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Material and electromagnetic energy flows generated in a spatially dispersive solid-state plasma by an electromagnetic wave impinging upon the surface at oblique incidence

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The anisotropic conductivity of a semi-infinite free-electron solid-state plasma having a sharp boundary is calculated assuming specular electron surface scattering. Explicit expressions for the conductivity-tensor components are presented for a fully degenerate plasma with special emphasis on the near-local regime. With the assumption that the monochromatic electromagnetic plane wave penetrates the surface at an oblique angle the reflected and transmitted fields are determined. By combining Maxwell's equations and the *nonlinear* Boltzmann equation the energy transport in the plasma is studied. With spatial dispersion effects taken into account, the cycle-averaged Poynting vectors of the plasma, originating in the induced dc mass transport of the conduction electrons, and of the electromagnetic field are determined. A particular investigation is devoted to the case of TE-mode propagation in the near-local regime, with emphasis on (i) a discussion of the angular deviation of the plasma Poynting vector from the electromagnetic Poynting vector and (ii) a calculation of the ratio between the magnitudes of the energy flows. The "problem" of the violation of the principle of energy conservation at a sharp boundary is considered.

I. INTRODUCTION

The main purposes of the present study are three. One object is to calculate the electromagnetic field inside a spatially dispersive solid-state plasma of semi-infinite extension, in the case where a monochromatic, plane electromagnetic wave is incident on the surface at an oblique angle. This part of the investigation has been undertaken to give a more general and sound basis for kinematic[1,2] and dynamic[3,4] nonthermal light-scattering studies in the cases where the scattering from the plasma bunching or the screening of the scattering from the lattice waves are of importance. The second aim is to investigate the stationary energy flow which is associated with the dc displacement of the free-carrier distribution, and which is induced by the penetrating electromagnetic wave. This part of the work is complementary to studies of energy flow associated with acoustic[5] and optical[6,7] lattice waves coupled to transverse or longitudinal electromagnetic fields in spatially dispersive absorbing media. The third purpose is to point out the crucial importance of the surface for nonlocal plasma-optic problems.

In Sec. II a rigorous calculation is given of the conductivity tensor $\overleftrightarrow{\sigma}(\vec{k},\omega)$ in the empty-lattice (or spherical Fermi surface) approximation assuming the conduction electrons to be specular scattered at the sharp surface of the semi-infinite solid-state plasma. The conduction-electron bunching is described by the Boltzmann transport equation and the arguments are generalizations of the "mirror-image" considerations given originally by Reuter and Sondheimer[8] for normal incidence. Explicit expressions for the components of the conductivity tensor are presented for a fully degenerate plasma in Sec. III, where a particular calculation of the conductivity tensor in what we shall call the near-local regime, i.e., the region where spatial dispersion effects are included in lowest (second) order, is also given. In Sec. IV the general expressions for the reflected and transmitted electromagnetic fields are derived, assuming the incoming TE or TM mode to penetrate the surface at an oblique angle. Simple, approximate formulas for oblique incidence have been presented recently by Hutchison and Hansen[9] in the case of free-electron metal films.

In Sec. V the cycle-averaged Poynting vectors of the plasma and the electromagnetic field are examined. The cycle-averaged Poynting vector of the plasma, originating in an induced dc electronic mass transport, is calculated on the basis of the *nonlinear* Boltzmann equation. Rigorous results are given for oblique incidence. Steady-state energy flow in absorbing dielectric as well as in conducting media, in which the effect of spatial dispersion is neglected, has been studied in considerable detail.[10,11] Previously, in spatially dispersive media only the energy flows carried by ultrasonic waves[5] and transverse optical lattice waves[6] generated by transverse radio-frequency and optical electromagnetic waves, and the energy flow carried by free carriers coupled to longitudinal, nondispersive lattice vibrations,[7] have been investigated. In all the above studies normal incidence has been assumed.

In Sec. VI we emphasize an investigation of TE-mode propagation in a fully degenerate plasma in the near-local optical regime. The explicit *mode expansion* of the electromagnetic field and the plasma bunching is discussed. The question of the determination of the appropriate mode expansion is analogous to the problem of additional boundary conditions (the so-called ABC) treated in different contexts.[12–15] The wave vectors of the modes are determined by the dispersion relation for TE-mode propagation in an unbounded medium in the case of specular electron surface scattering. In contrast to the studies in the domain of exciton polaritons, where *two* (transverse) allowed modes are propagating in the spatially dispersive crystal,[16] a *multimode* field pattern are, in general, obtained far from the local limit in the plasma-optic case. The wave-vector components parallel to the surface are equal, whereas the perpendicular components are different. In the near-local regime only a single transmitted plane-wave mode is excited. The Poynting vectors of the plasma and the electromagnetic field, as well as the ratio between these, are determined on explicit form. It is shown that, in general, the directions of the two Poynting vectors are different. The angular deviation is calculated and discussed. Finally, the "problem" of energy conservation at a sharp boundary is considered, and the question of second harmonic generation of light is touched in this connection.

II. THE CONDUCTIVITY TENSOR APPROPRIATE TO SPECULAR ELECTRON SURFACE SCATTERING

To determine, in the empty-lattice approximation, the electromagnetic field inside a solid-state plasma one has to solve the driven wave equation

$$\left[\vec{\nabla}\vec{\nabla}-\overleftrightarrow{1}\left[\nabla^2-\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right]\right]\cdot\vec{E}(\vec{r},t)=-\mu_0\frac{\partial\vec{J}(\vec{r},t)}{\partial t}\,, \tag{1}$$

where $\vec{E}(\vec{r},t)$ is the electric field, $\vec{J}(\vec{r},t)$ is the free-carrier current density, c_0 is the vacuum velocity of light, μ_0 is the vacuum permeability, $\vec{\nabla}$ is the gradient operator, and $\overleftrightarrow{1}$ is the unit tensor of dimension 3×3. Let us assume that the solid-state plasma occupy the half-space $z>0$, the rest of space being vacuum, and let a harmonic plane electromagnetic wave of angular frequency ω, i.e.,

$$\vec{E}_i(\vec{r},t)=\vec{E}_i(k_{\parallel},\omega)\exp[i(\vec{k}_{\parallel}\cdot\vec{r}-\omega t)]\times\exp\left\{i\left[\left[\frac{\omega}{c_0}\right]^2-k_{\parallel}^2\right]^{1/2}z\right\}\,, \tag{2}$$

be incident on the surface at an oblique angle. The component of the real wave vector parallel to the surface has been denoted by $\vec{k}_{\parallel}$. By assuming infinitesimal translational symmetry of the solid in directions parallel to the crystal boundary, the electric fields of the reflected (r) and transmitted (t) electromagnetic waves take the forms

$$\vec{E}_r(\vec{r},t)=\vec{E}_r(k_{\parallel},\omega)\exp[i(\vec{k}_{\parallel}\cdot\vec{r}-\omega t)]\times\exp\left\{-i\left[\left[\frac{\omega}{c_0}\right]^2-k_{\parallel}^2\right]^{1/2}z\right\}\,, \tag{3}$$

and

$$\vec{E}_t(\vec{r},t)=\vec{E}_t(k_{\parallel},\omega,z)\exp[i(\vec{k}_{\parallel}\cdot\vec{r}-\omega t)]\,. \tag{4}$$

By writing the free-carrier current density on the analogous form

$$\vec{J}(\vec{r},t)=\vec{J}(k_{\parallel},\omega,z)\exp[i(\vec{k}_{\parallel}\cdot\vec{r}-\omega t)]\,, \tag{5}$$

the Fourier transform of the driven wave equation is reduced to

$$\left\{\overleftrightarrow{1}\left[\frac{d^2}{dz^2}+\left[\frac{\omega}{c_0}\right]^2-k_{\parallel}^2\right]-\vec{O}\vec{O}\right\}\cdot\vec{E}_t(k_{\parallel},\omega,z)=-i\mu_0\omega\vec{J}(k_{\parallel},\omega,z),\quad z>0 \tag{6}$$

where the operator $\vec{O}$ is given by

$$\vec{O}=i\vec{k}_{\parallel}+\vec{e}_z\frac{d}{dz}\,, \tag{7}$$

$\vec{e}_z$ being a unit vector in the z direction.

