Feedback oscillations of stimulated Brillouin scattering in plasmas with supersonic flow

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Long-time stimulated Brillouin scattering oscillations may occur in subcritical plasmas with supersonic flow against the incident wave owing to a feedback of the scattered radiation. They are studied in the frame of both the parametric approximation and the mode-coupling theory.

In the present paper stimulated Brillouin scattering (SBS) in an underdense plasma is investigated under the condition of a supersonic plasma flow against the direction of the incident wave. The buildup of an underdense plasma plateau with supersonic flow velocity has been suggested by several theoretical models of laser-plasma interaction.[1–4] Although no direct experimental evidence has been given, the observation of blue-shifted SBS signals[5–7] may be interpreted as an indication of supersonic flow.

Treating SBS as resonant three-wave interaction between the electromagnetic (em) pump wave, the back-scattered em wave, and an ion-acoustic wave, supersonic plasma flow against the pump causes a change in the sign of the ion-wave group velocity. In terms of the parametric approximation this means that SBS changes from an absolute to a convective instability.

In this paper we show that a feedback, produced by a low-level double reflection of the scattered wave at the boundaries, changes the character of the instability to become absolute again. Moreover, for certain phase-jump conditions the feedback gives rise to long-time oscillations (compared with the ion-acoustic period) of the reflected signal. These oscillations are studied in terms of both the parametric approximation and the three-wave interaction model. Calculated periods for typical parameters cover the range of both SBS[6,7] and higher-harmonic intensity oscillations[8–10] observed at time-resolved measurements at laser-plasma interaction.

As shown in Fig. 1, we consider a uniform plasma region of length L and density $n_0 < n_c$ (n_c critical density with respect to the pump frequency ω_0). A supersonic plasma flow against the incident wave (velocity $v_0 < -v_s$, v_s ion-sound velocity) is assumed to be present.

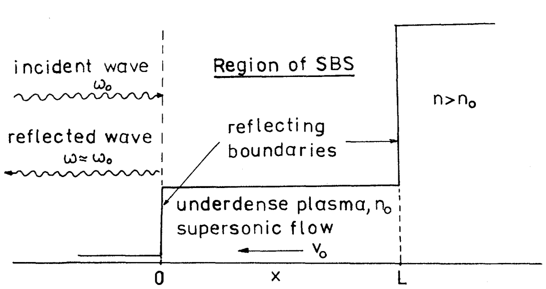

FIG. 1. One-dimensional plasma model.

The mode-coupling equations are

$$\begin{aligned}
&\frac{\partial E_+}{\partial t} + v_+ \frac{\partial E_+}{\partial x} = -\gamma_+ q E_- \ , \\
&\frac{\partial E_-}{\partial t} - v_- \frac{\partial E_-}{\partial x} = \gamma_- q^* E_+ \ , \\
&\frac{\partial q}{\partial t} + v_q \frac{\partial q}{\partial x} = \gamma_q E_+ E_-^* \ ,
\end{aligned} \tag{1}$$

where $v_+ = v_- = c\sqrt{\epsilon}$, $v_q = v_0 + v_s$ are the group velocities and $\gamma_+ = \gamma_- = 0.25\omega_0 n_0/n_c$, $\gamma_q = 0.25\omega_s$ are the coupling constants. E_+, E_- are normalized by $m_e\omega_0 v_T/e$; ω_s is the ion-sound frequency belonging to $k_q = 2k_0\sqrt{\epsilon}$, where k_0 is the pump vacuum wave number, v_T is the electron thermal velocity, and $\epsilon = 1 - n_0/n_c$. In deriving this system, the electric fields of the pump wave, the back-scattered wave, and the density perturbation $\delta n/n_0$ of the low-frequency wave have been assumed to vary as $\mathrm{Re}\{E_+(x,t)\exp[i(k_+x - \omega_+t)]\}$, $\mathrm{Re}\{-iE_-(x,t) \times \exp[i(k_-x - \omega_-t)]\}$, $\mathrm{Re}\{q(x,t)\exp[i(k_qx - \omega_qt)]\}$, respectively. The frequency and wave-number matching relations are

$$\begin{aligned}
&\omega_+ = \omega_0 = \omega_- + \omega_q \ , \\
&k_+ = k_0\sqrt{\epsilon} = k_- + k_q \simeq -k_- \ ,
\end{aligned}$$

with $\omega_q = \omega_s + k_q v_0$.

