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Population trapping in a multilevel system

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In a two-photon resonant three-level system, population can be trapped in a noninteracting superposition state. We demonstrate that this phenomenon persists in a multilevel system in which the final state is replaced by a number of nondegenerate states. The magnitude of the trapped population depends *only* on the matrix elements between the pair of two-photon resonant levels and the intermediate level. A dressed-state approach is also considered.

In a recent paper[1] the population dynamics of a three-level system driven by two applied fields has been examined using dressed-state techniques. Figure 1 shows the level scheme in which field $\vec{\epsilon}_1$ drives the transition $|a\rangle \rightarrow |b\rangle$, detuned Δ_1; field $\vec{\epsilon}_2$ drives the transition $|b\rangle \rightarrow |f\rangle$, detuned Δ_2; and γ represents incoherent loss from level $|b\rangle$ out of the system. A feature of this system is the phenomenon of population trapping[1-8] where, if states $|a\rangle$ and $|f\rangle$ are two-photon resonant, the populations in these states tend to nonzero values as $t \rightarrow \infty$, despite the loss γ. The dressed-state approach shows[1] that two-photon resonance dictates that one of the dressed states for the atom-plus-fields system contains no contribution from level $|b\rangle$ and hence does not decay. Asymptotically, the wave function is a multiple of this dressed state.

We now consider a generalization of this three-level system where the single level $|f\rangle$ is replaced by a discrete set of N levels ($f=1,2,\ldots,N$), one of which, say $|M\rangle$, is two-photon resonant with level $|a\rangle$. The system is assumed to be in the lambda configuration (as Fig. 1), but the results are valid also for the ladder configuration.[1] We show that trapped population exists in levels $|a\rangle$ and $|M\rangle$ with magnitudes independent of N and of the matrix elements between $|b\rangle$ and the other $N-1$ levels of the discrete set $\{\,|f\rangle\,\}$.

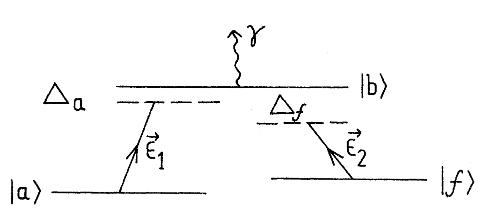

FIG. 1. The energy-level scheme.

We assume that field $\vec{\epsilon}_1$ drives the transition $|a\rangle \rightarrow |b\rangle$ at frequency ω_1, as in the three-level case, and that field $\vec{\epsilon}_2$ drives the transitions from $|b\rangle$ to any of the states $\{\,|f\rangle\,\}$ at frequency ω_2. Writing the wave function as

$$|\psi\rangle = c_a e^{-i\omega_a t}|a\rangle + c_b e^{-i\omega_b t}|b\rangle + \sum_{f=1}^{N} c_f e^{-i\omega_f t}|f\rangle \tag{1}$$

and substituting into the Schrödinger equation, gives the equations of motion (in the rotating-wave approximation)

$$i\dot{c}_a = -K_0 c_b e^{-i\Delta_a t}, \tag{2a}$$

$$i\dot{c}_b = -K_0^* c_a e^{i\Delta_a t} - i\gamma c_b - \sum_{f=1}^{N} K_f c_f e^{i\Delta_f t}, \tag{2b}$$

$$i\dot{c}_f = -K_f^* c_b e^{-i\Delta_f t}, \quad f=1,2,\cdots,N, \tag{2c}$$

where K_0 and K_f are the dipole matrix elements between $|a\rangle$ and $|b\rangle$ and between $|b\rangle$ and $|f\rangle$, respectively. The detunings are defined by $\omega_a+\omega_1+\Delta_a=\omega_b$ and $\omega_f+\omega_2+\Delta_f=\omega_b$. Throughout we shall consider the initial condition $c_a(0)=1$. We first investigate the nature of the solution for the amplitude c_a. By eliminating c_b and $\{\,c_f\,\}$ from (2), we obtain an integro-differential equation for c_a:

$$\ddot{c}_a+(\gamma+i\Delta_a)\dot{c}_a+|K_0|^2c_a = -\int_0^t \dot{c}_a(t')\sum_{f=1}^{N}|K_f|^2 e^{i(\Delta_f-\Delta_a)(t-t')}dt' . \tag{3}$$

Taking the Laplace transform of Eq. (3), using the initial conditions [note that $\dot{c}_a(0)=0$ from (2a)] and the convolution theorem, leads to an expression for $\bar{c}_a(s)$, the Laplace transform of c_a:

$$\bar{c}_a(s)=\frac{s+\gamma+i\Delta_a+\sum_{f=1}^{N}\frac{|K_f|^2}{s-i(\Delta_f-\Delta_a)}}{s^2+(\gamma+i\Delta_a)s+|K_0|^2+s\sum_{f=1}^{N}\frac{|K_f|^2}{s-i(\Delta_f-\Delta_a)}} . \tag{4}$$