Further progress in solving Eq. (6) is obtained by deriving a constitutive relation between the free-carrier current density and the self-consistent electric field. Restricting our treatment to electromagnetic wavelengths that are long in comparison to the characteristic electron deBroglie wavelength the constitutive relation is determined by means of the Boltzmann transport equation, which under the assumptions that a relaxation-time approximation holds, and that terms involving the magnetic field are negligible, takes the form

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial \vec{r}}-\frac{e}{m^*}\vec{E}\cdot\frac{\partial f}{\partial \vec{v}}=-\frac{f-f_0}{\tau}\ , \tag{8}$$

where $-e$, m^*, and τ are the charge, the effective mass, and the momentum relaxation time of the free carriers, respectively. Note that the empty-lattice assumption is modified slightly by introducing a scalar effective mass of the conduction electrons. The electronic relaxation time can depend on the free-carrier velocity, i.e., $\tau=\tau(\vec{v})$. By making the following ansatz for the distribution function of the conduction electrons:

$$f(\vec{r},\vec{v},t)=f_0(\vec{v})+f_1(k_{||},\omega,z,\vec{v})\exp[i(\vec{k}_{||}\cdot\vec{r}-\omega t)]\ , \tag{9}$$

where $f_0(\vec{v})$ is the thermal equilibrium distribution function, the *linearized* ac part of the Fourier-transformed Boltzmann equation takes the form

$$v_z\frac{\partial f_1(k_{||},\omega,z,\vec{v})}{\partial z}+[\tau^{-1}(\vec{v})+i(\vec{k}_{||}\cdot\vec{v}-\omega)]f_1(k_{||},\omega,z,\vec{v})=\frac{e}{m^*}\vec{E}_t(k_{||},\omega,z)\cdot\frac{\partial f_0(\vec{v})}{\partial \vec{v}},\quad z>0 \tag{10}$$

v_z being given by $v_z=\vec{v}\cdot\vec{e}_z$.

Let us assume that the electrons experience specular scattering at the surface ($z=0$). Redefining $\vec{E}(z)=\vec{E}(-z)$ for $z<0$, the distribution function satisfying the boundary condition $f_1(k_{||},\omega,z\rightarrow\infty,\vec{v})=0$ is given by

$$f_1(k_{||},\omega,z,\vec{v})=-\frac{e}{m^*v_z}\frac{\partial f_0(\vec{v})}{\partial \vec{v}}\cdot\left[\int_z^{\infty}\vec{E}_t(k_{||},\omega,z')\exp\left[\frac{z'-z}{v_z}[\tau^{-1}+i(\vec{k}_{||}\cdot\vec{v}-\omega)]\right]dz'\right],\quad v_z\le 0 \tag{11}$$

$$f_1(k_{||},\omega,z,\vec{v})=\frac{e}{m^*v_z}\frac{\partial f_0(\vec{v})}{\partial \vec{v}}\cdot\left[\int_{-\infty}^{z}\vec{E}_t(k_{||},\omega,z')\exp\left[\frac{z'-z}{v_z}[\tau^{-1}+i(\vec{k}_{||}\cdot\vec{v}-\omega)]\right]dz'\right],\quad v_z>0\ . \tag{12}$$

Inserting the above expression for the free-carrier distribution function into the equation for the Fourier transform of the current density

$$\vec{J}(k_{||},\omega,z)=-e\int\int\int_{-\infty}^{\infty}\vec{v}f_1(k_{||},\omega,z,\vec{v})d^3v\ , \tag{13}$$

the nonlocal constitutive relation between $\vec{J}$ and $\vec{E}_t$ can be written

$$\vec{J}(k_{||},\omega,z)=\int_{-\infty}^{\infty}\overleftrightarrow{\sigma}(k_{||},\omega,z-z')\cdot\vec{E}_t(k_{||},\omega,z')dz',\quad z>0 \tag{14}$$

with the ij-component of the response function $\overleftrightarrow{\sigma}(k_{||},\omega,z-z')$ (or conductivity-tensor kernel) is given by

$$[\overleftrightarrow{\sigma}(k_{||},\omega,z-z')]_{ij}=[\overleftrightarrow{\sigma}_{\infty}(k_{||},\omega,|z-z'|)]_{ij}[\overleftrightarrow{M}(z-z')]_{ij}\ , \tag{15}$$

where

$$\overleftrightarrow{\sigma}_{\infty}(k_{||},\omega,|z-z'|)=-\frac{e^2}{4\pi^3}\left[\frac{m^*}{\hbar}\right]^3\int\int\int_{\vec{v},v_z>0}\frac{\vec{v}\vec{v}}{v_z}\frac{\partial f_0(\mathscr{E})}{\partial \mathscr{E}}\times\exp\left[-\frac{|z-z'|}{v_z}[\tau^{-1}+i(\vec{k}_{||}\cdot\vec{v}-\omega)]\right]d^3v\ , \tag{16}$$

and

$$\overleftrightarrow{M}(z-z')=\begin{bmatrix}1 & 1 & \delta\\ 1 & 1 & \delta\\ \delta & \delta & 1\end{bmatrix}, \tag{17}$$

with

$$\delta(z-z')=\Theta(z-z')-\Theta(z'-z), \tag{18}$$

Θ being the Heaviside unit step function. In Eq. (16) we have introduced the Fermi-Dirac distribution function

$$f_0(\mathscr{E})=\left[\exp\left[\frac{\mathscr{E}-\mu}{k_BT}\right]+1\right]^{-1}, \tag{19}$$

μ being the chemical potential, and $\mathscr{E}=\frac{1}{2}m^*v^2$ the free-carrier kinetic energy. To derive Eq. (16) use has been made of the relation

$$\frac{\partial f_0(\vec{v})}{\partial\vec{v}}=\frac{2(m^*)^4}{h^3}\vec{v}\frac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}. \tag{20}$$

It should be noted that the integration in Eq. (16) extends over half the velocity space, i.e., $v_z>0$.

The explicit expression for the Fourier integral transform of real $k_\perp$,

$$\overleftrightarrow{\sigma}(k_{||},k_\perp,\omega)=\int_{-\infty}^{\infty}\overleftrightarrow{\sigma}(k_{||},\omega,z)\exp(-ik_\perp z)dz, \tag{21}$$

is obtained via Eq. (15). Thus,

$$\begin{aligned}\overleftrightarrow{\sigma}_\infty(k_{||},k_\perp,\omega)&=\int_{-\infty}^{\infty}\overleftrightarrow{\sigma}_\infty(k_{||},\omega,z)\exp(-ik_\perp z)dz\\ &=-\frac{e^2}{4\pi^3}\left[\frac{m^*}{\hbar}\right]^3\int\int\int_{\vec{v},v_z>0}\vec{v}\vec{v}\frac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}\left[\frac{1}{\tau^{-1}(\vec{v})+i[(\vec{k}_{||}+\vec{k}_\perp)\cdot\vec{v}-\omega]}\right.\\ &\qquad\left.+\frac{1}{\tau^{-1}(\vec{v})+i[(\vec{k}_{||}-\vec{k}_\perp)\cdot\vec{v}-\omega]}\right]d^3v\end{aligned} \tag{22}$$

and

$$\begin{aligned}[\overleftrightarrow{\sigma}(k_{||},k_\perp,\omega)]_{ij}&=\int_{-\infty}^{\infty}[\overleftrightarrow{\sigma}(k_{||},\omega,z)]_{ij}\exp(-ik_\perp z)dz\\ &=-\frac{e^2}{4\pi^3}\left[\frac{m^*}{\hbar}\right]^3\int\int\int_{\vec{v},v_z>0}v_iv_j\frac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}\left[\frac{1}{\tau^{-1}(\vec{v})+i[(\vec{k}_{||}+\vec{k}_\perp)\cdot\vec{v}-\omega]}\right.\\ &\qquad\left.-\frac{1}{\tau^{-1}(\vec{v})+i[(\vec{k}_{||}-\vec{k}_\perp)\cdot\vec{v}-\omega]}\right]d^3v\\ &\qquad (i,j)=(1,3);(3,1);(2,3);(3,2).\end{aligned} \tag{23}$$

