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Measurements of mode-splitting self-pulsing in a single-mode, Fabry-Perot laser

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Self-pulsing of a single-mode laser has been studied with the use of the inhomogeneously broadened 3.51-μm transition in xenon. Time-averaged intensity power spectra, average power output, and real-time observations of intensity fluctuations have been recorded for various detunings of the Fabry-Perot cavity. Two forms of self-pulsing have been observed, both of which can be explained in terms of the phase locking of several frequencies which simultaneously exist in the same spatial mode, made possible by the rapid variations of the index of refraction in the high-gain medium. One form of such mode splitting occurs in the wings of the atomic response due to the variations in the inhomogeneously broadened anomalous dispersion. The second form can occur anywhere in the tuning range of the laser as it is caused by distortions induced in the dispersion by the hole-burning effects of the oscillating mode. The pulsation frequency of this induced mode splitting varies with cavity tuning, shows a dip at the atomic line center, and may be absent for detuning of one or two homogeneous linewidths away from center. The instabilities have shown subharmonic bifurcations of the pulsation frequencies. Both high-Q pulsation frequencies and broad bands of frequencies in the power spectra have been observed for various operating conditions.

I. INTRODUCTION

Recent measurements[1] have found two regions in the frequency tuning of a high-gain, single-mode, helium-xenon laser for which the stable, cw output breaks up into periodic pulsations. Using the same laser and more sensitive diagnostic techniques, we have extended the range of such measurements to a predominantly inhomogeneously broadened medium.[2] For suitable excitation conditions we find that the two regions are no longer discrete and we observe an abrupt phase transition in the pulsing characteristics as the laser cavity is tuned. We have also observed the appearance of subharmonic bifurcations in which the pulsing frequency is joined by a weaker frequency component at exactly half of the dominant frequency. Studies of pulsing characteristics for different pressures and currents are reported.

II. BACKGROUND

Laser instabilities have been the subject of considerable experimental and theoretical interest.[1–16] Of particular concern has been the possibility that cw-excited, single-mode lasers might not remain stable as the population inversion is increased.[3,4,6,12–16] If one regards the onset of continuous-wave lasing action as a first-phase transition, then the question is raised as to the existence of other phases, their stability, and the thresholds for their appearance.[17,18] Because these phases involve the breakup of cw output, they are termed instabilities. As with many other nonlinear systems, one might well expect a sequence of these instabilities with each less homogeneous than the preceding one.[17–20] Such intermediate stages (involving spatial or temporal structures) have now commonly been observed in chemical reactions[21] and fluid dynamics[22] leading to a few universal descriptions of the intermediate transitions on the route from stability to turbulence or chaotic behavior as a suitable control parameter is adjusted.[23]

The term "self-pulsing" was first applied to such instabilities in lasers by Risken and Nummedal[3] whose studies of single-mode lasers found a threshold for periodic laser output above the threshold for the initial onset of laser action. In quite general terms, their pulsing solutions involved the determination that other optical frequencies satisfying the boundary conditions developed positive gain. These can be separated into two classes: multimode instabilities, wherein frequencies with different mode structure develop gain, leading to temporal or spatial pulsations, and mode-splitting instabilities, for which other frequencies having the same mode structure as the initial oscillating frequency find positive gain.

The former type of instabilities is very similar to the phenomenon of mode locking in multimode lasers.[5] Interaction between modes has been studied since the origins of laser theory[24] and this type of instability falls in the general class of passive mode locking (also called "self-induced mode locking").[5]

Our work concentrates on the less widely studied instabilities which can be attributed to mode splitting. Near threshold, this class of instabilities is caused by dispersive effects, as only a rapidly varying dispersion can cause more than one frequency to have the same wavelength. In 1970, Casperson and Yariv[25] predicted such mode splitting for suitably chosen, high-gain lasers operated near threshold. For inhomogeneously broadened media, frequencies nearly half of a full linewidth away from the resonance peak fall in a region of rapidly varying dispersion. Thus sufficiently tunable, high-gain lasers were predicted to show multifrequency, single-mode output in this part of their tuning range. A summary of the theory is captured by the solution of the phase-matching conditions represented by Eq. (1),

$$m\lambda/2=(L-l)+ln(\nu)\ , \tag{1}$$

where λ is the free-space wavelength, L is the length of the cavity, l is the length of the medium, m is the mode number, and $n(\nu)$ is the index of refraction at the laser oscillation frequency ν. For a Doppler-broadened medium, Eq. (1) may be written[25]

$$(x_m-x)=\beta F(x)\ , \tag{2}$$

where x is the laser oscillation frequency normalized as a detuning from the atomic resonance, x_m is

