Coherent γ radiation production by interaction between a relativistic electron beam and two interfering laser fields

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A method is proposed for obtaining coherent γ radiation through the interaction between a relativistic electron beam and two interfering laser fields. The periodic structure associated with the interference fringes gives rise to coherent effects of the radiation emitted by stimulated inverse Compton scattering, if suitable conditions are fulfilled.

I. INTRODUCTION

The possibility of producing radiation in the visible or shorter-wavelength region from electron beams is well known, and is used, for example, in synchrotron radiation.[1] Recently, the interaction of an electron beam and a suitable periodic magnetic field has been used in so-called *free-electron lasers* to obtain coherent radiation in the near-infrared or visible regions.[2] Shorter wavelengths in the γ-ray region in units of (MeV) have been obtained through inverse Compton scattering of a visible laser radiation onto an electron beam.[3] We think that this kind of interaction must be explored as a possible mechanism in obtaining quasicoherent sources of γ radiation.[4] In the following sections we wish to discuss a particular interaction geometry which should allow an increase in the production cross section of γ rays and provide a source of quasicoherent radiation through a cooperative emission process.

The geometry which we intend to study is obtained by making an electron beam cross a region where two laser fields interfere, giving rise to a periodic structure of interference fringes. The general idea can be understood with reference to Fig. 1. Two laser beams cross each other and produce a fringe system in space in a region where an electron beam is traveling. An electron of the beam, crossing the region intersected by the two laser beams, takes part in scattering processes in which a photon from the laser beam 1 or 2 is scattered in some arbitrary direction.

With reference to Fig. 1 let us turn our attention to a particular one of these scattering processes; namely, that in which the scattered photon is traveling along beam 2 but in the opposite direction (backscattering). The scattered photon gains energy in the process from the relativistic electron beam and can find itself, in the laboratory frame, in the γ region (when the electron energy is sufficiently high). We also have another backscattering process in which a photon is scattered along beam 1. In this case, of course, the photon loses energy.

We will show in this paper that these scattering processes have the highest probability of occurring in the space region where maxima of the intensity fringes occur. When these backscatterings are considered, an interference between the scattered fields

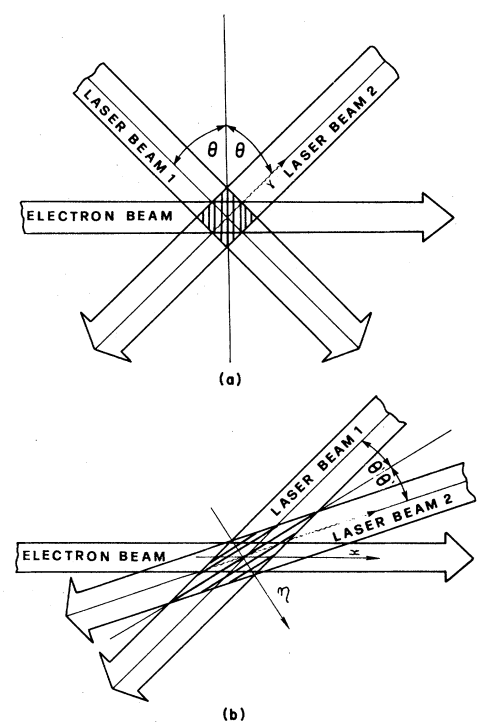

FIG. 1. Geometry for the production of γ rays: (a) in the laboratory frame and (b) in the electron rest frame.

occurs which increases the scattering cross section by a factor of 16 (if the two scattering beams have equal intensity), with respect to the case in which scattering from a single beam is considered (and a factor of 4 if the single beam has an intensity which is the sum of the intensity of the two laser beams). We will show that if particular angular conditions are fulfilled, the photons backscattered by each fringe may add in phase as in a Bragg scattering, so that the total scattered intensity is proportional to N^2, where N is the total number of crossed fringes, as it occurs in a coherent process.

Finally, we will show that the angular conditions which allow these circumstances to take place are not fulfilled in the case in which the two beams are at 180° with respect to each other as it occurs in the Weizsacher-Williams approximation of free-electron lasers, so that our geometry has distinct advantages over this conventional structure.

The interaction length for our process is rather short, being the region over which fringes are produced by laser beams of finite cross section, crossed by an electron beam of finite cross section so the total number of scattered photons cannot be very great; however, in our process a coherent effect is produced in the scattering from different fringes which is a way of producing a coherent or quasicoherent beam of γ rays.

We think that our discussion must be implemented by more refined calculations, taking into account the spread in energy of the electron beam, the linewidth of the interfering lasers, and so on, but we also think that the fundamental physics will not be changed by these more refined calculations.

To study our problem we use a quantum description in which an inverse Compton scattering occurs in the laboratory frame. In the electron rest frame, due to the very small energy exhange, the scattering can be considered quasielastic (we will call it Thomson scattering).

In Sec. II we treat this elastic scattering in the electron rest frame (ERF) by taking into account the particular fringe structure of the light field. A gain calculation is performed next, in Sec. III.

II. THOMSON SCATTERING OF ELECTRONS BY TWO INTERFERING LASER FIELDS

We have already shown[5] that the scattering of an electron in an interference region (a fringe) has a larger cross section than would be expected by assuming it as simply proportional to the square of the classical intensity of the field. It increases by a factor of 16.

This behavior is connected to a quantum description of the scattering process, which is of second order in the perturbation theory. It is connected to the uncertainty in the photon momentum in the fringes, and it is due to the coherence of the radiation fields. In the following we wish to study the effect in the case of Thomson scattering by N fringes for n_e electrons, showing the conditions under which the factor of 16 appears and the possibility of obtaining coherent scattering by the N fringes.