It follows by inspection of Eqs. (15), (22), and (23) that $\overleftrightarrow{\sigma}(k_{||},k_\perp,\omega)$ can be written in the compact form

$$\overleftrightarrow{\sigma}(\vec{k},\omega)=\frac{-e^2}{4\pi^3}\left[\frac{m^*}{\hbar}\right]^3\int\int\int_{-\infty}^{\infty}\frac{\frac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}\vec{v}\vec{v}\tau(\vec{v})d^3v}{1+i(\vec{k}\cdot\vec{v}-\omega)\tau(\vec{v})}, \tag{24}$$

where $\vec{k}=\vec{k}_{||}+\vec{k}_\perp$, $\vec{k}_\perp=k_\perp\vec{e}_z$.

It should be noticed that the integration extends over the entire velocity space. The result in Eq. (24) which has been obtained by a direct calculation could have been anticipated, since specular scattering of the electrons at the surface makes the semi-infinite solid-state plasma almost equivalent to an infinite extended plasma, for which an expression identical to that in Eq. (24) can easily be derived.

By rotating the coordinate system around the z axis until $\vec{k}_{||}$ lies in the xz plane, and by assuming that the electronic relaxation time depends on the energy of the free carrier only, i.e., $\tau=\tau(\mathscr{E})$, it follows readily from Eq. (24) that the conductivity tensor has the form

$$\overleftrightarrow{\sigma}(k_{||},k_{\perp},\omega)=\begin{bmatrix}\sigma_{xx}(k_{||},k_{\perp},\omega) & 0 & \sigma_{xz}(k_{||},k_{\perp},\omega)\\ 0 & \sigma_{yy}(k_{||},k_{\perp},\omega) & 0\\ \sigma_{zx}(k_{||},k_{\perp},\omega) & 0 & \sigma_{zz}(k_{||},k_{\perp},\omega)\end{bmatrix}, \quad (25)$$

and is symmetric, i.e., $\sigma_{xz}(k_{||},k_{\perp},\omega)=\sigma_{zx}(k_{||},k_{\perp},\omega)$. It should be noticed that the presence of the surface makes the conductivity *anisotropic* in the nonlocal regime.

III. ANISOTROPIC CONDUCTIVITY TENSOR OF A FULLY DEGENERATE SOLID-STATE PLASMA

In the limit where fully degenerate Fermi-Dirac statistics can be applied, i.e.,

$$\frac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}=-\delta(\mathscr{E}-\mathscr{E}_F)=\frac{-1}{m^*v_F}\delta(v-v_F)\ , \quad (26)$$

where δ is the Dirac delta function, and $\mathscr{E}_F$ and v_F are the Fermi energy and Fermi velocity of the conduction electrons, respectively, the tensor components of the conductivity given in Eqs. (24) and (25) can be expressed in terms of a single one-dimensional integral. Thus, by decomposing the wave vector into a component $(k_{||})$ parallel to the surface and a component $(k_{\perp})$ perpendicular to the surface

$$\vec{k}=k_{||}\vec{e}_x+k_{\perp}\vec{e}_z\ , \quad (27)$$

one obtains, as shown in Appendix A, if the free-carrier relaxation time is assumed to be energy independent,

$$\sigma_{xx}(k_{||},k_{\perp},\omega)=\frac{3\sigma_0 z_{||}^{-2}}{1-i\omega\tau}\left[1-\int_{-1}^{1}\frac{(c_2x+\frac{1}{2})^2dx}{(c_1x^2+c_2x+c_3)^{1/2}}\right], \quad (28)$$

$$\sigma_{yy}(k_{||},k_{\perp},\omega)=\frac{3\sigma_0 z_{||}^{-2}}{1-i\omega\tau}\left[\int_{-1}^{1}(c_1x^2+c_2x+c_3)^{1/2}dx-1\right], \quad (29)$$

$$\sigma_{zz}(k_{||},k_{\perp},\omega)=\frac{3\sigma_0}{4(1-i\omega\tau)}\int_{-1}^{1}\frac{x^2dx}{(c_1x^2+c_2x+c_3)^{1/2}}\ , \quad (30)$$

and

$$\sigma_{xz}(k_{||},k_{\perp},\omega)=\frac{3i\sigma_0 z_{||}^{-1}}{2(1-i\omega\tau)}\int_{-1}^{1}\frac{x(c_2x+\frac{1}{2})dx}{(c_1x^2+c_2x+c_3)^{1/2}}\ , \quad (31)$$

where we have introduced the abbreviations

$$c_1=-\tfrac{1}{4}(z_{||}^2+z_{\perp}^2)\ , \quad (32)$$

$$c_2=\tfrac{1}{2}iz_{\perp}\ , \quad (33)$$

and

$$c_3=\tfrac{1}{4}(1+z_{||}^2)\ . \quad (34)$$

The dimensionless complex quantitites $z_{||}$ and $z_{\perp}$ which are given by

$$z_{||}=\frac{k_{||}v_F\tau}{1-i\omega\tau}\ , \quad (35)$$

and

$$z_{\perp}=\frac{k_{\perp}v_F\tau}{1-i\omega\tau}\ , \quad (36)$$

are, as we shall see, very convenient as parameters in characterizing the qualitative behavior of the conductivity tensor. The dc free-carrier conductivity has been denoted by σ_0.

A. Perpendicular incidence

The integrals in Eqs. (28)–(31) can be solved explicitly, in the case where the electromagnetic wave is traveling perpendicular to the surface. Thus, for $k_{\parallel}\to 0$ (or equivalently, $z_{\parallel}\to 0$) one gets

$$(c_1x^2+c_2x+c_3)^{1/2}\cong c_2x+\tfrac{1}{2}+\frac{1-x^2}{1+2c_2x}\frac{z_{\parallel}^2}{4} . \quad (37)$$

By substituting this expression into Eqs. (28)–(31), performing the integrations and taking the limit $z_{\parallel}\to 0$ one obtains the longitudinal response function

$$\sigma_{zz}(k_{\perp},\omega)=\frac{3\sigma_0}{(1-i\omega\tau)z_{\perp}^3}(z_{\perp}-\arctan z_{\perp}) , \quad (38)$$

and the transverse response function

$$\sigma_{xx}(k_{\perp},\omega)=\sigma_{yy}(k_{\perp},\omega)=\frac{3\sigma_0}{2(1-i\omega\tau)z_{\perp}^3}\times[(1+z_{\perp}^2)\arctan z_{\perp}-z_{\perp}] , \quad (39)$$

in agreement with the results obtained by Lindhard.[17] No mixing of longitudinal and transverse modes occur at perpendicular incidence since

$$\sigma_{xz}(k_{\perp},\omega)=0 , \quad (40)$$

as expected.