PARAMETRIC APPROXIMATION

First, we consider the initial development of the instability, where the pump amplitude is much larger than that of the two other waves. In this approximation the system (1) is reduced to the pair of linear equations

$$\begin{aligned}
&\frac{\partial E_-}{\partial t} - v_- \frac{\partial E_-}{\partial x} = \gamma_- q^* E_0 \ , \\
&\frac{\partial q}{\partial t} + v_q \frac{\partial q}{\partial x} = \gamma_q E_-^* E_0 \ ,
\end{aligned} \tag{2}$$

where E_0 is the constant pump amplitude and v_q is negative (supersonic flow). For the boundary conditions $E(L,t)=E_L$ and $q(L,t)=q_L$, system (2) describes a convective instability where the stationary states are determined by the noise levels E_L and q_L.

Taking into account partial reflection of the backscattered wave at the boundaries leads to a feedback condition in the following way. Reflection at the left-hand boundary generates a wave with amplitude $R_0E_-(0,t)$ that propagates without interaction (no resonance) parallel to the pump. Another reflection of this wave at $x=L$ creates a nonzero boundary value

$$E_-(L,t)=R_0R_LE_-(0,t)=R_fE_-(0,t) \tag{3}$$

of the scattered wave at the right-hand boundary; in this way a feedback is established. [$R_0(R_L)$ is the left-hand side (right-hand side) amplitude reflection coefficient.] For nonabsorbing plasmas the feedback factor $R_f=R_0R_L$ is real ($-1\leq R_f\leq 1$). When a plasma of lower density is adjacent to the left-hand boundary, the sign of R_f only depends on whether the density at the opposite side jumps to a lower ($R_f>0$) or a higher ($R_f<0$) value. The latter case (Fig. 1) simulates a density configuration with lower (subcritical) and upper (supercritical) density shelves self-consistently coexisting with the incident radiation.[1-4]

To solve system (2) we use the ansatz $E_-, q^* \propto \exp[i(Kx-\Omega t)]$. Together with the feedback relation (3) and the boundary condition $q(L,t)=0$ we get from Eqs. (2) a linear homogeneous algebraic system which has a nontrivial solution only if the relation[11]

$$R_f\left(\cos p + i\frac{F}{p}\sin p\right)=\exp(i\alpha F) \tag{4}$$

is fulfilled. Here, the following notations are used:

$$p=(F^2-Q^2)^{1/2},\quad F=-\frac{\Omega L}{2}\frac{v_- - |v_q|}{v_-|v_q|}\ ,$$

$$Q^2=\frac{\gamma_-\gamma_q E_0^2L^2}{v_-|v_q|},\quad \alpha=\frac{v_- - |v_q|}{v_- + |v_q|}\ .$$

Because the ion-sound velocity is much less than $c\sqrt{\epsilon}$ for fairly underdense plasmas, we have $|v_q| << v_-$, e.g., $\alpha\simeq 1$. Introducing the expressions for the group velocities and coupling constants we get finally

$$F\simeq\frac{\Omega}{\omega_s}\frac{k_0L\sqrt{\epsilon}}{|1+M|},\quad Q^2=\frac{n_0}{n_c}\frac{E_0^2(k_0L)^2}{8|1+M|}\ , \tag{5}$$

where $M=v_0/v_s$ is the Mach number.