Expression (4) can be written as $Q(s)/R(s)$, where $Q(s)$ and $R(s)$ are polynomials of order $N+1$ and $N+2$, respectively, by multiplying the numerator and the denominator by the product of N factors $s-i(\Delta_1-\Delta_a)$, $s-i(\Delta_2-\Delta_a),\ldots,$ $s-i(\Delta_N-\Delta_a)$. Note that one of these factors is s since $\Delta_M=\Delta_a$; further, the denominator of (4) does not contain a term in s^{-1} to cancel this factor and therefore $R(s)$ itself has a factor s. When Q/R is put into partial fractions, we obtain a result of the form

$$\bar{c}_a(s)=\frac{A}{s}+\sum_{i=1}^{N+1}\frac{B_i}{(s-s_i)} , \tag{5}$$

where A and $\{B_i\}$ are constants and $\{s_i\}$ are the remaining zeros of R (apart from $s=0$). The full solution for $c_a(t)$ is thus the sum of a constant A and $N+1$ exponential terms. In fact, these latter terms must be exponentially *decreasing,* as can be seen by setting the denominator of (4) to zero and substituting $s=ir$ where r is real and nonzero. The imaginary part of the resulting expression states the contradiction $\gamma r=0$. Hence all values s_i have a nonzero real part which must be such that $c_a(t)$ remains finite. The Laplace inversion of each of the terms $(s-s_i)^{-1}$ in (5) therefore gives a decaying exponential, and $c_a(\infty)=A$. From (4) and (5)

$$A=\lim_{s\to 0}s\bar{c}_a(s)=\frac{|K_M|^2}{|K_0|^2+|K_M|^2} , \tag{6}$$

and the trapped population in $|a\rangle$ is given by

$$P_a(\infty)=\frac{|K_M|^4}{(|K_0|^2+|K_M|^2)^2} . \tag{7}$$

With $t\to\infty$ in Eq. (2b) and setting $0=\dot{c}_b(\infty)$ $=c_f(\infty)$ for $f\neq M$, gives

$$K_0^*c_a(\infty)+K_Mc_M(\infty)=0 , \tag{8}$$

and using (7) we find

$$P_M(\infty)=\frac{|K_0|^2|K_M|^2}{(|K_0|^2+|K_M|^2)^2} . \tag{9}$$

Results (7) and (9) are clearly independent of the non-two-photon resonant levels and agree with those found previously in the three-level system.[1] In fact, a simple derivation can be obtained by using the equation

$$K_M^*c_a-K_0c_M=K_M^* , \tag{10}$$

obtained from (2a) and (2c) with $f=M$, integrating once.[6] The pair of equations, (10) with $t\to\infty$ and (8), then give the trapped populations (7) and (9). The Laplace-transform method, however, reveals the crucial role of the two-photon resonance, which leads to a constant term in c_a, and the loss γ which leads to the exponential decay of all other terms in c_a.

The above results lead one to suspect that for two-photon resonance between $|a\rangle$ and $|M\rangle$ one dressed state of the system contains contributions from only $|a\rangle$ and $|M\rangle$, independently of the other levels $\{\,|f\rangle\,\}$, $f\neq M$, and that this dressed state corresponds to a zero eigenvalue. We now show that this is the case. In the undressed state basis $|a\rangle|n_1\rangle|n_2\rangle$, $|b\rangle|n_1-1\rangle|n_2\rangle$, $|f\rangle|n_1-1\rangle|n_2+1\rangle$, $f=1$ to N, where $|n_1\rangle$ and $|n_2\rangle$ are the photon number states associated with fields $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$, the Hamiltonian can be written in matrix form as

$$H=\begin{bmatrix} 0 & K_0 & 0 & 0 & 0 & \cdots \\ K_0^* & \Delta_a & K_1 & K_2 & K_3 & \cdots \\ 0 & K_1^* & \Delta_a-\Delta_1 & 0 & 0 & \cdots \\ 0 & K_2^* & 0 & \Delta_a-\Delta_2 & 0 & \cdots \\ 0 & K_3^* & 0 & 0 & \Delta_a-\Delta_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{bmatrix} . \tag{11}$$

The eigenvalues λ are given by

$$\lambda \begin{vmatrix} \Delta_a-\lambda & K_1 & K_2 & \cdots \\ K_1^* & \Delta_a-\Delta_1-\lambda & 0 & \cdots \\ K_2^* & 0 & \Delta_a-\Delta_2-\lambda & \cdots \\ \vdots & 0 & 0 & \\ & \vdots & \vdots & \end{vmatrix} + |K_0|^2 \prod_{i=1}^{N}(\Delta_a-\Delta_i-\lambda)=0 \quad (12)$$

and since $\Delta_M=\Delta_a$, $\lambda=0$ solves (12). The eigenvector

$$\chi = \begin{bmatrix} \chi_a \\ \chi_b \\ \chi_1 \\ \chi_2 \\ \vdots \end{bmatrix} \quad (13)$$

of H corresponding to $\lambda=0$ solves $H\chi=0$ which, from (11) and (13) leads to

$$\begin{bmatrix} K_0\chi_b \\ K_0^*\chi_a+\Delta_a\chi_b+\sum_{i=1}^{N}K_i\chi_i \\ K_1^*\chi_b+(\Delta_a-\Delta_1)\chi_1 \\ \vdots \\ K_{M-1}^*\chi_b+(\Delta_a-\Delta_{M-1})\chi_{M-1} \\ K_M^*\chi_b \\ K_{M+1}^*\chi_b+(\Delta_a-\Delta_{M+1})\chi_{M+1} \\ \vdots \end{bmatrix} = 0 . \quad (14)$$

Each element of this column vector is then equated to zero. Immediately we see that $\chi_b=0$ and therefore

$$\chi_1=\chi_2=\cdots=\chi_{M-1}=\chi_{M+1}=\cdots=\chi_N=0 .$$

The only nonzero elements of χ are χ_a and χ_M which, from the second element of (14), satisfy $K_0^*\chi_a+K_M\chi_M=0$. This eigenvector represents a dressed state, corresponding to a zero eigenvalue, with contributions from $|a\rangle$ and $|M\rangle$ only, weighted in the ratio of the trapped amplitudes as required. As $t\to\infty$ the wave function tends to a multiple of this dressed state.

It may be interesting to investigate these results in the light of conserved quantities derived for three-level[9] and multilevel[10] systems.

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