Average muonic Coulomb capture probabilities for 65 elements

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A total of 146 measured muonic Coulomb capture ratios of anhydrous solid binary compounds has been used to derive average muonic Coulomb capture probabilities for 65 elements. The measured capture ratios can be well reproduced by ratios of these capture probabilities. This is also true for ratios in ternary compounds, in glass, and in alloys. These muonic capture probabilities can serve as a basis for the application of muonic x rays in elemental analysis.

I. INTRODUCTION

Muonic per atom Coulomb capture ratios in binary and more complex compounds have been measured by many experimental groups for more than twenty years. These experiments showed that the probability P for Coulomb capture in an element does not depend in a simple way on the atomic number Z, as suggested, e.g., by Fermi and Teller[1]; on the contrary, it exhibits a rather pronounced periodic variation with Z.[2,3] Only recently theories have been developed which reproduce this periodic behavior of P by relating it to the atomic radius[4] or to the number of not too strongly bound electrons.[5]

A quantitative description of muonic Coulomb capture probabilities is not only an interesting physical problem on its own, but it is also the basis for the application of muonic atoms in nondestructive elemental analysis.[6–8] The total K-series intensities from the different elements of a sample can be used as a quantitative measure for its elemental composition. However, these x-ray intensities have to be corrected with the respective per atom capture probabilities in order to obtain atomic abundances.

The determination of the average per atom capture probability for an element is only meaningful if this probability depends mainly on Z and only to a small extent on other properties of the sample as concentration or valence of the element Z or on the other components. Naumann and Daniel[9] recently demonstrated that, for simple alkali halides, the muonic capture probability of one component does not depend on the other component. It has been shown experimentally in the cases of NbV,[10] CuAl, and AgZn alloys, NaCl-NaBr and KCl-KBr solid solutions,[11] and oxides,[3] all with different atomic ratios, that per atom capture ratios in solids vary only at a percent level if the concentrations vary up to a factor of 400. Several experiments indicated that per atom capture ratios are not strongly influenced (mostly $<10\%$) by the chemical bond and the valence of the constituent.[3,12,13] Therefore, an attempt was made to calculate average capture probabilities for homogeneous anhydrous solid binary compounds through the use of all available experimental data on muonic Coulomb capture ratios. It is not expected, however, that the capture probabilities thus derived can be applied to inhomogeneous matter, gas mixtures, or materials containing hydrogen. Inhomogeneities affect capture ratios,[14] whereas the per atom capture ratios in gases depend upon the concentration.[15] Transfer effects, finally, may influence capture probabilities in hydrogeneous compounds.

II. CALCULATION OF AVERAGE EFFECTIVE CAPTURE PROBABILITIES

The muonic per atom Coulomb capture ratio $A(Z,Z')$ of a binary compound $Z_k Z'_m$ is given by

$$A(Z,Z')=R(Z,Z')m/k\ ,$$

where $R(Z,Z')$ is the measured muonic capture ratio in the compound of the elements Z and Z' and k/m is the atomic ratio. In order to calculate average capture probabilities $P(Z)$ for a great number of elements, we assume that capture ratios are ratios of capture probabilities:

$$A(Z,Z')=P(Z)/P(Z')\ .$$

A least-squares computer program was developed to fit capture probabilities $P(Z)$ of 65 elements to 146 measured capture ratios $A(Z,Z')$. Since only rela-

TABLE I. Average muonic capture probabilities and measured capture ratios.