We turn our attention to the Thomson scattering (elastic scattering) because for relativistic electrons an elastic process occurs in the electron rest frame when the energy of the photons is much smaller than the rest energy of the electron. In the ERF the scattering is therefore a nonrelativistic one. The nonrelativistic interaction Hamiltonian for elastic scattering is given by

$$H_{\rm int}(\vec{r},t)=\frac{e^2}{2mc^2}\vec{A}(\vec{r},t)\vec{A}(\vec{r},t)\ , \tag{1}$$

where e and m are the charge and mass of the electron, respectively, c is the light velocity, and $\vec{A}$ is the usual vector potential of the electromagnetic field. The scattering probability, for n_e electrons, is given by

$$\begin{aligned}P_{\rm tot}&=\frac{n_e}{\hbar^2}\sum_f\left|\int_{t_0}^t dt'\int_V d\vec{r}\,'[\langle f\,|\,H_{\rm int}(\vec{r},t)\,|\,i\,\rangle]\right|^2\\&=\sum_f\frac{n_e}{\hbar^2}\left|\left[\frac{e^2}{2mc^2}\right]\int_{t_0}^t dt'\int_V d\vec{r}\,'[\langle f'\,|\,\vec{A}(\vec{r}\,',t')\vec{A}(\vec{r}\,',t')\,|\,i\,\rangle]\langle f''\,|\,i''\rangle\right|^2 .\end{aligned} \tag{2}$$

In our case the wave field is given by two interfering modes:

$$\begin{aligned}\vec{A}(\vec{r},t)&=\vec{A}_1(\vec{r},t)+\vec{A}_2(\vec{r}\,',t)\\&=[A_1\exp(-i\omega_1 t+i\vec{K}_1\cdot\vec{r})\hat{a}_1+A_1^*\exp(i\omega_1 t-i\vec{K}_1\cdot\vec{r})\hat{a}_1^\dagger]+[A_2\exp(-i\omega_2 t+i\vec{K}_2\cdot\vec{r})\hat{a}_2\\&\quad+A_2^*\exp(i\omega_2 t-i\vec{K}_2\cdot\vec{r})\hat{a}_2^\dagger]\ ,\end{aligned} \tag{3}$$

where

$$\vec{A}=c\left[\frac{2\pi\hbar}{\omega V}\right]^{1/2}\vec{\epsilon}\ ,$$

$$\langle f''\,|\,i''\rangle=\frac{\mu(\vec{r}_i)}{\sqrt{V}}e^{-i\vec{P}_0\cdot\vec{r}}\frac{u^*(\vec{r}_f)}{\sqrt{V}}e^{+i\vec{P}_f\cdot\vec{r}}$$

$$=|\,M_{fi}\,|\ .$$

V is the normalization volume given by the superposition volume of the two interfering waves.

The space-time integration is performed on a time smaller than the coherence time of the fields, and on a volume smaller than the coherence volume. We put as initial state of the field

$$|\,i'\rangle=|\,R_0\rangle=|\,R_{01}\rangle\,|\,R_{02}\rangle\ , \tag{4}$$

where $|\,\{R_0\}\,\rangle$ are coherent states, and as final state

$$|\,f'\rangle=|\,R_f\rangle=|\,R_{f1}\rangle\,|\,R_{f2}\rangle\ , \tag{5}$$

$|\,i''\rangle$ and $|\,f''\rangle$ are the initial and final electron states, given by

$$|\,i''\rangle=\frac{1}{\sqrt{V}}\mu(\vec{r}_i)\exp(i\vec{P}_0\cdot\vec{r})\ ,$$

$$|\,f''\rangle=\frac{1}{\sqrt{V}}\mu(\vec{r}_f)\exp(-i\vec{P}_f\cdot\vec{r})\ ,$$

$$\mu(\vec{r}_i)\mu(\vec{r}_f)=\delta_{if}\ .$$

From the momentum conservation we have

$$P_{\rm tot}=\sum_f\left[\frac{e^2}{mc^2}\right]^2\frac{n_e}{\hbar^2}\left[\left|\int_{t_0}^{t}dt'\int_V d\vec{r}'\,|\,M_{fi}\,|\,\langle R_f\,|\,\hat{A}_s^{(-)}(\vec{r}',t')\hat{A}_i^{(+)}(\vec{r}',t')\,|\,R_0\rangle\right|^2\right]\ , \tag{6}$$

where the subscripts i and s refer to the incident and scattered fields.

By squaring and summing on the final states of the radiation field, we have

$$P_{\rm tot}=\left[\frac{e^2}{mc^2}\right]^2\frac{n_e}{\hbar^2}\left[\int_{t_0}^{t}dt'\int_{t_0}^{t}dt''\int_V d\vec{r}'\int_V d\vec{r}''\left[\sum_f|\,M_{fi}\,|^2\right]\right.$$
$$\times\cos^2\alpha[\langle R_0\,|\,\hat{A}_i^{(-)}(\vec{r}'',t'')\hat{A}_s^{(+)}(\vec{r}'',t'')$$
$$\left.\times\hat{A}_s^{(-)}(\vec{r}',t')\hat{A}_i^{(+)}(\vec{r}',t')\,|\,R_0\rangle]\right]\ . \tag{7}$$

By using the commutation rules and from Eq. (3) we have

$$P_{\rm tot}=\left[\frac{e^2}{mc^2}\right]^2\frac{n_e}{\hbar^2}$$
$$\times\left[\int_{t_0}^{t}dt'\int_{t_0}^{t}dt''\int_V d\vec{r}'\int_V d\vec{r}''\left[\sum_f|\,M_{fi}\,|^2\right]\right.$$
$$\times\cos^2\alpha\{\,\langle R_0\,|\,[\hat{A}_1^{(+)}(\vec{r}'',t'')+\hat{A}_2^{(+)}(\vec{r}'',t'')]_j$$
$$\times[\hat{A}_1^{(-)}(\vec{r}',t')+\hat{A}_2^{(-)}(\vec{r}',t')]_s[\hat{A}_1^{(-)}(\vec{r}'',t'')+\hat{A}_2^{(-)}(\vec{r}'',t'')]_i$$
$$\left.\times[\hat{A}_1^{(+)}(\vec{r}',t')+\hat{A}_2^{(+)}(\vec{r}',t')]_i\,\,|\,R_0\rangle\}\right]\ , \tag{8}$$

where $\cos^2\alpha=|\,\vec{\epsilon}_s\cdot\vec{\epsilon}_i\,|^2$, i.e., α is the angle between the polarization vectors of scattered and incident field.