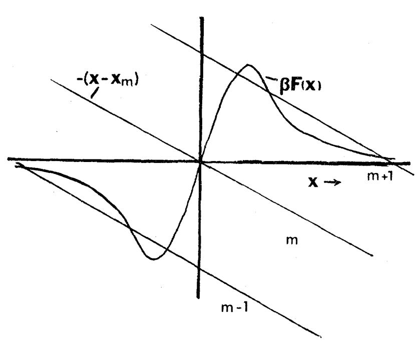

FIG. 1. Graphical solution of Eq. (2) as proposed in Ref. 25 for several mode numbers (m). Multiple intersections of a mode line with the dispersion curve indicate spontaneous mode splitting in which several frequencies having the same mode index are resonant for a particular cavity length.

the empty cavity resonant frequency for mode index m, and $F(x)$ is Dawson's integral. The dimensionless parameter β is given by

$$\beta=\frac{r\Delta\nu_c}{\Delta\nu_D}\frac{4(\ln 2)^{1/2}}{\pi^{1/2}}\ , \tag{3}$$

where r is the laser threshold parameter defined by Casperson,[14] $\Delta\nu_c$ is the cavity linewidth, and $\Delta\nu_D$ is the Doppler-broadened linewidth. For sufficiently large β (i.e., relatively low-Q cavities or lasers well above threshold) Eq. (2) has three solutions indicating that several frequencies simultaneously satisfy the boundary conditions for the same mode index m. This can be seen most easily in the graphical solutions[25] as shown in Fig. 1. The multiple solutions (when they occur) are all well away from the center of the atomic resonance and thus can only be observed when a laser is significantly detuned from the atomic line center.

We call this phenomenon spontaneous mode splitting. The major requirement for simultaneous oscillation of all of these frequencies is the existence of sufficient gain, which should be present in inhomogeneously broadened media. For extremely inhomogeneously broadened lasers there should be little interfrequency coupling, but any significant degree of homogeneous broadening may lead to suppression of the weaker frequencies due to hole burning (gain reduction). Coupling might also induce phase locking of the frequencies in the manner well understood for passive phase locking in multimode lasers, with periodic pulsations as a result. For the spontaneous mode-splitting case, the shared spatial mode structure should enhance the phase locking of the several frequencies, virtually guaranteeing pulsed behavior.

The only major effort to observe this predicted passive mode splitting was limited by extreme mode pulling, which prevented the tuning of a single mode sufficiently far from line center.[26] In addition, the use of a large helium pressure in the xenon-helium mixture led to a high degree of homogeneous broadening and caused a large hole to be burned in the gain profile by the dominant frequency. Thus the weaker frequencies expected from mode splitting were suppressed on two accounts. In our recently reported initial experiments,[1] we were able to observe the predicted mode splitting through careful selection of laser tuning range and limitations placed on the pressure broadening.

A second mode-splitting instability has been more recently explained[13–16] in terms of distortions in the dispersion caused by hole burning in the gain

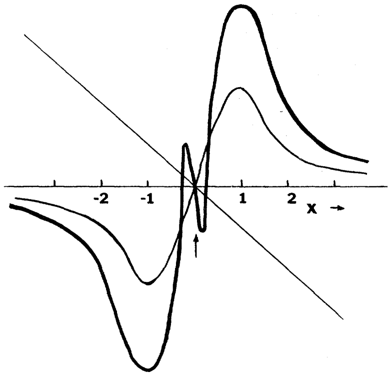

FIG. 2. Equivalent graphical solution for the mode splitting as proposed in Ref. 14 for the induced mode splitting at line center.