Element	Z	Average capture probability relative to oxygen $P(Z)$	Measured capture ratios: Oxides	$A(Z,Z')$	Fluorides	$A(Z,Z')$	Chlorides	$A(Z,Z')$	Other compounds	$A(Z,Z')$
Li	3	0.18 ±0.04			LiF	0.10±0.08[a]	LiCl	0.19 ±0.08[b]	LiI	0.063±0.008[c]
Be	4	0.075±0.010	BeO	0.12±0.04[d]			BeCl	0.056±0.005[b]	{BN	0.25 ±0.02[e]
B	5	0.25 ±0.07	B_2O_3	0.22±0.05[d]					B_2S_4}	0.45 ±0.12[a]
N	7	1.02 ±0.30								
F	9	0.994±0.030							{NaBr	0.37 ±0.02[g]
Na	11	1.00 ±0.04	Na_2O_2	0.99±0.05[f]	NaF	0.97±0.05[a]	NaCl	0.79 ±0.03[b]	NaI}	0.29 ±0.04[d]
Mg	12	0.93 ±0.04	MgO	0.89±0.05[f]	MgF_2	0.92±0.03[a]	$MgCl_2$	0.76 ±0.04[b]		
Al	13	0.76 ±0.06	Al_2O_3	0.74±0.04[f]	AlF_3	0.95±0.17[a]	$AlCl_3$	0.66 ±0.04[h]	AlI_3	0.48 ±0.18[i]
Si	14	0.84 ±0.06	SiO_2	0.84±0.04[f]						
P	15	1.04 ±0.06	P_2O_5	1.00±0.05[f]			PCl_5	0.82 ±0.04[b]		
S	16	1.23 ±0.05								
Cl	17	1.32 ±0.04							{KBr	0.583±0.008[g]
K	19	1.54 ±0.05			KF	1.89±0.19[j]	KCl	1.14 ±0.02[g]	KI}	0.54 ±0.06[j]
Ca	20	1.90 ±0.09	CaO	1.71±0.09[k]	CaF_2	1.47±0.26[a]	$CaCl_2$	1.41 ±0.07[b]	CaS	1.57 ±0.04[h]
Sc	21	2.78 ±0.31	Sc_2O_3	2.78±0.20[d]						
Ti	22	2.66 ±0.19	TiO_2	2.70±0.13[f]	TiF_4	2.44±0.31[a]				
V	23	2.76 ±0.17	V_2O_4	2.70±0.19[f]	VF_4	3.20±0.29[a]	VCl_3	2.03 ±0.11[b]		
Cr	24	2.98 ±0.19	Cr_2O_3	3.45±0.25[f]			$CrCl_3$	2.15 ±0.10[b]		
Mn	25	2.73 ±0.18	MnO_2	2.60±0.19[f]	MnF_2	2.16±0.21[a]	$MnCl_2$	2.48 ±0.14[b]		
Fe	26	3.28 ±0.21	Fe_2O_3	3.21±0.20[f]	Fe_2F_3	2.44±0.39[a]	$FeCl_3$	2.01 ±0.35[b]	FeS	3.03 ±0.17[a]
Co	27	2.94 ±0.27	Co_3O_4	3.35±0.25[f]	CoF_2	2.38±0.32[a]	$CoCl_2$	2.09 ±0.29[b]		
Ni	28	2.88 ±0.22	Ni_2O_3	2.66±0.20[f]	NiF_2	3.09±0.23[a]	NiCl	2.34 ±0.31[b]		
Cu	29	3.26 ±0.36	CuO	3.26±0.23[f]						
Zn	30	3.20 ±0.19	ZnO	3.06±0.24[f]	ZnF_2	3.28±0.20[a]	$ZnCl_2$	2.17 ±0.30[b]	ZnS	2.69 ±0.17[a]
Ga	31	2.77 ±0.31	Ga_2O_3	2.77±0.20[f]						
Ge	32	2.90 ±0.33	GeO_2	2.90±0.21[f]						
As	33	2.89 ±0.26	As_2O_3	3.39±0.25[f]					As_2S_3	2.01 ±0.18[a]
Se	34	2.72 ±0.31	SeO_2	2.72±0.20[f]						
Br	35	2.70 ±0.14								
Rb	37	2.39 ±0.19	RbO_2	2.58±0.20[k]	RbF	2.41±0.18[a]	RbCl	1.53 ±0.19[b]		
Sr	38	2.13 ±0.14	SrO	2.12±0.11[k]	SrF_2	1.83±0.28[a]			SrS	1.90 ±0.17[a]
Y	39	2.34 ±0.19	Y_2O_3	2.19±0.16[f]	YF_3	2.54±0.19[a]				
Zr	40	2.60 ±0.26	ZrO_2	2.62±0.19[f]	ZrF_4	2.33±0.49[a]	$ZrCl_4$	2.06 ±0.43[b]		
Nb	41	3.06 ±0.34	Nb_2O_5	2.95±0.23[f]			$NbCl_5$	3.25 ±0.58[b]		
Mo	42	3.48 ±0.29	MoO_3	3.60±0.29[f]			$MoCl_5$	3.22 ±0.58[b]	MoS_2	2.70 ±0.20[a]