Performing the time integration over a long time (but shorter than the coherence time of the fields) and using the statistical independence between scattered and incident fields,

$$P_{\mathrm{tot}}=\frac{2\pi}{\hbar}n_e t\left[\frac{e^2}{mc^2}\right]^2\left[\int_V d\vec{r}'\int_V d\vec{r}''\left[\sum_f |M_{fi}|^2\right]\cos^2\alpha(A_s)^2(A_i)^2\right.$$
$$\times[\langle R_0|(\hat{a}_{1s}e^{i\vec{K}_{1s}\cdot\vec{r}''}\hat{a}_{1s}^{\dagger}e^{-i\vec{K}_{1s}\cdot\vec{r}'})+(\hat{a}_{2s}e^{i\vec{K}_{2s}\cdot\vec{r}''}\hat{a}_{2s}^{\dagger}e^{-i\vec{K}_{2s}\cdot\vec{r}'})$$
$$+(\hat{a}_{1s}e^{i\vec{K}_{1s}\cdot\vec{r}''}\hat{a}_{2s}^{\dagger}e^{-i\vec{K}_{2s}\cdot\vec{r}'})+(\hat{a}_{2s}e^{-i\vec{K}_{2s}\cdot\vec{r}''}\hat{a}_{1s}^{\dagger}e^{-i\vec{K}_{1s}\cdot\vec{r}'})|R_0\rangle]$$
$$\times[\langle R_0|(\hat{a}_{1i}^{\dagger}e^{-i\vec{K}_{1i}\cdot\vec{r}''}\hat{a}_{1i}e^{i\vec{K}_{1i}\cdot\vec{r}'})+(\hat{a}_{1i}^{\dagger}e^{-i\vec{K}_{1i}\cdot\vec{r}''}\hat{a}_{2i}e^{i\vec{K}_{2i}\cdot\vec{r}'})$$
$$\left.+(\hat{a}_{2i}^{\dagger}e^{-i\vec{K}_{2i}\cdot\vec{r}''}\hat{a}_{1i}e^{i\vec{K}_{1i}\cdot\vec{r}'})+(\hat{a}_{2i}^{\dagger}e^{-i\vec{K}_{2i}\cdot\vec{r}''}\hat{a}_{2i}e^{i\vec{K}_{2i}\cdot\vec{r}'})|R_0\rangle]\right]$$
$$\times\delta(E_f-E_i)\,, \tag{9}$$

where $A_s=A_i=(2\pi\hbar c^2/\omega V)^{1/2}$ for elastic scattering. If we study a backscattering process we have $\cos^2\alpha=1$ and therefore

$$P_{\mathrm{tot}}=\frac{2\pi}{\hbar}n_e t\left[\int_V d\vec{r}'\int_V d\vec{r}''\left[\sum_f |M_{fi}|^2\right](A_i)^2(A_s)^2(e^{i\vec{K}_{1s}\cdot(\vec{r}''-\vec{r}')}\langle\hat{a}_{1s}\hat{a}_{1s}^{\dagger}\rangle+e^{i\vec{K}_{2s}\cdot(\vec{r}''-\vec{r}')}\langle\hat{a}_{2s}\hat{a}_{2s}^{\dagger}\rangle\right.$$
$$+e^{i\vec{K}_{1s}\cdot\vec{r}''}e^{-i\vec{K}_{2s}\cdot\vec{r}''}\langle\hat{a}_{1s}\hat{a}_{2s}^{\dagger}\rangle$$
$$+e^{i\vec{K}_{2s}\cdot\vec{r}''}e^{-i\vec{K}_{1s}\cdot\vec{r}'}\langle a_{2s}\hat{a}_{1s}^{\dagger}\rangle)$$
$$\times(e^{-i\vec{K}_{1i}\cdot(\vec{r}''-\vec{r}')}\langle\hat{a}_{1i}^{\dagger}\hat{a}_{1i}\rangle+e^{-i\vec{K}_{2i}\cdot(\vec{r}''-\vec{r}')}\langle\hat{a}_{2i}^{\dagger}\hat{a}_{2i}\rangle+e^{-i\vec{K}_{2i}\cdot\vec{r}''}e^{i\vec{K}_{1i}\cdot\vec{r}'}\langle\hat{a}_{2i}^{\dagger}\hat{a}_{1i}\rangle$$
$$\left.+e^{-i\vec{K}_{1i}\cdot\vec{r}''}e^{i\vec{K}_{2i}\cdot\vec{r}'}\langle\hat{a}_{1i}^{\dagger}\hat{a}_{2i}\rangle)\delta(E_f-E_i)\right]. \tag{10}$$

The field-operator average is made on coherent states for a time smaller than the coherent time of the field: the interference terms of the incident field are not zero.[5,6] Moreover, by using the uncertainty in the modes we have

$$\langle\hat{a}_{1i}^{\dagger}\hat{a}_{1i}\rangle=\langle\hat{a}_{2i}^{\dagger}\hat{a}_{2i}\rangle=\langle\hat{a}_{2i}^{\dagger}\hat{a}_{1i}\rangle=\langle\hat{a}_{1i}^{\dagger}\hat{a}_{2i}\rangle=|\alpha_i|^2\,.$$

Generally, the interference terms of the scattered field are zero, but if the emitted field is a coherent field it is possible to have

$$\langle\hat{a}_{1s}\hat{a}_{2s}^{\dagger}\rangle=\langle\hat{a}_{2s}\hat{a}_{1s}^{\dagger}\rangle\neq 0\,,$$

with

$$\langle\hat{a}_{2s}\hat{a}_{1s}^{\dagger}\rangle=\langle\hat{a}_{1s}\rangle\langle\hat{a}_{2s}^{\dagger}\rangle=|\alpha_{1s}|\,|\alpha_{2s}|=|\alpha_s|^2$$

(see Refs. 5 and 6).