profile by a single mode operating above threshold. For a narrow homogeneous linewidth, the distortions occur only for frequencies near the laser oscillation frequency (and its mirror image about the center frequency, caused by Doppler shifts and the down-and-back nature of a Fabry-Perot cavity). An example of a graphical solution for this case is shown in Fig. 2. The dispersion may be so distorted that several new frequencies now satisfy the boundary conditions. The onset of these (probably phase-locked) sideband frequencies gives rise to pulsations in the intensity output of the laser. Because the dispersive effects are caused by the oscillating mode, we term this instability "induced mode splitting." This effect should be strongest at line center and is likely to exist over much of the laser tuning range.

Experimental observations of pulse trains attributed to this effect were first reported by Casperson in his initial paper analyzing this phenomenon.[13] He has since extended the theory in a careful analysis of Doppler-broadened and non-Doppler single-mode lasers in both ring cavity and Fabry-Perot configurations, but limited to cavities tuned to the atomic resonance. His work shows that the simplest hole-burning models predict instabilities for many partially homogeneously broadened lasers. When population pulsations are included, even homogeneously broadened lasers are found to be unstable above sufficiently high thresholds.[15,16] The first work on laser instabilities for cavities detuned from resonance has just been reported,[16(b)] but these techniques have not yet been applied to inhomogeneously broadened Fabry-Perot lasers.

Casperson's experimental results show variation of the pulsation frequency with gain and also give evidence of some alternation in the heights of successive pulses. Single pulses give evidence of coherent ringing or some form of relaxation oscillation as well. In our studies, both those recently reported[1] and the more extensive results presented here, the power spectra of the intensity pulsations have been carefully monitored so that the details of pulsing thresholds, pulsing frequencies, and thresholds for more complicated pulsations can be determined.

III. EXPERIMENT DESIGN

These studies were made using the high-gain 3.51-μm transition in xenon. Laser tubes were filled with single-isotope gas (90% enriched ^{136}Xe) and the discharge was dc excited with a series ballast resistor of 150 $k\Omega$ to prevent plasma oscillations. The single-isotope xenon medium operating under low pressure and discharge currents of 2–10 mA was inhomogeneously broadened by Doppler shifts to a full width at half maximum (FWHM) of approximately 120 MHz. The homogeneous linewidths of the medium are a combination of the natural linewidth (4.6 ± 0.2 MHz),[27,28] pressure broadening due to the xenon gas (10.9 ± 1.3 MHz/Torr,[28] and pressure broadening due to the helium buffer gas (18.6 ± 0.7 MHz/Torr.[28] As operating pressures ranged from 10–200 mTorr of xenon and 0–5 Torr of helium, the dominant variations in homogeneous linewidth were introduced by changes in the helium pressure.

The laser used for these studies, shown schematically in Fig. 3, was specially designed to achieve both the high gain and the wide tuning range needed to observe the spontaneous mode splitting. The 1-mm capillary for the active medium was selected to ensure high gain.[29] A large gas reservoir surrounding the aluminum cold cathode served to offset the gas cleanup that plagues work with low-pressure xenon discharges. Because of the relatively high current densities in the capillary tube, a nonexcited return path was provided between anode and

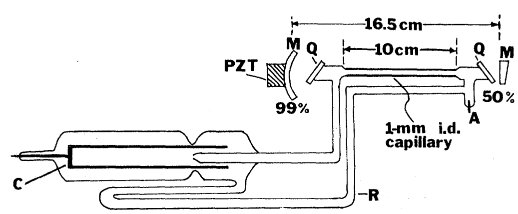

FIG. 3. Schematic drawing of the high-gain, dc-excited helium-xenon laser. PZT, piezoelectric crystal; M, mirrors; Q, quartz windows; R, nonexcited pressure equilization path; A, anode; C, cold cathode.

cathode to avoid developing pressure gradients.