TABLE I. (*Continued.*)

Element	Z	Average capture probability relative to oxygen $P(Z)$	Oxides	$A(Z,Z')$	Fluorides	$A(Z,Z')$	Chlorides	$A(Z,Z')$	Other compounds	$A(Z,Z')$
Tc	43	3.3 ±0.5	TcO_2	3.26±0.31[f]						
Pd	46	3.60 ±0.50	PdO	3.57±0.44[f]			$PdCl_2$	2.75 ±0.36[b]	Ag_2S	2.28 ±0.14[a]
Ag	47	3.14 ±0.19	Ag_2O	3.83±0.32[f]	AgF_2	3.20±0.23[a]	AgCl	2.50 ±0.25[b]	AgI	1.45 ±0.25[d]
									CdS	2.50 ±0.16[a]
Cd	48	3.01 ±0.18	CdO	3.14±0.25[f]	CdF_2	3.25±0.37[a]	$CdCl_2$	2.26 ±0.26[i]	$CdBr_2$	0.95 ±0.12[i]
In	49	2.9 ±0.5	In_2O_3	2.92±0.31[f]					CdI_2	1.00 ±0.12[i]
Sn	50	2.54 ±0.18	SnO_2	3.02±0.23[f]	SnF_2	2.30±0.15[a]	$SnCl_2$	1.98 ±0.22[i]		
Sb	51	3.16 ±0.23	Sb_2O_3	3.52±0.28[f]	SbF_3	2.98±0.30[a,i]	$SbCl_3$	2.55 ±0.39[i]	Sb_2S_3	2.43 ±0.18[a]
Te	52	3.16 ±0.36	TeO_2	3.22±0.25[f]			$TeCl_4$	2.07 ±0.50[b]		
I	53	2.97 ±0.26								
Cs	55	3.25 ±0.27	CsO_2	3.25±0.26[k]	CsF	3.65±0.35[a]	CsCl	2.21 ±0.23[b]		
Ba	56	3.76 ±0.27	BaO_2	2.84±0.36[f]	BaF_2	3.32±0.40[a]	$BaCl_2$	2.67 ±0.33[b]	BaS	3.59 ±0.20[a]
La	57	3.43 ±0.33	La_2O_3	2.73±0.33[f]	LaF_3	3.93±0.28[a]				
Ce	58	5.2 ±0.5	CeO_2	4.50±0.55[f]	CeF_3	5.40±0.34[a]				
Nd	60	5.8 ±0.4	Nd_2O_3	5.13±0.35[f]	NdF_3	6.72±0.44[a]				
Sm	62	4.4 ±1.1	Sm_2O_3	4.4 ±0.7[f]						
Eu	63	4.3 ±0.8	Eu_2O_3	4.34±0.46[f]						
Gd	64	5.8 ±0.5	Gd_2O_3	5.52±0.33[f]	GdF_3	7.3 ±0.7[a]				
Dy	66	6.4 ±0.6	Dy_2O_3	5.8 ±0.6[f]	DyF_3	6.8 ±0.5[a]				
Er	68	6.8 ±0.8			ErF_3	6.8 ±0.5[a]				
Yb	70	6.2 ±0.5	Yb_2O_3	6.85±0.42[f]	YbF_3	5.1 ±0.6[a]				
Lu	71	5.3 ±0.9	Lu_2O_3	5.3 ±0.6[f]						
Ta	73	6.0 ±0.5	Ta_2O_5	7.2 ±0.7[f]	TaF_5	5.92±0.41[a]	$TaCl_5$	3.7 ±0.6[b]		
W	74	6.6 ±0.9	WO_3	5.8 ±0.7[f]			WCl_6	7.2 ±0.9[b]		
Hg	80	4.8 ±0.5			HgF_2	4.64±0.33[a]	$HgCl_2$	3.91 ±0.44[b]		
Tl	81	5.0 ±0.5	Tl_2O_3	4.8 ±0.6[f]	TlF	5.2 ±0.6[a]	TlCl	3.69 ±0.37[b]		
Pb	82	3.69 ±0.24	PbO_2	5.0 ±0.6[f]	PbF_2	4.2 ±0.5[a]	$PbCl_2$	3.16 ±0.24[d]	PbS	2.67 ±0.16[a]
									PbI_2	1.22 ±0.11[d]
Bi	83	3.78 ±0.39	Bi_2O_3	3.77±0.43[f]	BiF_3	3.94±0.36[a]	$BiCl_3$	2.55 ±0.41[b]		
Th	90	3.0 ±0.6	ThO_2	3.6 ±0.5[f]	ThF_4	2.3 ±0.5[a]				
U	92	4.7 ±0.9	UO_2	4.7 ±0.6[f]						