If backscattering is considered, we can assume that the backscattered fields along directions 1 and 2 also interfere. In this case both the incident and scattered photon momenta are undetermined, and the momentum conservation writes

$$\Delta\vec{K}_i+\frac{\vec{P}_0}{\hbar}=\Delta\vec{K}_s+\frac{\vec{P}_f}{\hbar}\,.$$

The scattering probability in modes $1s$ and $2s$ becomes

$$P_{\rm tot}=\frac{2\pi}{\hbar}tn_e\left[\frac{e^2}{mc^2}\right]^2(A_i)^2(A_s)^2|\alpha_i|^2(1+|\alpha_s|^2)$$
$$\times\left[\int_V d\vec{r}'\int_V d\vec{r}''\left[\sum_f|M_{fi}|^2\right]e^{i\vec{K}_{1s}\cdot\vec{r}''}e^{-i\vec{K}_{1i}\cdot\vec{r}''}(e^{-i\vec{K}_{1s}\cdot\vec{r}'}+e^{-i\vec{K}_{2s}\cdot\vec{r}'}e^{i(\vec{K}_{2s}-\vec{K}_{1s})\cdot\vec{r}''}+e^{-i\vec{K}_{2s}\cdot\vec{r}'}+e^{-i\vec{K}_{1s}\cdot\vec{r}'}e^{i(\vec{K}_{2s}-\vec{K}_{1s})\cdot\vec{r}''})\right.$$
$$\left.\times(e^{i\vec{K}_{1i}\cdot\vec{r}'}+e^{i\vec{K}_{2i}\cdot\vec{r}'}e^{-i(\vec{K}_{2i}-\vec{K}_{1i})\cdot\vec{r}''}+e^{i\vec{K}_{1i}\cdot\vec{r}'}e^{-i(\vec{K}_{2i}-\vec{K}_{1i})\cdot\vec{r}''}+e^{i\vec{K}_{2i}\cdot\vec{r}'})\delta(E_f-E_i)\right]. \quad (11)$$

From Eq. (11) we have also

$$P_{\rm tot}=\frac{2\pi}{\hbar}tn_e\left[\frac{e^2}{mc^2}\right]^2(A_i)^2(A_s)^2(|\alpha_i|^2)(1+|\alpha_s|^2)$$
$$\times\left[\int_V d\vec{r}'\int_V d\vec{r}''\left[\sum_f|M_{fi}|^2\right]e^{i\vec{K}_{1s}\cdot\vec{r}''}e^{-i\vec{K}_{1i}\cdot\vec{r}'}\right.$$
$$\times[(e^{-i\vec{K}_{2s}\cdot\vec{r}'}+e^{-i\vec{K}_{1s}\cdot\vec{r}'})(e^{i(\vec{K}_{2s}-\vec{K}_{1s})\cdot\vec{r}''}+1)]$$
$$\left.\times[(e^{i\vec{K}_{2i}\cdot\vec{r}'}+e^{i\vec{K}_{2i}\cdot\vec{r}'})(e^{-i(\vec{K}_{2i}-\vec{K}_{1i})\cdot\vec{r}''}+1)]\delta(E_f-E_i)\right]. \quad (12)$$

By making M_{fi} explicit, Eq. (12) becomes

$$P_{\rm tot}=\frac{2\pi}{\hbar}tn_e\left[\frac{e^2}{mc^2}\right]^2(A_i)^2(A_s)^2|\alpha_i|^2(1+|\alpha_s|^2)\frac{1}{V^2}$$
$$\times\sum_{P_f}\left[\int_V d\vec{r}'e^{i[(\vec{P}_0-\vec{P}_f)\cdot\vec{r}'/\hbar]}[(e^{-i\vec{K}_{2s}\cdot\vec{r}'}+e^{-i\vec{K}_{1s}\cdot\vec{r}'})(e^{i\vec{K}_{2i}\cdot\vec{r}'}+e^{i\vec{K}_{1i}\cdot\vec{r}'})]\right.$$
$$\times\int_V d\vec{r}''e^{-i[(\vec{P}_0-\vec{P}_f)/\hbar]\cdot\vec{r}''}e^{i\vec{K}_{1s}\cdot\vec{r}''}e^{-i\vec{K}_{1i}\cdot\vec{r}''}$$
$$\left.\times[(e^{i(\vec{K}_{2s}-\vec{K}_{1s})\cdot\vec{r}''}+1)(e^{-i(\vec{K}_{2i}-\vec{K}_{1i})\cdot\vec{r}''}+1)]\delta(E_f-E_i)\right]. \quad (13)$$