In the local limit ($z_{\perp}\to 0$) we retain the isotropic result

$$\sigma_{xx}(\omega)=\sigma_{yy}(\omega)=\sigma_{zz}(\omega)=\sigma_0/(1-i\omega\tau) .$$

B. Long-wavelength region

Of special importance for plasma-optical experiments at oblique incidence is the behavior of the conductivity tensor at long wavelengths ($|z_{\perp}|,|z_{\parallel}|<<1$). In this case explicit expressions can be derived by making a Taylor expansion on the right-hand side of the equation

$$(c_1x^2+c_2x+c_3)^{1/2}=\tfrac{1}{2}[1+2iz_{\perp}x+z_{\parallel}^2-(z_{\perp}^2+z_{\parallel}^2)x^2]^{1/2} \quad (41)$$

and then performing the integrations in Eqs. (28)–(31). Thus, after somewhat tedious, but straightforward calculations one obtains to second order in $z_{\perp}$, $z_{\parallel}$, and $(z_{\perp}z_{\parallel})^{1/2}$ the result

$$\overleftrightarrow{\sigma}(k_{\parallel},k_{\perp},\omega)=\begin{bmatrix} 1-\frac{1}{5}(3z_{\parallel}^2+z_{\perp}^2) & 0 & -\frac{2}{5}z_{\parallel}z_{\perp} \\ 0 & 1-\frac{1}{5}(z_{\parallel}^2+z_{\perp}^2) & 0 \\ -\frac{2}{5}z_{\parallel}z_{\perp} & 0 & 1-\frac{1}{5}(z_{\parallel}^2+3z_{\perp}^2) \end{bmatrix}\frac{\sigma_0}{1-i\omega\tau} . \quad (42)$$

In the limit of local optics the anisotropies of the conductivity stemming from the presence of the boundary and from the distinction between longitudinal and transverse modes disappear.

IV. REFLECTED AND TRANSMITTED ELECTROMAGNETIC FIELDS AT OBLIQUE INCIDENCE

To determine the transmitted and the reflected electromagnetic fields, it follows by combining Eqs. (6) and (14) that one has to solve the integro-differential equation

$$\overleftrightarrow{\mathscr{L}}\left[k_{\parallel},\omega,\frac{d}{dz},\frac{d^2}{dz^2}\right]\cdot\vec{E}_t(k_{\parallel},\omega,z)=-i\mu_0\omega\int_{-\infty}^{\infty}\overleftrightarrow{\sigma}(k_{\parallel},\omega,z-z')\cdot\vec{E}_t(k_{\parallel},\omega,z')dz' , \quad (43)$$

where the linear tensorial operator $\overleftrightarrow{\mathscr{L}}$ is given by

$$\overleftrightarrow{\mathscr{L}}\left[k_{\parallel},\omega,\frac{d}{dz},\frac{d^2}{dz^2}\right]=\overleftrightarrow{1}\left[\frac{d^2}{dz^2}+\left[\frac{\omega}{c_0}\right]^2-k_{\parallel}^2\right]-\vec{O}\vec{O} . \quad (44)$$

Now, Eq. (43) implies that

$$\int_{-\infty}^{\infty}\overleftrightarrow{\mathscr{L}}\left[k_{||},\omega,\frac{d}{dz},\frac{d^2}{dz^2}\right]\cdot\vec{E}_t(k_{||},\omega,z)\exp(-ik_\perp z)dz$$
$$=-i\mu_0\omega\int_{-\infty}^{\infty}\exp(-ik_\perp z)\int_{-\infty}^{\infty}\overleftrightarrow{\sigma}(k_{||},\omega,z-z')\cdot\vec{E}_t(k_{||},\omega,z')dz'\,dz\;. \tag{45}$$

By integrating by parts the left-hand side of this equation once and twice, and by inverting the order of integration on the right-hand side one obtains in matrix notation the following set of inhomogeneous algebraic equations for the Fourier amplitudes of the electric field:

$$\left\{\left[\left[\frac{\omega}{c_0}\right]^2-k^2\right]\overleftrightarrow{1}+i\mu_0\omega\overleftrightarrow{\sigma}(k_{||},k_\perp,\omega)+\vec{k}\vec{k}\right\}\cdot\vec{E}_t(k_{||},k_\perp,\omega)=2\begin{bmatrix}\dfrac{\partial E_{t,x}(k_{||},\omega,z\to 0+)}{\partial z}\\ \dfrac{\partial E_{t,y}(k_{||},\omega,z\to 0+)}{\partial z}\\ 0\end{bmatrix}. \tag{46}$$

Thus, it is realized that the total electric field in the solid-state plasma is given by

$$\vec{E}_t(\vec{r},t)=\frac{1}{2\pi}\exp[i(\vec{k}_{||}\cdot\vec{r}-\omega t)]\int_{-\infty}^{\infty}\vec{E}_t(k_{||},k_\perp,\omega)\exp(ik_\perp z)dk_\perp\;, \tag{47}$$

where

$$\vec{E}_t(k_{||},k_\perp,\omega)=2\left\{\left[\left[\frac{\omega}{c_0}\right]^2-k^2\right]\overleftrightarrow{1}+\vec{k}\vec{k}+i\mu_0\omega\overleftrightarrow{\sigma}(k_{||},k_\perp,\omega)\right\}^{-1}\cdot\begin{bmatrix}\dfrac{\partial E_{t,x}(k_{||},\omega,z\to 0+)}{\partial z}\\ \dfrac{\partial E_{t,y}(k_{||},\omega,z\to 0+)}{\partial z}\\ 0\end{bmatrix}. \tag{48}$$

The unknown field derivatives $\partial E_{t,x}(k_{||},\omega,z\to 0^+)/\partial z$ and $\partial E_{t,y}(k_{||},\omega,z\to 0^+)/\partial z$, and the Fourier amplitude of the reflected field $\vec{E}_r(k_{||},\omega)$ can be determined in terms of the amplitude of the incident electric field via the usual boundary conditions for the electromagnetic field at a sharp, nonmoving surface. By decomposing the incident and the reflected fields into their TE and TM components, continuity of the tangential component of the electric field implies the following relations among the Fourier amplitudes:

$$E_i^{\mathrm{TE}}(k_{||},\omega)+E_r^{\mathrm{TE}}(k_{||},\omega)=\frac{1}{2\pi}\vec{e}_y\cdot\left[\int_{-\infty}^{\infty}\vec{E}_t(k_{||},k_\perp,\omega)dk_\perp\right], \tag{49}$$

and

$$[E_i^{\mathrm{TM}}(k_{||},\omega)-E_r^{\mathrm{TM}}(k_{||},\omega)]\left[1-\left[\frac{c_0k_{||}}{\omega}\right]^2\right]^{1/2}=\frac{1}{2\pi}\vec{e}_x\cdot\left[\int_{-\infty}^{\infty}\vec{E}_t(k_{||},k_\perp,\omega)dk_\perp\right], \tag{50}$$

where $\vec{e}_i$ denotes a unit vector along the Cartesian axis $i=x,y$. The continuity of the tangential component of the magnetic field

$$\vec{B}=(i\omega)^{-1}\vec{\nabla}\times\vec{E}=(i\omega)^{-1}\left[-\frac{\partial E_y}{\partial z}\vec{e}_x-\left[\frac{\partial E_z}{\partial x}-\frac{\partial E_x}{\partial z}\right]\vec{e}_y+\frac{\partial E_y}{\partial x}\vec{e}_z\right]$$

across the boundary gives by means of Eqs. (2), (3), and (47) the conditions

$$[E_i^{\mathrm{TE}}(k_{||},\omega)-E_r^{\mathrm{TE}}(k_{||},\omega)]\left[\left[\frac{\omega}{c_0}\right]^2-k_{||}^2\right]^{1/2}=\frac{1}{2\pi}\vec{e}_y\cdot\left[\int_{-\infty}^{\infty}\vec{E}_t(k_{||},k_\perp,\omega)k_\perp dk_\perp\right], \tag{51}$$

and

$$\frac{\omega}{c_0}[E_i^{\mathrm{TM}}(k_{||},\omega)+E_r^{\mathrm{TM}}(k_{||},\omega)]=\frac{1}{2\pi}\int_{-\infty}^{\infty}(k_\perp\vec{e}_x-k_{||}\vec{e}_z)\cdot\vec{E}_t(k_{||},k_\perp,\omega)dk_\perp\;. \tag{52}$$