[a]Reference 16 (Hartmann *et al.*).
[b]Reference 17 (Daniel *et al.*).
[c]Reference 18 (Lathrop *et al.*).
[d]Reference 2 (Zinov *et al.*).
[e]Reference 19 (Knight *et al.*).
[f]Reference 3 (von Egidy *et al.*).
[g]Reference 11 (Naumann *et al.*).
[h]Reference 20 (Mausner *et al.*).
[i]Reference 21 (Brandão d'Oliveira *et al.*).
[j]Reference 22 (Naumann *et al.*).
[k]Reference 23 (von Egidy *et al.*).

tive values of $P(Z)$ can be determined, the value for oxygen was set to $P(Z'=8)=1.00$. Table I lists all elements, the calculated capture probabilities $P(Z)$, and all used experimental capture ratios $A(Z,Z')$. The errors of $P(Z)$ represent the internal errors derived from the fit multiplied by the normalized $\bar{\chi}$. A value $\bar{\chi}=1.56$ was obtained which shows that the agreement between the measured and calculated capture ratios $A(Z,Z')$ is satisfactory. The result also supports the assumptions that capture proba-

TABLE II. Capture ratios in ternary compounds, glass, and alloys.

Compound and Composition	Z,Z'	measured ratio $A(Z,Z')$	calculated ratio $P(Z)/P(Z')$
Ternary Compounds			
$NaNO_2$[a]	Na,O	0.89±0.04	1.00±0.04
	N,O	0.95±0.04	1.02±0.30
$NaNO_3$[a]	Na,O	0.88±0.04	1.00±0.04
	N,O	0.76±0.04	1.02±0.30
Na_2SO_3[a]	Na,O	0.95±0.04	1.00±0.04
	S,O	1.18±0.05	1.23±0.05
Na_2SO_4[a]	Na,O	0.91±0.05	1.00±0.04
	S,O	1.04±0.04	1.23±0.05
Na_2SeO_3[a]	Na,O	0.93±0.04	1.00±0.04
	Se,O	2.92±0.15	2.72±0.31
Na_2SeO_4[a]	Na,O	0.94±0.04	1.00±0.04
	Se,O	2.90±0.15	2.72±0.31
$NaClO_4$[b]	Na,Cl	0.70±0.06	0.76±0.05
$MgSO_4$[b]	Mg,S	0.83±0.02	0.76±0.06
$CaSO_4$[b]	Ca,S	1.89±0.06	1.54±0.10
$AlPO_4$[b]	Al,P	0.89±0.03	0.73±0.07
Glass[c]			
3.8 at. % Na	Na,O	1.14±0.03	1.00±0.04
2.1 at. % Mg	Mg,O	0.92±0.05	0.93±0.04
0.5 at. % Al	Al,O	0.73±0.09	0.76±0.06
26.1 at. % Si	Si,O	0.88±0.03	0.84±0.06
2.3 at. % K	K,O	1.62±0.10	1.54±0.05
2.3 at. % Ca	Ca,O	1.89±0.10	1.90±0.09
0.3 at. % Ti	Ti,O	2.64±0.18	2.66±0.19
0.3 at. % Fe	Fe,O	3.24±0.26	3.28±0.21
0.4 at. %. Ba	Ba,O	3.43±0.34	3.76±0.27
Alloys			
Cd-Mg[d]	Cd,Mg	3.14±0.25	3.24±0.24
Sn-Mg[e]	Sn,Mg	2.73±0.30	2.73±0.23
Cu-Al[f]	Cu,Al	3.51±0.05	4.29±0.58
Ni-Ca[e]	Ni,Ca	1.64±0.16	1.52±0.14
Nb-V[g]	Nb,V	1.21±0.04	1.11±0.14
Cu-Ni[d]	Cu,Ni	1.08±0.05	1.13±0.15
Y-Ni[e]	Y,Ni	0.77±0.11	0.81±0.09
Ag-Zn[f]	Ag,Zn	0.98±0.04	0.98±0.08
Te-Se[d]	Te,Se	1.02±0.08	1.16±0.19