If we set $\vec{\Delta}=\vec{P}_0/\hbar+\vec{K}_{1i}-\vec{P}_f/\hbar-\vec{K}_{1s}$, $\vec{\Delta}_i=\vec{K}_{2i}-\vec{K}_{1i}$, which is the spatial periodicity of the fringes in the incident field, and $\vec{\Delta}_s=\vec{K}_{2s}-\vec{K}_{1s}$, which is the spatial periodicity of the fringes in the scattered field, we have

$$P_{\rm tot}=\frac{2\pi}{\hbar}n_e t\left[\frac{e^2}{mc^2}\right]^2(A_i)^2(A_s)^2|\alpha_i|^2(1+|\alpha_s|^2)\frac{1}{V^2}$$
$$\times\left[\sum_{P_f}\int_V d\vec{r}'e^{i\vec{\Delta}\cdot\vec{r}'}(1+e^{-i(\vec{K}_{2s}-\vec{K}_{1s})\cdot\vec{r}'})(1+e^{i(\vec{K}_{2i}-\vec{K}_{1i})\cdot\vec{r}'})\right.$$
$$\left.\times\int_V d\vec{r}''e^{-i\vec{\Delta}\cdot\vec{r}''}(1+e^{i(\vec{K}_{2s}-\vec{K}_{1s})\cdot\vec{r}''})(1+e^{-i(\vec{K}_{2i}-\vec{K}_{1i})\cdot\vec{r}''})\right]\delta(E_f-E_i)\,. \quad (14)$$

If we take into account the fact that in the case of backscattering $\vec{K}_{2s}=-\vec{K}_{2i}$, $\vec{K}_{1s}=-\vec{K}_{1i}$, or $\vec{\Delta}_i=\vec{\Delta}_s$, Eq. (14) becomes

$$P_{\rm tot}=\frac{2\pi}{\hbar}n_e t\left[\frac{e^2}{mc^2}\right]^2(A_i)^2(A_s)^2\,|\,\alpha_i\,|^2(1+|\,\alpha_s\,|^2)\frac{1}{V^2}\,.$$
$$\times\left[\sum_{P_f}\int_V d\vec{r}\,'e^{i\vec{\Delta}\cdot\vec{r}\,'}(1+e^{i\vec{\Delta}_i\cdot\vec{r}\,'})(1+e^{i\vec{\Delta}_i\cdot\vec{r}\,'})\int_V d\vec{r}\,''e^{-i\vec{\Delta}\cdot\vec{r}\,''}(1+e^{-i\vec{\Delta}_i\cdot\vec{r}\,''})(1+e^{-i\vec{\Delta}_i\cdot\vec{r}\,''})\right]\delta(E_f-E_i)\,. \tag{15}$$

Equation (15) can be written also as

$$P_{\rm tot}=\frac{2\pi}{\hbar}n_e t\left[\frac{e^2}{mc^2}\right]^2(A_i)^2(A_s)^2\,|\,\alpha_i\,|^2(1+|\,\alpha_s\,|^2)\frac{1}{V^2}$$
$$\times\sum_{P_f}\left|\int_V d\vec{r}\,''(2+2e^{-i\vec{\Delta}_i\cdot\vec{r}\,''})e^{-i\vec{\Delta}\cdot\vec{r}\,''}\right|^2\delta(E_f-E_i)\,, \tag{16}$$

i.e.,

$$P_{\rm tot}=\frac{2\pi}{\hbar}n_e t\left[\frac{e^2}{mc^2}\right]^2(A_i)^2(A_s)^2\,|\,\alpha_i\,|^2(1+|\,\alpha_s\,|^2)\frac{1}{V^2}$$
$$\times\sum_{P_f}\left|2\int_V d\vec{r}\,''e^{-i\vec{\Delta}\cdot\vec{r}\,''}+2\int_V d\vec{r}\,''e^{-i(\vec{\Delta}+\vec{\Delta}_i)\cdot\vec{r}\,''}\right|^2\delta(E_f-E_i)\,. \tag{17}$$

Equation (17) contains a first term, which represents the scattering by the whole interaction region, and a second term which represents the scattering contribution due to the periodic distribution of the fringes. In the second term the momentum conservation requires that both interfering beams contribute to the scattering in a fixed photon and electron final state, but it needs a "periodic vector" ($\vec{\Delta}_i$) to assure it. Coherent effects can be shown to exist if some periodicity condition is fulfilled, or if

$$e^{-i\vec{\Delta}\cdot\vec{r}}=f(\vec{r}) \tag{18}$$

is a periodic function.

We consider what happens along the plane in which fringes are produced, η being the coordinate along the fringes. Through the use of a Fourier expansion, we can write

$$[f(\vec{r})]_\eta=f(\eta)=f\left[\eta+\frac{1}{\Delta_i}\right]$$
$$=\sum_n A_n\exp(2\pi i n\eta/P)$$
$$=\sum_{\Delta_i}A_{\Delta_i}e^{i\Delta_i\eta}$$

with $\Delta_i=2\pi/P$; P being the fringe periodicity, and with

$$A_{\Delta_i}=\frac{1}{P}\int_P f(\eta)e^{-i\Delta_i\eta}d\eta$$
$$=\frac{1}{P}\int_P e^{-i\Delta\eta}e^{-i\Delta_i\eta}d\eta$$
$$=\frac{1}{P}\left[\frac{e^{-i(\Delta+\Delta_i)P}-1}{-i(\Delta+\Delta_i)}\right]. \tag{19}$$

From Eq. (19) we obtain

$$f(\eta)=e^{-i\Delta\eta}=\sum_{\Delta_i}A_{\Delta_i}e^{i\Delta_i\eta}$$
$$=\sum_{\Delta_i}\frac{1}{P}\left[\frac{e^{-i(\Delta+\Delta_i)P}-1}{-i(\Delta+\Delta_i)}\right]e^{i\Delta_i\eta}\,. \tag{20}$$

Now, we introduce Eq. (20) into Eq. (17), which becomes

$$P_{\rm tot}=\frac{2\pi}{\hbar}n_e t\left[\frac{e^2}{mc^2}\right]^2(A_i)^2(A_s)^2|\alpha_i|^2(1+|\alpha_s|^2)\frac{1}{L^2}$$
$$\times\sum_{P_f}\left|\left\{2\int_L d\eta''\left[\sum_{\Delta_i}\frac{1}{P}\left[\frac{e^{-i(\Delta+\Delta_i)P}-1}{-i(\Delta+\Delta_i)}\right]\right]e^{i\Delta_i\eta''}\right.\right.$$
$$\left.\left.+2\sum_{\Delta_i}\frac{1}{P}\left[\frac{e^{-i(\Delta+\Delta_i)P}-1}{-i(\Delta+\Delta_i)}\right]\int_L e^{i\Delta_i\eta''}e^{-i\Delta_i\eta''}d\eta''\right\}\delta(\Delta\zeta)\delta(\Delta\xi)\right|^2\delta(E_f-E_i)\ , \tag{21}$$