The laser was designed with a short cavity (16.5 cm) resulting in a free-cavity mode spacing of 909 MHz. This was necessary to maintain single-mode operation over the full tuning range of the laser in the presence of a mode-pulling factor (β) in excess of 3.5 as required for the desired spontaneous mode splitting.[25]

The laser cavity length was fixed by 4-in.-diam cylindrical Pyrex spacer. The output coupling mir-

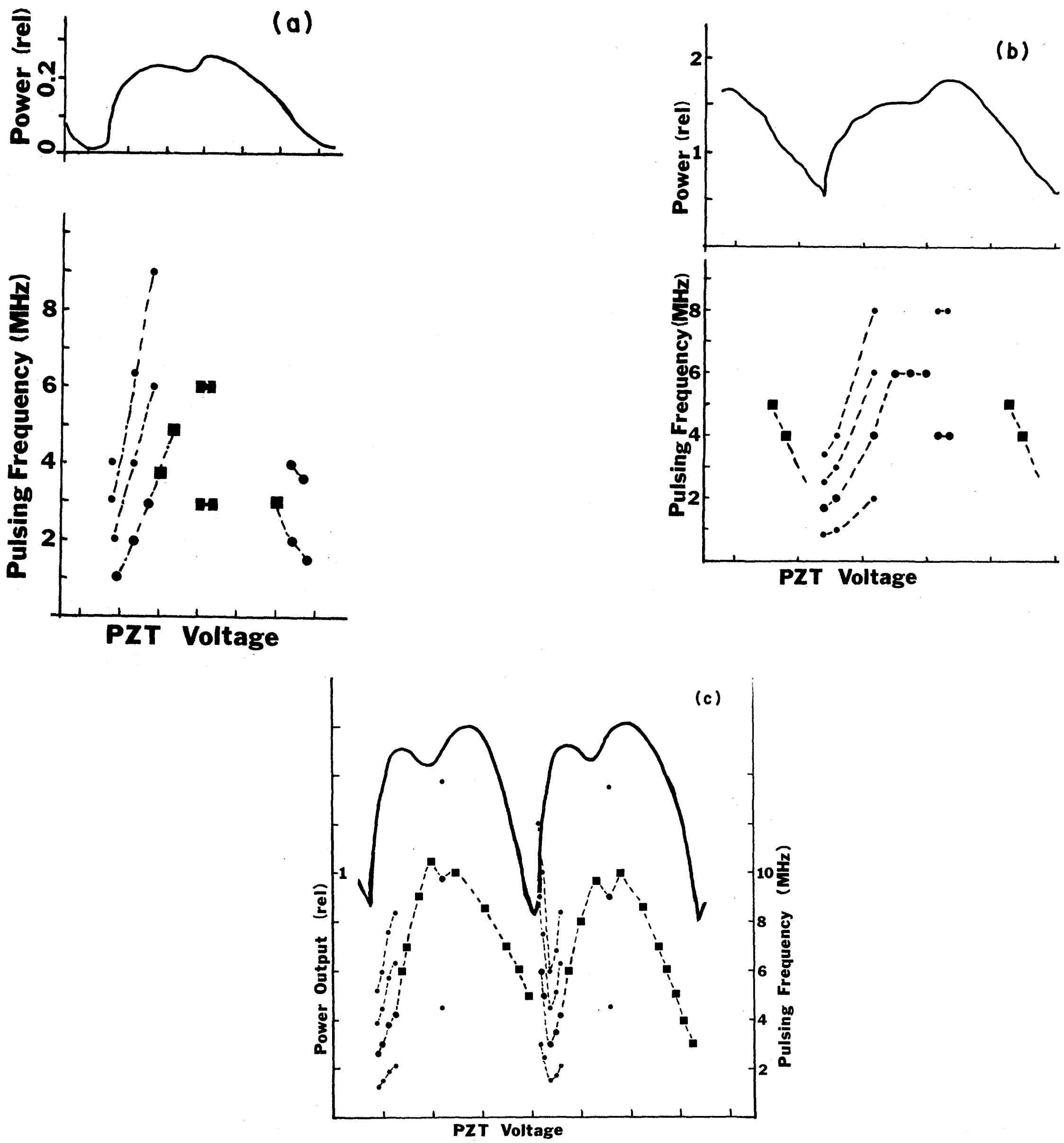

FIG. 4. Sample plots of laser power output (solid curves) and peaks in laser power spectra vs cavity tuning for 98-mTorr Xe and 0-Torr He at discharge currents of 4 mA (a), 6 mA (b), and 8 mA (c). Broad peaks are indicated by (■), indicating a 2—3 MHz width for the peak. Narrow peaks are indicated by large (●). Smaller dots indicate weaker frequencies in the power spectra. These include harmonic overtones as well as subharmonics and their overtones when they occur. (a) shows a narrow range of high-Q pulsations and subharmonics at line center. Note the reduction of the fundamental pulsing frequency in correlation with the reduced output intensity at the Lamb dip. Unusual features near mode switching may be a combination of spontaneous mode splitting and locking of two modes symmetrically placed about line center.

ror was a quartz wedge coated for 50% reflectivity with a broadband dielectric layer. The other mirror was a first-surface, aluminum-coated spherical mirror with a 30-cm radius of curvature giving overall a nearly confocal geometry. The spherical mirror was mounted on a piezoelectric transducer so that the cavity spacing could be varied over approximately three free spectral ranges.

Now that the induced mode splitting is reasonably well understood, it is clear that the design considerations for observing it are not nearly so stringent. In fact, observed output fluctuations for lasers with higher loss cavities and 4-mm-diam discharges which were previously attributed to mechanical or discharge instabilities can now, in retrospect, be properly identified as manifestations of the induced mode splitting. Details of these observations will be reported elsewhere.

Longitudinal mode operation was achieved by the insertion of apertures at each end of the laser cavity between the mirrors and the discharge tube. The windows on the discharge tube were mounted at Brewster's angle to minimize losses for vertically polarized light. These quartz windows appeared to suppress the other high-gain xenon laser line at 5.57 μm. The absence of other spectral content in the laser output indicated that other weak laser lines in the xenon spectrum were significantly below threshold.