For shortness, we introduce

$$\overleftrightarrow{\Xi}(\vec{k},\omega)=2\left\{\left[\left[\frac{\omega}{c_0}\right]^2-k^2\right]\overleftrightarrow{1}+\vec{k}\vec{k}+i\mu_0\omega\overleftrightarrow{\sigma}(\vec{k},\omega)\right\}^{-1}, \tag{53}$$

and note that $\overleftrightarrow{\Xi}$ has the general matrix form

$$\overleftrightarrow{\Xi}(\vec{k},\omega)=\begin{bmatrix}\Xi_{xx} & 0 & \Xi_{xz}\\ 0 & \Xi_{yy} & 0\\ \Xi_{zx} & 0 & \Xi_{zz}\end{bmatrix}, \tag{54}$$

where $\Xi_{xz}=\Xi_{zx}$. In terms of $\overleftrightarrow{\Xi}$ the transmitted field in Eq. (48) takes the form

$$\vec{E}_t(k_{\parallel},k_{\perp},\omega)=\begin{bmatrix}\Xi_{xx}(\vec{k},\omega)\dfrac{\partial E_{t,x}(k_{\parallel},\omega,z\to 0+)}{\partial z}\\ \Xi_{yy}(\vec{k},\omega)\dfrac{\partial E_{t,y}(k_{\parallel},\omega,z\to 0+)}{\partial z}\\ \Xi_{xz}(\vec{k},\omega)\dfrac{\partial E_{t,x}(k_{\parallel},\omega,z\to 0+)}{\partial z}\end{bmatrix}. \tag{55}$$

It follows directly from inspection of Eqs. (49)–(52) and Eq. (55) that $E_r^{\rm TE}(k_{\parallel},\omega)$ and $\partial E_{t,y}(k_{\parallel},\omega,z\to 0+)/\partial z$ are determined by solving Eqs. (49) and (51) separately, and $E_r^{\rm TM}(k_{\parallel},\omega)$ and $\partial E_{t,x}(k_{\parallel},\omega,z\to 0+)/\partial z$ by solving Eqs. (50) and (52). Thus, as one would expect, the components of the electric field parallel and perpendicular to the plane of incidence are uncoupled. For TE-mode propagation the final expression for the amplitude reflection coefficient ($r^{\rm TE}$) becomes

$$r^{\rm TE}(k_{\parallel},\omega)\equiv\frac{E_r^{\rm TE}}{E_i^{\rm TE}}=\frac{\displaystyle\int_{-\infty}^{\infty}\left[1-\frac{k_\perp}{k_\perp^0}\right]\Xi_{yy}(k_{\parallel},k_\perp,\omega)dk_\perp}{\displaystyle\int_{-\infty}^{\infty}\left[1+\frac{k_\perp}{k_\perp^0}\right]\Xi_{yy}(k_{\parallel},k_\perp,\omega)dk_\perp}, \tag{56}$$

where $k_\perp^0=[(\omega/c_0)^2-k_{\parallel}^2]^{1/2}$. On normalized form the corresponding *field gradient* is given by

$$t^{\rm TE}(k_{\parallel},\omega)\equiv\frac{\dfrac{\partial E_{t,y}(k_{\parallel},\omega,z\to 0+)}{\partial z}}{E_i^{\rm TE}}=\frac{4\pi}{\displaystyle\int_{-\infty}^{\infty}\left[1+\frac{k_\perp}{k_\perp^0}\right]\Xi_{yy}(k_{\parallel},k_\perp,\omega)dk_\perp}. \tag{57}$$

For TM-mode propagation the amplitude reflection coefficient takes the form

$$r^{\rm TM}(k_{\parallel},\omega)\equiv\frac{E_r^{\rm TM}}{E_i^{\rm TM}}=\frac{\displaystyle\int_{-\infty}^{\infty}\left[\left[\frac{c_0k_\perp}{\omega}-\frac{\omega}{c_0k_\perp^0}\right]\Xi_{xx}(k_{\parallel},k_\perp,\omega)-\frac{c_0k_{\parallel}}{\omega}\Xi_{xz}(k_{\parallel},k_\perp,\omega)\right]dk_\perp}{\displaystyle\int_{-\infty}^{\infty}\left[\left[\frac{c_0k_\perp}{\omega}+\frac{\omega}{c_0k_\perp^0}\right]\Xi_{xx}(k_{\parallel},k_\perp,\omega)-\frac{c_0k_{\parallel}}{\omega}\Xi_{xz}(k_{\parallel},k_\perp,\omega)\right]dk_\perp}, \tag{58}$$

and the appropriate normalized *field gradient* is given by

$$t^{\rm TM}(k_{\parallel},\omega)\equiv\frac{\dfrac{\partial E_{t,x}(k_{\parallel},\omega,z\to 0+)}{\partial z}}{E_i^{\rm TM}}=\frac{4\pi}{\displaystyle\int_{-\infty}^{\infty}\left[\left[\frac{c_0k_\perp}{\omega}+\frac{\omega}{c_0k_\perp^0}\right]\Xi_{xx}(k_{\parallel},k_\perp,\omega)-\frac{c_0k_{\parallel}}{\omega}\Xi_{xz}(k_{\parallel},k_\perp,\omega)\right]dk_\perp}. \tag{59}$$

V. POYNTING VECTORS OF THE SOLID-STATE PLASMA AND THE ELECTROMAGNETIC FIELD

A. General considerations

In a spatially dispersive solid-state plasma the total energy-flux density vector $\vec{S}$ is composed of a contribution from the electromagnetic field and a contribution from the conduction-electron system, i.e.,

$$\vec{S}(\vec{r},t)=\vec{S}_t^e(\vec{r},t)+\vec{S}^f(\vec{r},t) \tag{60}$$

in the empty-lattice approximation. Of practical interest for monochromatic modes is the cycle-average Poynting vector

$$\langle \vec{S} \rangle_T \equiv \vec{S}(k_{\|},\omega,z) .$$

The contribution from the solution [Eq. (9)] of the *linearized* Boltzmann equation to the cycle-average plasma Poynting vector is zero. Thus, to calculate the energy flow in the plasma one cannot neglect the nonlinearities in the transport equation. In the present work we shall examine the energy transport by retaining in the Boltzmann equation the contribution from the nonlinear term

$$\vec{E}_t(\vec{r},t)\cdot\partial f_1(\vec{r},\vec{v},t)/\partial\vec{v}$$

only. Thus second and higher harmonics in the free-carrier distribution function and the electric field are neglected. In consequence of the above considerations it follows that the cycle-average free-carrier Poynting vector is given by

$$\langle \vec{S}^f(\vec{r},t)\rangle_T \equiv \vec{S}^f(k_{\|},\omega,z)=\frac{m^*}{2}\int\int\int_{-\infty}^{\infty} v^2\vec{v} f_{\mathrm{dc}}^{\mathrm{NL}}(k_{\|},\omega,z,\vec{v})d^3v , \tag{61}$$

where the *nonlinear* (NL) dc contribution to the free-carrier distribution function is

$$f_{\mathrm{dc}}^{\mathrm{NL}}(k_{\|},\omega,z,\vec{v})=\frac{e\tau}{4m^*}\vec{E}_t^*(k_{\|},\omega,z)\cdot\vec{\nabla}_v f_1(k_{\|},\omega,z,\vec{v})+\mathrm{c.c.} \tag{62}$$