[a]Reference 13 (Schneuwly *et al.*).
[b]Reference 20 (Mausner *et al.*).
[c]Reference 24 (Köhler *et al.*).
[d]Reference 25 (Bergmann *et al.*).
[e]Reference 14 (Bergmann *et al.*).
[f]Reference 11 (Naumann *et al.*).
[g]Reference 10 (Bergmann *et al.*).

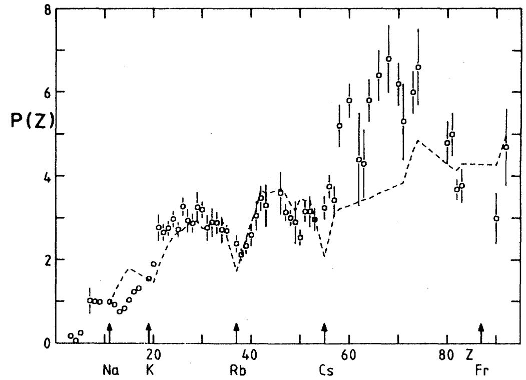

FIG. 1. Calculated values of capture probabilities $P(Z)$ vs Z. Values are normalized to $P(8)=1.00$. Dashed curve: Theoretical prediction of Ref. 4.

bilities $P(Z)$ do not strongly depend on the second component Z', the concentration, or the nature of the chemical bond. The capture probabilities $P(Z)$ in Table I can be compared with the measured capture ratios in oxides, since the $P(Z)$ are normalized to $P(8)=1.00$. Figure 1 shows $P(Z)$ as a function of Z together with theoretical predictions for oxides calculated with the formula[4]

$$A(Z,Z')=\frac{Z^{1/3}\ln(0.57Z)r(Z')}{Z'^{1/3}\ln(0.57Z')r(Z)},$$

$r(Z)$ being the atomic radius of element Z. The periodicity of $P(Z)$ is evident.

III. APPLICATION TO TERNARY COMPOUNDS, GLASS, AND ALLOYS

Although $P(Z)$ values were derived only from binary compounds, it can be asked if they are valid also for a wider class of materials. In this case, they would represent an essential help for the application of muonic x rays to elemental analysis of matter. In order to answer this question, measured capture ratios in ternary compounds, glass, and alloys are compared in Table II with calculated ratios of $P(Z)$. The agreement is very good. This demonstrates that the average capture probabilities are a powerful tool if muonic x rays are applied to nondestructive chemical analysis. In particular, the glass data show that for elements with an abundance as low as 0.1 at. % the concentration can be determined with a relative error smaller than 10%. However, it should be kept in mind that the capture probabilities are valid only for homogeneous anhydrous and solid matter.

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