell T matrix requires special consideration. We shall not deal with the $l=0$ case here for that reason. However, we emphasize that the various values of l are experimentally distinguishable. We have changed the normalization of the expressions given by a factor $(2\pi)^3$ and specialized to a proton target.

[9]See Eq. (135) of Ref. 7.

[10]See Ref. 6 or Eq. (215) of Ref. 8.

[11]The existence of this term was first suggested to us by Larry Spruch.