By inserting Eq. (62) into Eq. (61) and integrating by parts one obtains, assuming τ to be velocity independent, the alternative expression

$$\vec{S}^f(k_{\|},\omega,z)=-\frac{e\tau}{4}\vec{E}_t^*(k_{\|},\omega,z)\cdot\left[\int\int\int_{-\infty}^{\infty}\left[\frac{v^2}{2}\overleftrightarrow{1}+\vec{v}\vec{v}\right]f_1(k_{\|},\omega,z,\vec{v})d^3v\right]+\mathrm{c.c.} \tag{63}$$

To proceed further we transform the expressions for $f_1(k_{\|},\omega,z,\vec{v})$ given in Eqs. (11) and (12) by inserting in these the Fourier integral transformations of $\vec{E}_t(k_{\|},\omega,z')$. By interchanging then the order of integration and performing the straightforward integration over z', and by making use of Eq. (20), one gets for *all* $\vec{v}$ the following expression for the ac part of the conduction-electron distribution function:

$$f_1(k_{\|},\omega,z,\vec{v})=\frac{e\tau}{4\pi^3}\left[\frac{m^*}{\hbar}\right]^3\frac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}\vec{v}\cdot\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\vec{E}_t(k_{\|},k_{\perp},\omega)\exp(ik_{\perp}z)dk_{\perp}}{1+i(\vec{k}\cdot\vec{v}-\omega)\tau(\vec{v})}\right] . \tag{64}$$

Finally, by combining Eqs. (63) and (64) the time-average material Poynting vector can be written

$$\vec{S}^f(k_{\|},\omega,z)=-\frac{e^2\tau^2(m^*)^3}{8\pi^2h^3}\int\int_{-\infty}^{\infty}\vec{E}_t^*(k_{\|},k_{\perp}',\omega)\cdot\overleftrightarrow{T}(k_{\|},k_{\perp},\omega)\cdot\vec{E}_t(k_{\|},k_{\perp},\omega)$$
$$\times\exp\{i[k_{\perp}-(k_{\perp}')^*]z\}dk_{\perp}'dk_{\perp}+\mathrm{c.c.} , \tag{65}$$

where the third-order tensor $\overleftrightarrow{T}$ is given by

$$\overleftrightarrow{T}(k_{\|},k_{\perp},\omega)=\int\int\int_{-\infty}^{\infty}\frac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}\frac{\frac{v^2}{2}\vec{v}\overleftrightarrow{1}+\vec{v}\vec{v}\vec{v}}{1+i(\vec{k}\cdot\vec{v}-\omega)\tau}d^3v \ . \tag{66}$$

Note that $\overleftrightarrow{T}_{ijk}=\overleftrightarrow{T}_{ikj}$.

The cycle-averaged electromagnetic Poynting vector of the transmitted field is given by the expression

$$\langle\vec{S}_t^e(\vec{r},t)\rangle_T\equiv\vec{S}_t^e(k_{\|},\omega,z)=\tfrac{1}{4}\vec{E}_t^*(k_{\|},\omega,z)\times\vec{H}_t(k_{\|},\omega,z)+\text{c.c.} \ , \tag{67}$$

where

$$\vec{E}_t(k_{\|},\omega,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{E}_t(k_{\|},k_{\perp},\omega)\exp(ik_{\perp}z)dk_{\perp} \ , \tag{68}$$

and

$$\vec{H}_t(k_{\|},\omega,z)=\frac{k_{\|}}{\mu_0\omega}(\vec{e}_z\vec{e}_y-\vec{e}_y\vec{e}_z)\cdot\vec{E}_t(k_{\|},\omega,z) +\frac{1}{2\pi\mu_0\omega}(\vec{e}_y\vec{e}_x-\vec{e}_x\vec{e}_y)\cdot\left[\int_{-\infty}^{\infty}\vec{E}_t(k_{\|},k_{\perp},\omega)k_{\perp}\exp(ik_{\perp}z)dk_{\perp}\right] . \tag{69}$$

B. TE-mode propagation

Let us assume that a TE mode is incident on the plasma. By inserting

$$\vec{E}_t(k_{\|},k_{\perp},\omega)=\Xi_{yy}(\vec{k},\omega)t^{\text{TE}}(k_{\|},\omega)E_i^{\text{TE}}(k_{\|},\omega)\vec{e}_y$$

into Eqs. (68) and (69) one obtains via Eq. (67) the following expression for the cycle-averaged electromagnetic Poynting vector of the transmitted field:

$$\vec{S}_t^{e,\text{TE}}(k_{\|},\omega,z)=\frac{|t^{\text{TE}}(k_{\|},\omega)|^2|E_i^{\text{TE}}(k_{\|},\omega)|^2}{(4\pi)^2\mu_0\omega} \times\int\int_{-\infty}^{\infty}(k_{\|}\vec{e}_x+k_{\perp}\vec{e}_z)\Xi_{yy}^*(k_{\|},k_{\perp}',\omega)\Xi_{yy}(k_{\|},k_{\perp},\omega)\exp\{i[k_{\perp}-(k_{\perp}')^*]z\}dk_{\perp}'dk_{\perp}+\text{c.c.} \tag{70}$$

The cycle-averaged plasma Poynting vector corresponding to TE-mode propagation can readily be obtained by combining Eqs. (65) and (66). Thus, one gets

$$\vec{S}^{f,\text{TE}}(k_{\|},\omega,z)=-\frac{e^2\tau^2(m^*)^3}{8\pi^2h^3}|t^{\text{TE}}(k_{\|},\omega)|^2|E_i^{\text{TE}}(k_{\|},\omega)|^2 \times\int\int_{-\infty}^{\infty}\Xi_{yy}^*(k_{\|},k_{\perp}',\omega)\Xi_{yy}(k_{\|},k_{\perp},\omega)\vec{W}^{yy}(k_{\|},k_{\perp},\omega)\exp\{i[k_{\perp}-(k_{\perp}')^*]z\}dk_{\perp}'dk_{\perp}+\text{c.c.} \ , \tag{71}$$

where

$$\vec{W}_{yy}=\int\int\int_{-\infty}^{\infty}\frac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}\frac{v_y^2(v_x\vec{e}_x+v_z\vec{e}_z)d^3v}{1+i(\vec{k}\cdot\vec{v}-\omega)\tau} \ . \tag{72}$$

As expected, it is realized from Eqs. (70)—(72) that the Poynting vectors are confined to the xz plane.

VI. EXAMPLE: TE-MODE PROPAGATION IN A FULLY DEGENERATE SOLID-STATE PLASMA IN THE NEAR-LOCAL REGIME

In the following, the general theory described in the preceding sections is applied to a study of the amplitude reflection coefficient (r^{TE}), the normalized field gradient (t^{TE}) at the surface, and the cycle-averaged electromagnetic ($\vec{S}_t^{e,\mathrm{TE}}$) and plasma ($\vec{S}^{f,\mathrm{TE}}$) Poynting vectors in the case where an electromagnetic TE mode is incident on the surface at an oblique angle. The main emphasis will be given to a calculation of the above quantities in the limit where nonlocal transport effects are included in lowest order. It will be assumed that fully degenerate Fermi-Dirac statistics can be applied.