Figure 2 shows solutions of Eq. (4) in the form $F=f(Q)$ with R_f as parameter. In all cases we found absolutely growing solutions ($\mathrm{Im}\,\Omega>0$) if the pump intensity exceeds a threshold. The sign of R_f, however, has an essential influence on the nature of the instability. Whereas for $R_f>0$ (and $Q \gtrsim \ln 2/R_f$) a purely growing mode ($\mathrm{Re}\,\Omega\simeq 0$) dominates [Fig. 2(b)], the most unstable mode in the case $R_f<0$ is accompanied by a nonzero real part $|\mathrm{Re}\,\Omega|\gtrsim \mathrm{Im}\,\Omega$ [Fig. 2(a)]. That is, the intensity of the scattered radiation oscillates for $R_f<0$ with a period T given by $T=\pi/\mathrm{Re}\,\Omega$.

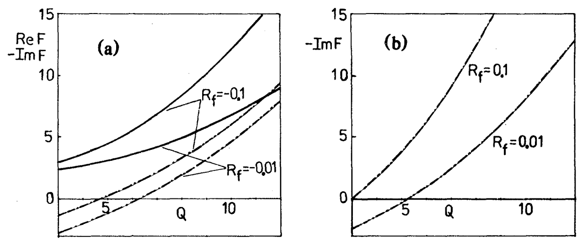

FIG. 2. Solutions $F=f(Q)$ of Eq. (4) with R_f as parameter (ReF—solid line, ImF—dashed line). (a) $R_f<0$, (b) $R_f>0$; in this case ReF is zero.

To illustrate the results we chose the following typical parameters:

$$R_f=-0.01,\quad M=-2,\quad n_0/n_c=0.3\ ,$$
$$E_0=0.25,\quad k_0L=25(2\pi)\ . \tag{6}$$

This set gives $Q\simeq 7.5$; from Fig. 2(a) we find then $\mathrm{Re}F\simeq 4.5$, $\mathrm{Im}F\simeq -1.1$ which corresponds to a period and a growth rate of

$$\omega_sT\simeq 80,\quad \gamma/\omega_s\simeq 0.07\ , \tag{7}$$

respectively. A direct integration of system (2) with the parameters (6) and $q(x,0)=10^{-4}$ produces the temporal evolution of the reflection coefficient $|R(t)|^2=|E_-(0,t)/E_0|^2$ shown in Fig. 3. After a transient phase both period and growth rate are in accordance with (7). Figure 4 shows the spatial amplitude structures of the decay waves at maximum and minimum reflectivity. At maximum reflectivity the amplitudes decrease almost exponentially [Fig. 4(a)],

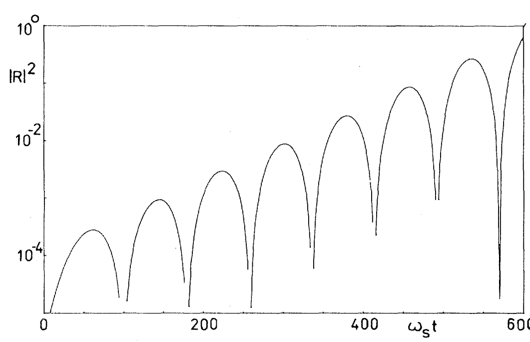

FIG. 3. Temporal evolution of the reflection coefficient $|R(t)|^2=|E_-(0,t)|^2/E_0^2$ obtained by numerical integration of the parametric equations (2) with the parameters (6).

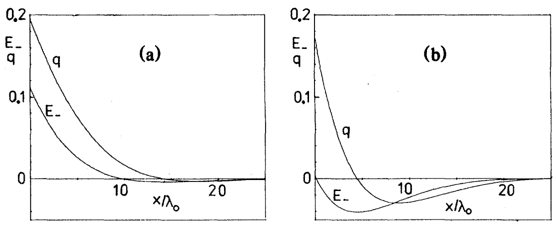

FIG. 4. Spatial structure of both the scattered wave amplitude E_- and the density perturbation q at (a) maximum ($\omega_s t = 350$) and (b) minimum reflectivity ($\omega_s t = 394$); λ_0 is the pump vacuum wavelength.

but note the small negative values of both quantities caused by the feedback condition (3). In the subsequent time evolution the negative density perturbation grows simultaneously in space and time owing to the combined action of negative group velocity and the instability mechanism, shifting the zero of q with nearly ion-wave group velocity towards the left-hand boundary. As a result reflectivity decreases. At the minimum ($|R|^2 \simeq 0$) the competing contributions of the density perturbation cause complete phase annihilation of the scattered radiation.