By use of a beam splitter, the laser output was simultaneously monitored by two InAs photovoltaic detectors. One detector was reverse biased and integrally mounted on a preamplifier, resulting in a detector bandwidth in excess of 100 MHz. The output was fed to one or both of a fast oscilloscope and a scanning rf spectrum analyzer (Tektronix 1401A). The laser beam to the other detector was chopped and the signal was detected using a lock-in amplifier.

IV. LASER OUTPUT CHARACTERISTICS

A. Power output versus cavity length

Typical laser power output curves are shown as the continuous curves in Figs. 4(a)–4(c). The Lamb dip, which is pronounced near the peak of the output curves, washes out as the helium pressure is increased. The asymmetry of the Lamb dips may be explained in terms of a combination of frequency-dependent dispersion focusing[30] and a radial dependence of the excitation of the medium.[31] In addition, the apertures in the laser cavity introduce a slight frequency dependence for the cavity losses.[32] Explanation of the specific asymmetries would require detailed radial gain profile measurements for this discharge tube. Similar measurements and the determination of their consequences for laser power output are the subject of a separate study. It is sufficient to note here that the observed asymmetries are not inconsistent with these predictions.[33]

B. Dependence of laser instabilities on cavity tuning

Typical plots of frequency peaks in the laser output power spectra as a function of the cavity length are shown with the corresponding average laser power output curves in Figs. 4(a)–4(c). Figures 4(a) and 4(b) show pulsing exactly on line center and away from line center. The gaps in the curves are real and are evidence that no pulsing occurred in those regions near line center. Figure 4(c) shows pulsing for all frequencies and also shows a dip in the pulsing frequency at line center. These instabilities can be attributed to the induced mode splitting and the variation of pulsing frequency with cavity tuning is caused by the intensity-dependent power broadening of the holes in the gain profile (hence the dip in frequency at the Lamb dip in power). The regions of no pulsing occur because the partial overlapping of holes can prevent any sideband frequencies from becoming resonant.[34]

Some of the multiple peaks indicated on the power spectra are harmonics of the fundamental pulsing frequency. They arise from the nonlinearities in the field-atom interaction and the response to beat frequencies in the field. For most cases, the sidebands are nearly symmetrically located about the principal oscillating frequency. The fundamental pulsing frequency is probably the beat between the principal oscillating mode and the sidebands, while the difference between the sidebands provides a beat frequency that enhances the harmonic of that fundamental. Examples of typical power output pulsations are shown in real time in Fig. 5.