ζ and ξ being the other two Cartesian coordinates of the photon beams. Setting $(e^2/mc^2)=r_0^2$,

$$P_{\rm tot}=\frac{2\pi}{\hbar}n_e t r_0^2(A_i)^2(A_s)^2|\alpha_i|^2(1+|\alpha_s|^2)\frac{1}{L^2}$$
$$\times\sum_{P_f}\left|\left[2f(P)\frac{e^{i\Delta_i L}-1}{i\Delta_i}+2f(P)L\right]\delta(\Delta\xi)\delta(\Delta\zeta)\right|^2\delta(E_f-E_i)\ , \tag{22}$$

where

$$f(P)=\sum_{\Delta_i}\frac{1}{P}\left[\frac{e^{-i(\Delta+\Delta_i)P}-1}{-i(\Delta+\Delta_i)}\right]\ .$$

Now,

$$\Delta_i L=\Delta_i PN=\frac{2\pi}{P}PN=2\pi N\ . \tag{23}$$

If N is an integer number, then

$$e^{i\Delta_i L}-1=0\ ; \tag{24}$$

therefore, by observing that $\sum_{\Delta_i}=\sum_N=N$, Eq. (22) becomes

$$P_{\rm tot}=16n_e\frac{2\pi}{\hbar}tr_0^2(A_i)^2(A_s)^2(1+|\alpha_s|^2)|\alpha_i|^2\sum_{P_f}\frac{N^2}{P^2}\frac{\sin^2[(\Delta+\Delta_i)P/2]}{(\Delta+\Delta_i)^2}\delta(\Delta\zeta)\delta(\Delta\xi)\delta(E_f-E_i)\ . \tag{25}$$

Equation (25) represents the contribution to the scattering by each fringe, with a cooperative effect shown by the term N^2.

In the limit in which $P\to\infty$, the function

$$\frac{\sin^2[(\Delta+\Delta_i)P/2]}{P^2(\Delta+\Delta_i)^2}\to\delta((\Delta+\Delta_i)\eta)\ .$$

Since $P=\lambda'/2\sin\theta'$ the condition $P\to\infty$ becomes an "angle" condition, i.e., $\sin\theta'\to O(\lambda'=\lambda/\gamma)$. Now we observe that, by using the relation $\Delta_i=2\pi/P$, we can write Eq. (25) as

$$W_{\rm tot}=\frac{P_{\rm tot}}{t}=n_e 16\frac{2\pi}{\hbar}r_0^2(A_i)^2(A_s)^2(1+|\alpha_s|^2)|\alpha_i|^2\sum_{P_f}N^2\frac{\sin^2\left[\frac{\Delta P}{2}\right]}{(\Delta P+2\pi)^2}\delta(\Delta\zeta)\delta(\Delta\xi)\delta(E_f-E_i)\ , \tag{26}$$

which has a maximum for

$$\frac{\Delta P}{2}=\frac{\pi}{2}\ , \tag{26'}$$

or

$$\Delta=\frac{\Delta_i}{2}\ . \tag{27}$$

It can easily be seen that

$$\Delta=\Delta\omega_i'/c\ ,$$

where $\Delta\omega_i'$ is the linewidth of the incident radiation from (26′) and (27) we derive a condition for the angle θ' (in ERF)

$$\sin\theta' \simeq \frac{\Delta\omega_i'}{\omega_i'} . \tag{28}$$

III. GAIN OF THE SYSTEM

Now we study the gain in the system discussed in the previous section in which an electron beam travels through the interference fringes produced by two crossing beams.

We suppose that the laser sources are equal, i.e., we consider two monochromatic plane waves (with the same polarization) crossing each other at a small angle [Fig. 1(a)]. In the ERF [Fig. 1(b)] fringes are produced along the η axis with periodicity

$$P \sim \frac{\lambda_i'}{2\sin\theta'}, \quad \lambda_i' \sim \frac{\lambda_i}{\gamma} . \tag{29}$$

Electrons traveling along the x axis are scattered by the photons in the beams. For relativistic electrons ($\gamma \gg 1$) the scattering, in ERF, as already noted is practically an elastic scattering (stimulated Thomson effect).

In the ERF the incident frequencies are given by

$$\omega_1' = \gamma\omega_1(1-\beta\cos\psi_1) , \tag{30}$$

$$\omega_2' = \gamma\omega_2(1+\beta\cos\psi_2) , \tag{31}$$

where $\psi_1 = \psi_2 = 90° - \theta$.

To obtain stable fringes in ERF it is necessary that

$$\omega_1' = \omega_2' ,$$

i.e.,

$$\omega_1(1-\beta\cos\psi_1) = \omega_2(1+\beta\cos\psi_2) . \tag{32}$$

We observe that for small θ in the laboratory frame we have $\omega_1 \simeq \omega_2$.

In the laboratory frame (LF) we study backscattering. The scattered frequencies are given by

$$\begin{aligned} \omega_{1s}' &= \gamma^2\omega(1-\beta\cos\psi_1)(1+\beta\cos\psi_1'') , \\ \omega_{2s}' &= \gamma^2\omega(1+\beta\cos\psi_2)(1+\beta\cos\psi_2'') , \\ \psi_1'' &= -\psi_1', \quad \psi_1' = 2\theta' + \psi_2' , \\ \psi_2'' &= -\psi_2', \quad \psi_2' = \psi_1' - 2\theta' . \end{aligned} \tag{33}$$

It is possible to consider the scattering as a stimulated process because the emitted photons travel in the opposite direction with respect to the incident photons, therefore they can produce stimulated emission of photons while crossing other fringes. Moreover, we remember that the presence of the N^2 term in the scattering probability means a cooperative effect by the N fringes.