According to the relation

$$R = \frac{\gamma_-}{v_-} \int_0^L q(x)\,dx \ ,$$

following from Eq. (2) with $\partial E_-/\partial t = 0$, $|R|^2 = 0$ is equivalent to

$$\int_0^L q(x)dx = 0 \ .$$

For $R_f > 0$, obviously, phase-reversed density perturbations do not develop and consequently system (2) admits only purely growing solutions.

RESULTS OF MODE-COUPLING THEORY

If the unstable decay waves become comparable in amplitude to the pump, the parametric approximation breaks down and the subsequent evolution must be described by the full mode-coupling system (1), which particularly includes pump depletion. Since Q is proportional to E_0, pump depletion is expected to produce lower values of Q, that is, larger oscillation periods. Indeed, a numerical solution of the full system (1) for the parameters (6) yields a period of $\omega_s T \simeq 110$ after saturation, in contrast to $\omega_s T \simeq 80$ during the initial phase. Figure 5 shows calculated

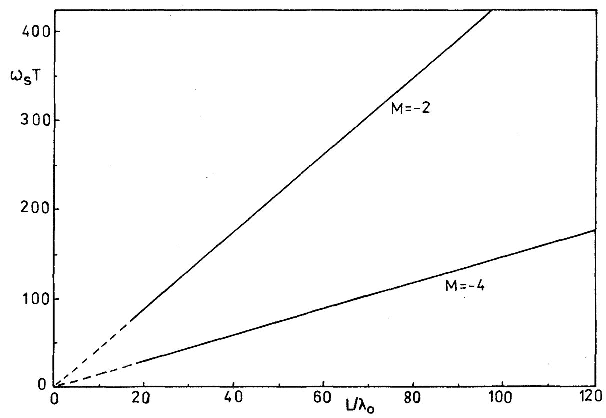

FIG. 5. Saturated oscillation period vs plasma length for two flow velocities ($M = -2, -4$); other parameters as in (6).

oscillation periods after saturation as a function of the plasma length L (linear dependence). The group velocity scaling proved to be $T \propto |1+M|^{-1}$. These results can well be fitted by the relation

$$\omega_s T \simeq \frac{\pi}{4} \frac{k_0 L \sqrt{\epsilon}}{|1+M|} \ . \tag{8}$$

In terms of (5), this means $|\mathrm{Re}F| \simeq 4$. From Fig. 2(a) ($R_f = -0.01$) we see that $|\mathrm{Re}F| \simeq 4$ is just reached near to the threshold ($\mathrm{Im}F \simeq 0$). Actually, with increasing R_f the threshold is shifted to lower values; $\mathrm{Re}F$ at threshold, however, remains nearly unchanged. Therefore, formula (8) provides an acceptable estimation of the saturated oscillation periods independently of the feedback factor R_f.

In addition to the mode-coupling calculations the fully nonlinear basic system (HF wave equation and ion fluid equations) has been solved numerically for various parameter combinations. The results confirm the main characteristics of the long-time SBS oscillations discussed above.

DISCUSSION

Values of $\omega_s T \simeq 100$ correspond to periods of $T \simeq 25$ ps for Nd and $T \simeq 250$ ps for CO_2 laser light, respectively, assuming $v_s/c = 10^{-3}$. Such oscillations have been observed in several laser-target experiments at time-resolved measurements of both SBS[6,7] and harmonic radiation $\frac{3}{2}\omega_0$, $2\omega_0$, ...[8-10] Therefore, we suggest that SBS feedback oscillations may be responsible for these burstlike phenomena.

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