Both at line center and away from line center the instability showed the appearance of exact half frequencies (and their harmonics) in the power spectra. These peaks ranged in height up to 80% (in some cases) of the height of the peak for the main pulsation frequency in the power spectra. Although we have not systematically mapped the multiparameter space that governs the laser operation, our observations indicate that these subharmonics appear after the dominant pulsation frequency as the excitation conditions are increased. Real-time oscilloscope

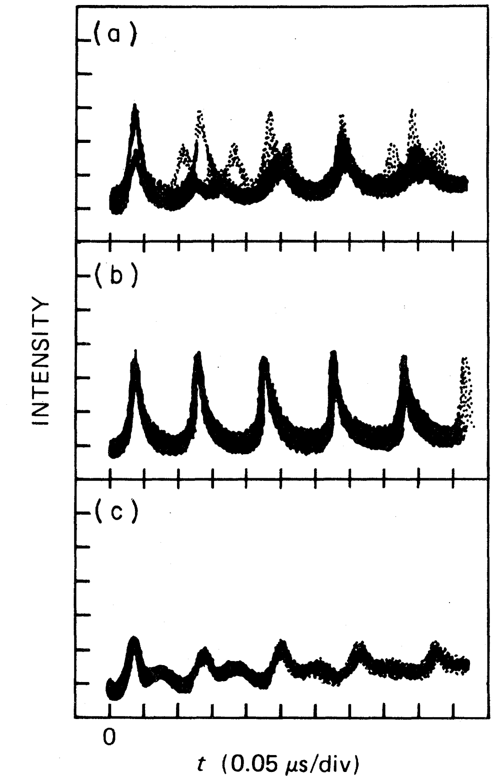

FIG. 5. Real-time traces of laser pulsing output for 70-mTorr Xe, 0.9-Torr He, and 4 mA. Shown at 50 ns/div are the coexistence of two frequencies at the transition from spontaneous to induced mode splitting (a), high-Q pulsing (b), and pulsing with subharmonic modulation and ringing (c). Note the higher pulsation frequencies ($\sim 10-20$ MHz) foir this case of $\Delta\nu_h = 22$ MHz.

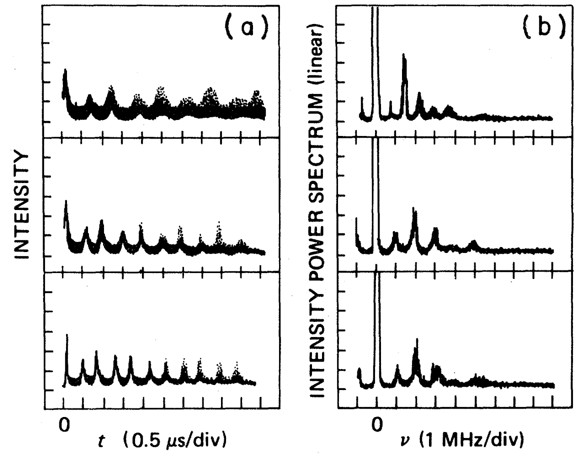

FIG. 6. Paired real-time displays (a) and power spectra (b) of detected laser power output for the period-doubled instability observed for changing cavity length with 70-mTorr Xe and 6 mA. Real-time traces are shown at 0.5 μsec/div and power spectra are shown at 1 MHz/div, with 10-kHz resolution.

traces are given in Fig. 6 showing pulse trains composed of pulses which alternate in height and the corresponding high-resolution power spectra. There is some evidence that pairs of alternating pulses are slightly bunched.

Figure 7 shows a series of drawings of the time-averaged power spectrum of the detected laser signal as the laser cavity length was decreased through one free spectral range. In the wings of the profile [7(a)—7(e)] a high-frequency, relatively "noisy" pulsation was observed. This may be an example of the spontaneous mode splitting expected in the wings of the tuning range. The noisiness can be attributed to the lack of definite phase locking of these frequencies. In 7(f) a relatively abrupt transition from the low-Q pulsations to higher-Q and lower frequency pulsations is observed. Two independent fundamental frequencies appear here as in the real-time photograph in Fig. 5(a). The transition is completed by 7(h) where the new fundamental frequency at 4.4 MHz shows both harmonics and a subharmonic.