In the following we calculate the gain for photons emitted in the direction opposite to the photons of beam 2 (see Fig. 1). The rate equation for the gain is

$$\frac{d\bar{n}_s}{dt} = G\bar{n}_s c = (G^{(a)} - G^{(b)})c\bar{n}_{1s} \tag{34}$$

(in analogy to the Sukhatme and Wolff calculation) where $G^{(a)}$ is the gain for the backscattering process between the electrons and photons 2; $G^{(b)}$ refers to scattering processes between the scattered photons and electrons, i.e., the scattering process which absorb useful high-energy photons to accelerate electrons. The quasielastic scattering in the ERF, due to the recoil of the electrons, assures the gain. We write

$$G^{(a)} = \frac{\bar{n}_e V}{\bar{n}_s c}\left[\frac{d\bar{n}_s}{dt}\right]_a , \tag{35}$$

where $\bar{n}_e$ is the electron density, and

$$\left[\frac{d\bar{n}_s}{dt}\right] = \int d^3P_0 f(P_{0x})\delta(P_{0y})\delta(P_{0z}) \times \sum_{P_f} W_{\text{tot}} , \tag{36}$$

where we have integrated over the incoming electron momentum distribution $f(P_{0x})$ and summed over the final electron momentum.

We analyze the gain along the x direction, in the case in which there is interference in the scattered fields. If we put $\bar{n}_0 = (\bar{n}_i/v)c$ we have

$$G^{(a)} = 16\bar{n}_e\bar{n}_0 c^2 4N^2 \frac{r_0^2(2\pi)^2}{\omega\omega_s} I , \tag{37}$$

where from Eq. (26)

$$I=16\int dP_{0x}f(P_{0x})\int dP_{fx}\frac{\left[\sin\left[\frac{P_{0x}}{\hbar}+K_{0x}-\frac{P_{fx}}{\hbar}-K_{sx}\right]L_f\right]^2}{\left[\left[\frac{P_{0x}}{\hbar}+K_{0x}-\frac{P_{fx}}{\hbar}-K_{sx}\right]2L_f+2\pi\right]^2}$$

$$\times\delta((P_{fx}^2c^2+m^2c^4)^{1/2}+\hbar\omega_s-(P_{0z}^2c^2+m^2c^4)^{1/2}-\hbar\omega)\ , \tag{38}$$

where $L_f=P_x/2$ is the fringe dimension, in the LF along the $\hat{x}$ direction.

The gain $G^{(a)}$ is obtained in the LF. N in Eq. (37) is the fringe number along the $\hat{x}$ direction. In Eq. (38) we have used the relation

$$\sum_{P_{fx}}\rightarrow\int dP_{fx}\frac{L_x}{(2\pi\hbar)}\ .$$

The first integral in Eq. (38) can now be resolved by using the δ-function properties.

In the same way it is possible to obtain the reverse gain $G^{(b)}$. The difference between $G^{(b)}$ and $G^{(a)}$ lies in the inversion between the initial and final photon-electron states, with a frequency shift in the final state with respect to the initial photon state of the forward process, which gives the $G^{(a)}$ gain.

After some algebra and setting[7]

$$\beta_0=\frac{v_0}{c}=P_{0x}\ ,$$

$$\tilde{\beta}_0=\frac{\omega_s-\omega}{\omega_s+\omega}\ ,$$

$$\Delta\beta=\beta_0-\tilde{\beta}_0,\ \ \tilde{\gamma}_0=\frac{1}{(1-\tilde{\beta}_0^2)^{1/2}}\ ,$$

$F'(\beta_0)$ is asymptotically equal to the derivative of the electron distribution function, and we get the difference between $G^{(a)}$ and $G^{(b)}$ which gives the total gain

$$G_{\text{tot}}=\frac{\bar{n}_e16N^2r_0^2\bar{n}_0c^2(2\pi)^2\hbar\omega_sf(\psi_2,\psi_2')}{c\omega\omega_s\tilde{\gamma}_0^5Lmc^2}$$

$$\times\int d\beta_0F'(\beta_0)\left[\frac{\sin(K_s\,\Delta\beta\,L_f)}{2K_s\,\Delta\beta\,L_f+2\pi}\right]^2\ , \tag{39}$$

where $f(\psi_2,\psi_2')$ is a function which contains the interference angle contribution.

The integral appearing in the gain expression depends critically upon a comparison between the widths of the sine function $(2\pi/K_sL_f)$ and the width $\Delta\beta$ of the electron velocity distribution.[7] In fact, when $\Delta\beta>(2\pi/K_sL_f)$ we are in the "large-cavity limit," and the maximum gain is given by (in cm^{-1})

$$G_{\max}=\frac{16N^2r_0^2}{(1+\beta\cos\psi_2)}\frac{\hbar\omega_s}{\Delta E}\frac{\tilde{E}_0}{\Delta E}(2\pi)^3\bar{n}_0\bar{n}_e\lambda_i\lambda_s^2$$

$$\times(1+\beta\cos\psi'2)\ . \tag{40}$$

In the opposite limit $(\Delta\beta<2\pi/K_sL_f)$ the maximum gain becomes (in cm^{-1})

$$G_{\max}=16r_0^2\frac{1+\beta\cos\psi_2'}{1+\beta\cos\psi_2}\frac{\hbar\omega_s}{\Delta E}\frac{\tilde{E}_0}{\Delta E}$$

$$\times(2\pi)^3\bar{n}_0\bar{n}_e\lambda_i\lambda_s^2\ , \tag{41}$$

i.e., the gain does not depend on N^2.