A. The transmitted electromagnetic field

The transmitted field is given by

$$\vec{E}_t^{\mathrm{TE}}(k_{\|},\omega,z)=\vec{e}_y E_i^{\mathrm{TE}}(k_{\|},\omega)t^{\mathrm{TE}}(k_{\|},\omega)(2\pi)^{-1}\int_{-\infty}^{\infty}\Xi_{yy}(k_{\|},k_{\perp},\omega)\exp(ik_{\perp}z)dk_{\perp},\quad z>0 \tag{73}$$

where the normalized field gradient (t^{TE}) is given by Eq. (57). The function Ξ_{yy} is readily obtained by combining Eqs. (25) and (53). Thus,

$$\Xi_{yy}(k_{\|},k_{\perp},\omega)=\frac{2}{\left[\dfrac{\omega}{c_0}\right]^2-k_{\|}^2-k_{\perp}^2+i\mu_0\omega\sigma_{yy}(k_{\|},k_{\perp},\omega)}\ . \tag{74}$$

The integral along the real $k_{\perp}$ axis in Eq. (73), in principle, can be evaluated by contour integration in the upper part ($\mathrm{Im}k_{\perp}>0$) of the complex $k_{\perp}$ plane (see Fig. 1). The poles of the integrand, which for a given frequency ω contribute in the residue calculation, are determined by the dispersion relation for TE-mode propagation in an unbounded medium, i.e.,

$$\left[\frac{c_0 k}{\omega}\right]^2=1+i\frac{\sigma_{yy}(\vec{k},\omega)}{\epsilon_0\omega} \tag{75}$$

in combination with $k_{\perp}=(k^2-k_{\|}^2)^{1/2}$. By denoting the complex wave numbers obtained from Eq. (75),

$$k_{\perp,n}^{\mathrm{res}}=[(k_n^{\mathrm{res}})^2-k_{\|}^2]^{1/2},\quad (\mathrm{Im}k_{\perp,n}^{\mathrm{res}}>0)$$

the transmitted electromagnetic field becomes

$$\vec{E}_t^{\mathrm{TE}}(\vec{r},t)=\vec{e}_y E_i^{\mathrm{TE}}(k_{\|},\omega)t^{\mathrm{TE}}(k_{\|},\omega)\times\frac{1}{2\pi}\sum_n a_n(k_{\|},\omega)\exp[-(\mathrm{Im}k_{\perp,n}^{\mathrm{res}})z]\exp\{i[(\vec{k}_{\|}+\vec{e}_z\mathrm{Re}k_{\perp,n}^{\mathrm{res}})\cdot\vec{r}-\omega t]\}\ , \tag{76}$$

where the amplitudes $a_n(k_{\|},\omega)$ in the *mode expansion* (see Fig. 2) of the transmitted electromagnetic field are given by

$$a_n(k_{\|},\omega)=4\pi i\lim_{k_{\perp}\to k_{\perp,n}^{\mathrm{res}}}\frac{k_{\perp}-k_{\perp,n}^{\mathrm{res}}}{\left[\dfrac{\omega}{c_0}\right]^2\left[1+\dfrac{i\sigma_{yy}(k_{\|},k_{\perp},\omega)}{\epsilon_0\omega}\right]-k_{\|}^2-k_{\perp}^2} \tag{77}$$

assuming simple first-order poles.

Let us consider the mode expansion close to the local region in the fully degenerate plasma, where according to Eq. (42) one has

$$\sigma_{yy}(k_{\|},k_{\perp},\omega)=\sigma_0[1-\tfrac{1}{5}(z_{\|}^2+z_{\perp}^2)]/(1-i\omega\tau)\ .$$

The explicit expressions for r^{TE}, t^{TE}, $k_{\perp,n}^{\mathrm{res}}$ and a_n can be derived in this near-local regime by noting that the poles are determined by the equation

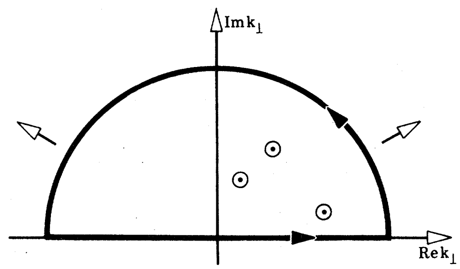

FIG. 1. Contour integration along an expanding semicircle in the upper half ($\mathrm{Im}k_\perp > 0$) of the complex $k_\perp$ plane. The poles are determined by the dispersion relation for electromagnetic-mode propagation in an unbounded medium, and lie in the first quadrant, because the amplitude attenuation coefficients and the real parts of the wave vectors are positive.

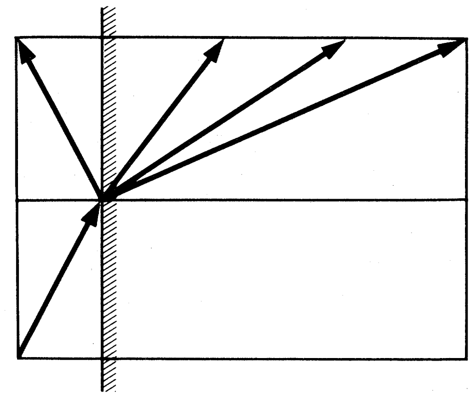

FIG. 2. Mode expansion of the transmitted electromagnetic field in a case where, for a given frequency, three wave vectors are obtained from the dispersion relation for TE-mode propagation. Note that the components of the real parts of the incident, reflected, and transmitted wave vectors parallel to the surface are equal.

$$(k_\perp^{\mathrm{res}})^2=\frac{A(k_{\parallel},\omega)}{B(\omega)}, \quad \mathrm{Im}k_\perp^{\mathrm{res}}>0 \tag{78}$$

where

$$A(k_{\parallel},\omega)=\left[\frac{\omega}{c_0}\right]^2-k_{\parallel}^2+i\mu_0\omega\frac{\sigma_0}{1-i\omega\tau}\left[1-\frac{1}{5}\left[\frac{k_{\parallel}v_F\tau}{1-i\omega\tau}\right]^2\right], \tag{79}$$

and

$$B(\omega)=1+\frac{i}{5}\frac{\mu_0\omega\sigma_0(v_F\tau)^2}{(1-i\omega\tau)^3} . \tag{80}$$

Contour integration along an expanding semicircle in the upper half plane (see Fig. 1) applied to Eqs. (56) and (57) shows that the amplitude reflection coefficient is given by

$$r^{\mathrm{TE}}(k_{\parallel},\omega)=\frac{\left[\left[\frac{\omega}{c_0}\right]^2-k_{\parallel}^2\right]^{1/2}-k_\perp^{\mathrm{res}}}{\left[\left[\frac{\omega}{c_0}\right]^2-k_{\parallel}^2\right]^{1/2}+k_\perp^{\mathrm{res}}} . \tag{81}$$

and the normalized field gradient at the surface is given by

$$t^{\mathrm{TE}}(k_{\parallel},\omega)=2i(AB)^{1/2}\frac{\left[\left[\frac{\omega}{c_0}\right]^2-k_{\parallel}^2\right]^{1/2}}{\left[\left[\frac{\omega}{c_0}\right]^2-k_{\parallel}^2\right]^{1/2}+k_\perp^{\mathrm{res}}} . \tag{82}$$

In the present case the transmitted field consists of a single mode. Thus,

$$\vec{\mathrm{E}}_t^{\mathrm{TE}}(\vec{r},t)=\frac{t^{\mathrm{TE}}E_i^{\mathrm{TE}}}{i(AB)^{1/2}}\exp(-z\,\mathrm{Im}k_\perp^{\mathrm{res}})\exp\{i[(\vec{k}_{\parallel}+\vec{e}_z\mathrm{Re}k_\perp^{\mathrm{res}})\cdot\vec{r}-\omega t]\} . \tag{83}$$

It is a straightforward matter to show that one obtains for the amplitude reflection (r^{TE}) and transmission $[t^{\mathrm{TE}}/(iA^{1/2}B^{1/2})]$ coefficients the well-known results from local optics[18] in the limit $l=v_F\tau\to 0$.