The laser frequency evolves smoothly with detuning from 7(h) to 7(j), but another sharp transition occurs at 7(k). The subharmonics that appear in 7(1) grow to a peak amplitude in 7(m) and then vanish as the fundamental pulsing frequency also moves toward zero, reaching less than 1 MHz in 7(q) before disappearing. A broad, noisy pulsation, perhaps a return of the spontaneous mode splitting, appears at about 6 MHz in Figs. 7(n)—7(r).

C. Range of pulsation frequencies at line center

Theories of the laser pulsing instabilities at line center[13,14] predict that the pulsations can occur only when the parameter δ,

$$\delta = 2\pi(\Delta\nu_h)t_c \ , \tag{4}$$

is less than unity. Here $\Delta\nu_h$ is the homogeneous linewidth and t_c is the cavity lifetime.

Figure 8 shows a plot of pulsation frequencies observed at line center versus the homogeneous linewidth. The range of pulsation frequencies, indicated by vertical lines, was obtained by adjusting the gain parameter through changes in the discharge current.

Given the cavity losses and mirror reflectivities,

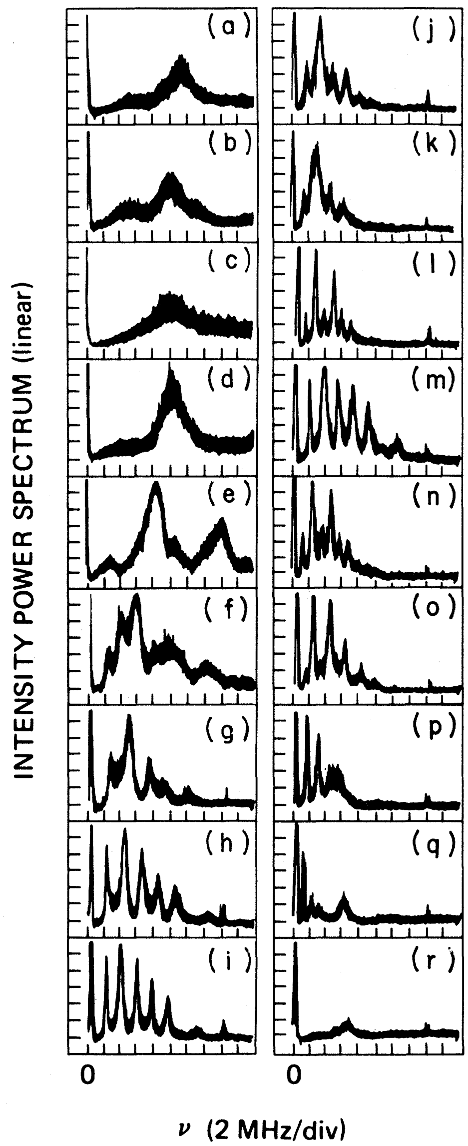

FIG. 7. Typical traces from rf spectrum analysis of detected laser power output for 70-mTorr Xe and 3 mA. Spectra are displayed linearly at 2 MHz/div with a resolution of 0.1 MHz. System resolution is indicated by the width of the sharp, zero-frequency peak (largest peak) and by the spike due to reflections at 8 MHz. The 18 frames from (a) to (r) follow a single mode across the gain profile as the PZT voltage is increased. The Lamb dip occurs at (k). Note in this case broad chaotic pulsing [(a)−(d)] with a subharmonic [(b)] and harmonics [(d)]. A transition in pulsing frequency and degree of chaos is evident in (f) and (g). Relatively narrow pulsing frequencies appear in (h) with a strong subharmonic. The subharmonic structure breaks up near the Lamb dip [(j) and (k)] then recovers in (m). Fundamental pulsing frequency proceeds monotonically to near zero in (m) to (q).

our cavity lifetime was determined to be 4.2×10^{-10} s. Thus Fig. 8 shows instabilities for $\delta=(2.7\times10^{-9})\Delta\nu_h$, which spans the range of $\delta=0.013$ to $\delta=0.12$.