The condition having $\Delta\beta>(2\pi/K_sL_f)$ becomes an "angle condition"; in fact,

$$L_f\sim\frac{\lambda_i}{4\sin\theta},\ \ \theta=\frac{\pi}{2}-\psi_2\Longrightarrow L_f=\frac{\lambda_i}{4\cos\psi_2}\ ,$$

$$K_s=\frac{2\pi}{\lambda_s}=\frac{2\pi}{\lambda_i}\gamma^2(1+\beta\cos\psi_2)(1+\beta\cos\psi_2'')\ ,$$

but $\cos\psi_2=\cos(\pi/2-\theta)=\sin\theta$ and

$$\cos\psi_2''=+\cos(-\psi_2')=\frac{\beta+\cos\psi_2}{1+\beta\cos\psi_2}\ , \tag{42}$$

where Eq. (42) is a result of the "relativistic aberration formula."[8] With Eq. (42), K_s becomes

$$K_s=\frac{2\pi}{\lambda_i}\gamma^2(1+\beta)(1+\cos\psi_2)\ , \tag{43}$$

and we have

$$\Delta\beta=1-\frac{\omega_s-\omega}{\omega_s+\omega}=\frac{2\omega}{\omega+\omega_s}$$
$$=\frac{2}{1+\gamma^2(1+\beta\cos\psi_2)(1+\beta\cos\psi_2'')}$$
$$=\frac{2}{1+\gamma^2(1+\beta)(1+\cos\psi_2)}\ , \tag{44}$$

which, after some algebra gives

$$\cos\psi_2 \ll \beta\ , \tag{45}$$

i.e.,

$$\sin\theta \ll \beta \Longrightarrow \theta \ll \pi/2\ .$$

Relation (45) must be fulfilled together with Eq. (28), from which we have that (in the LF)

$$\gamma\frac{\Delta\omega_i}{\omega_i}<\beta\ . \tag{46}$$

IV. CONCLUSIONS

We have discussed a particular geometry for the interaction between a relativistic electron beam and two crossed laser fields which allows us to obtain coherent γ photons.

The main results of our calculations are expressed by Eqs. (26), (28), (40), (41), and (46). Equation (26) gives the total probability of production of γ photons, while Eqs. (40) and (41) are the total gain in case of large-cavity or small-cavity limits respectively. Equations (28) and (46) are two conditions which must be fulfilled by the angle and frequency widths of interfering laser beams to have the cooperative effect described by Eq. (26) and to be in the condition of the large-cavity limit. Some numerical examples may now give a hint to the possibility of a real experiment.

Let us assume we have a relativistic electron beam with $\gamma=10^2$, or $\beta=(1-1/\gamma^2)^{1/2}=9.9995\times10^{-1}$. To have conditions (45) and (46) fulfilled in the LF we may take $\theta\sim1°$. If one of the two laser beams is a CO_2 ($\lambda=10.6\ \mu$m), we have

$$P=\lambda/2\sin\theta=3\times10^{-2}\ \text{cm}\ .$$

By assuming an interaction length $L=1$ cm, we have $N=33$. We also have $\omega=1.7\times10^{14}$ Hz and $E=51.1$ MeV, while $\Delta\omega/\omega\sim10^{-2}$ and $\Delta E=10^5$ eV, ($E/\Delta=10^2$ eV). The scattered beam has $\omega_s=3.10^{18}$ Hz and $\lambda_s=2.6\times10^{-8}$ cm. By assuming $n_e/V=10^{11}$ electrons per cm^{-3} and $n_0/V=10^{20}$ photons per cm^{-3} we have a total gain of $G_m\sim0.4-4$ cm^{-1}. If we take $\theta=5°$ which still fulfills conditions (46) and (28) we have $p=6.10^{-3}$ cm, $N=100$, and $G_m\sim4.40$ cm^{-1}.

The number of scattered photons n_s is easily calculated from the relation

$$n_s=\frac{LGP_iV}{cA\hbar\omega}\ , \tag{47}$$

where P_i is the incident laser power V the interaction volume, A the laser-beam cross section and $\hbar\omega$ is the energy of the incident laser photons.

A comparison of relation (47) with the number of photons obtained in ADONE experiment[3] can be useful. In this case the number of scattered photons per second and per unit frequency is given by

$$\frac{d\dot{n}_s}{d\omega_s}=P_i\left[\frac{d\sigma_T}{d\omega_s}\frac{1}{\sigma_T}\right]\frac{1}{2\pi R}\frac{4N_e\gamma r_0^2}{S_e\hbar\omega}\ , \tag{48}$$

where N_e is the number of electrons circulating in the storage ring, S_e is the average cross section of the electron beam, and R is the average radius of the equilibrium orbit.

By using the ADONE experimental values $N_e=2\times10^{11}$ which gives $\bar{n}_e=N_e/2\pi RS_e=10^9$ cm^{-3}, $S_e=10^{-2}$ cm^2, $R=1670$ cm, $\gamma=3\times10^3$, $P_i=250$ W, $\lambda_i=4.9\times10^{-5}$ cm, $\hbar w=2.5\hbar$ eV, $A=10^{-2}$ cm^2, $4\gamma^2r_0^2\sim10^{-18}$ cm^2, we have from Eq. (48)

$$\dot{n}_s=\frac{c}{L}n_s\simeq10^5-10^7\ \text{sec}^{-1}\ ,$$

and from Eq. (47)

$$\dot{n}_s\simeq10^9-10^{10}\ \text{sec}^{-1}\ ,$$

with $G\sim5\times10^{-13}$ cm^{-1}, $E/\Delta=10^3$, and $N=3000$ fringes.

This comparison shows in an impressive way the advantage of our geometry.

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