Where pulsations are observed, pulsation frequencies at line center are predicted to range from zero to well in excess of $\Delta\nu_h$ depending on how far the laser is operated above threshold (cf. Fig. 1 in Ref. 14). For example, for $\delta=0.01$ the pulsations should begin almost at the lasting threshold ($r=1$), while for $\delta=0.1$ pulsations require $r=1.2$, and for $\delta=0.3$, pulsations require $r=1.8$. As the gain is raised above this value for the onset of pulsations, the increasing laser intensity burns a deeper and wider hole in the gain profile increasing the pulsation frequency.

Since the pulsing frequency ν_p is predicted to vary quite rapidly with changes in the gain for $\nu_p\le\frac{1}{2}\Delta\nu_h$, the absence of low pulsation frequencies in our data for line center may be attributed to our not making sufficiently small adjustments in the discharge current which controls the gain. Other data indicates a rather critical threshold by the absence of pulsing at line center in some cases. This may be an indication that the pulsing region at line center spans such a narrow range of frequencies that it cannot be observed with our laser which has short-term stability of approximately $\pm\frac{1}{4}$ MHz.

As we were unable to stabilize the laser frequency more precisely, measurements of pulsing frequency versus gain were not attempted directly. With improved frequency monitoring and detector sensitivity, such experiments should be possible and will be undertaken.

The threshold parameter for the single lasing mode could be smoothly varied in the scans of the

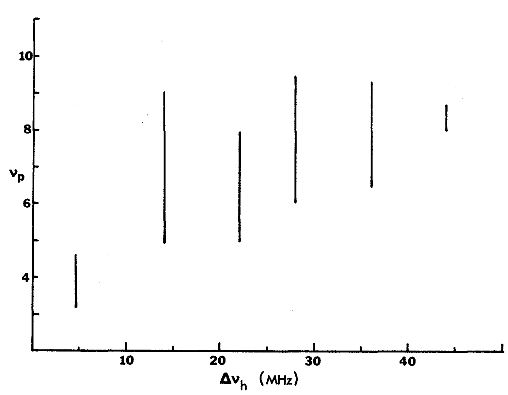

FIG. 8. Range of observed pulsation frequencies ν_p at line center vs the homogeneous linewidth. No pulsing was observed for $\Delta\nu_h\ge49$ MHz.

cavity length. The sequence of drawings in Fig. 7 shows the approach of the pulsing frequency to zero at some critical threshold value which was still above the lasing threshold.

The exact pulsation frequency is difficult to predict theoretically as present models cannot easily go beyond forecasting the frequency of the instability when it begins to break up the single-frequency state. Such predications involve a linear stability analysis of the single-frequency state. The steady-state pulsation frequency could only be determined by detailed numerical calculations not presently available.

V. SUMMARY

High-gain, inhomogeneously broadened lasers have been found to display a range of instabilities that include periodic pulsation and subharmonic bifurcation of those pulsation frequencies. Physical models that explain the pulsation frequencies in terms of mode splitting due to the anomalous dispersion are consistent with the experimental observations. The dependence of pulsation frequencies on the cavity tuning is in agreement with the expected effects of intensity broadening which include a reduced pulsation frequency at line center. Further measurements and analysis of variations in the pulsing frequency with laser parameters are in progress and will be reported elsewhere.

In these experiments we have observed cw operation, stable periodic output, and period-doubled behavior with a wide range of strengths for the subharmonic frequency. We have not seen transitions from period two to period four, or transitions from period two to chaotic behavior. The broad peaks in the power spectra may represent chaotic behavior, but we cannot assert that from our present work. Routes to chaos that differ from the Feigenbaum sequence of period doublings do exist and have been studied in lasers with impressed modulation of the field or the medium.[35,36] We are compelled in this work to admit that definite routes to chaos have not yet been established experimentally. This matter is, of course, a subject of intense, on-going investigations.

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