B. Poynting vectors of the plasma and the electromagnetic field

The general expression for the cycle-averaged electromagnetic energy flux in the plasma in the case of TE-mode propagation in the near-local region is obtained from Eq. (70) by contour integrations (see Fig. 1) quite analogous to those leading to the results of Eqs. (81)–(83). Thus, one obtains

$$\vec{S}_t^{e,\mathrm{TE}}(k_{||},\omega,z)=\frac{2\,|E_i^{\mathrm{TE}}(k_{||},\omega)|^2}{\mu_0\omega}\frac{(k_\perp^0)^2}{(k_\perp^0+\mathrm{Re}k_\perp^{\mathrm{res}})^2+(\mathrm{Im}k_\perp^{\mathrm{res}})^2}[(\vec{e}_x k_{||}+\vec{e}_z\mathrm{Re}k_\perp^{\mathrm{res}})\exp(-2z\,\mathrm{Im}k_\perp^{\mathrm{res}})]\ , \qquad (84)$$

In the local limit $l\rightarrow 0$ the above expression for the electromagnetic energy flux reduces to the well-known elementary result.[19]

To determine the mass flow an explicit calculation of $\vec{W}^{yy}=W_x^{yy}\vec{e}_x + W_z^{yy}\vec{e}_z$ for a fully degenerate solid-plasma, i.e.,

$$\partial f_0(\mathscr{E})/\partial\mathscr{E}=-\delta(\mathscr{E}-\mathscr{E}_F)$$

has to be carried out. This tedious calculation, of which a few details are pointed out in Appendix B, leads to the following final expressions:

$$W_x^{yy}(k_{||},k_\perp,\omega)=\frac{4\pi v_F^4}{im^* z_{||}^3(1-i\omega\tau)}\left[\int_{-1}^{1}(1+2c_2x)(c_1x^2+c_2x+c_3)^{1/2}dx+\tfrac{1}{3}(z_\perp^2-z_{||}^2)-1\right]\ , \qquad (85)$$

and

$$W_z^{yy}(k_{||},k_\perp,\omega)=\frac{4\pi v_F^4}{m^* z_{||}^2(1-i\omega\tau)}\left[\frac{i}{3}z_\perp-\int_{-1}^{1}x(c_1x^2+c_2x+c_3)^{1/2}dx\right]\ , \qquad (86)$$

where the abbreviations introduced in Eqs. (32)–(36) have been used.

In the near-local regime $|z_\perp|,\ |z_{||}|<<1$ one can make a Taylor expansion of the square roots in Eqs. (85) and (86) as shown in Eq. (41). Doing this one obtains to lowest order in the nonlocal effects

$$W_x^{yy}(k_{||},k_\perp,\omega)=\frac{4\pi i}{15}\frac{v_F^4 z_{||}}{m^*(1-i\omega\tau)}+O(|z_{||}|^2,|z_\perp|^2)\ , \qquad (87)$$

and

$$W_z^{yy}(k_{||},k_\perp,\omega)=\frac{4\pi i}{15}\frac{v_F^4 z_\perp}{m^*(1-i\omega\tau)}+O(|z_{||}|^2,|z_\perp|^2)\ . \qquad (88)$$

Two features should be remarked about these results. The one is that W_x^{yy} and W_z^{yy} are of *first order* in the nonlocal parameters $z_\perp$ and $z_{||}$, in contrast to the nonlocal corrections to the conductivity tensor [see Eq. (42)], which are of *second order* in these parameters. The second thing is that W_x^{yy} depends on $k_{||}$, and W_z^{yy} on $k_\perp$, only.

The integrals in Eqs. (85) and (86) can be evaluated exactly in the case where the electromagnetic wave penetrates perpendicular into the plasma. Thus, for $k_{||}\rightarrow 0$ (i.e., $z_{||}\rightarrow 0$) one obtains, by making use of the expansion in Eq. (37), $W_x^{yy}=0$, as expected, and

$$W_z^{yy}(k_\perp,\omega)=\frac{2\pi i v_F^4}{m^*(1-i\omega\tau)z_\perp^4}[\tfrac{2}{3}z_\perp^3+z_\perp-(1+z_\perp^2)\arctan z_\perp]\ . \qquad (89)$$

Finally, by inserting Eqs. (87) and (88) into Eq. (71) one obtains after contour integrations the plasma Poynting vector

$$\vec{S}^{f,\mathrm{TE}}(k_{||},\omega,z)=\frac{16\pi}{15}\frac{e^2(m^*)^2\tau^3v_F^5}{h^3[1+(\omega\tau)^2]^2}|E_i^{\mathrm{TE}}(k_{||},\omega)|^2\frac{(k_\perp^0)^2}{(k_\perp^0+\mathrm{Re}k_\perp^{\mathrm{res}})^2+(\mathrm{Im}k_\perp^{\mathrm{res}})^2}\exp(-2z\,\mathrm{Im}k_\perp^{\mathrm{res}})$$

$$\times(2\omega\tau k_{||}\vec{e}_x+\{2\omega\tau\,\mathrm{Re}k_\perp^{\mathrm{res}}+[1-(\omega\tau)^2]\mathrm{Im}k_\perp^{\mathrm{res}}\}\vec{e}_z)\ , \qquad (90)$$

C. Angular splitting of the electromagnetic and the material Poynting vectors

Since $\mathrm{Re}k_\perp^{\mathrm{res}} > 0$, it follows from Eq. (84) that the component of the transmitted, cycle-averaged electromagnetic energy flux perpendicular to the surface is *always* directed *away* from the surface. This result is, of course, not restricted to the near-local regime, but holds in general. In contrast, the component of the time-averaged material energy flux perpendicular to the surface can have either sign. Thus, if the inequality

$$\frac{\mathrm{Re}k_\perp^{\mathrm{res}}}{\mathrm{Im}k_\perp^{\mathrm{res}}} < \frac{(\omega\tau)^2-1}{2\omega\tau} \tag{91}$$

holds, the material energy flow is directed *towards* the surface at an oblique angle, in general. If the above inequality is not fulfilled the plasma energy flow is *away* from the boundary. Let us choose $k_\parallel > 0$. The angular deviation of the Poynting vectors is thus

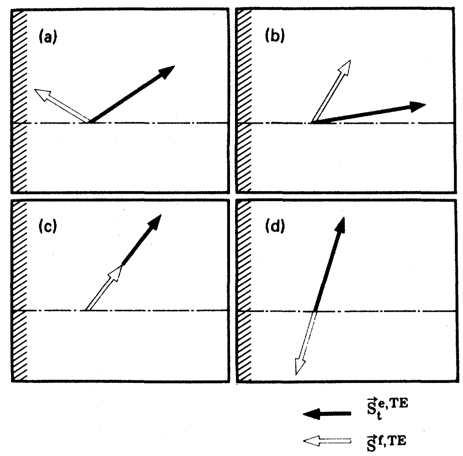

FIG. 3. Schematic diagrams showing the Poynting vectors of the transmitted electromagnetic TE field and the solid-state plasma in some characteristic cases. (a) Angular splitting of the Poynting vectors implies that the sharp surface acts as a sink of energy, whereas in (b) it acts as a source. (c) Two Poynting vectors are shown in a collinear configuration which, in the near-local regime, corresponds to $\omega\tau=1$. (d) Geometry is anticollinear.

$$\Delta \equiv \theta^f - \theta^e = \arctan\left[\frac{k_\parallel[1-(\omega\tau)^2]\mathrm{Im}k_\perp^{\mathrm{res}}}{2[k_\parallel^2+(\mathrm{Re}k_\perp^{\mathrm{res}})^2]\omega\tau+[1-(\omega\tau)^2]\mathrm{Re}k_\perp^{\mathrm{res}}\mathrm{Im}k_\perp^{\mathrm{res}}}\right], \quad k_\parallel > 0, \quad -\pi \le \Delta \le \tfrac{1}{2}\pi \tag{92}$$

where θ^f and θ^e are the angles between the surface plane, and the plasma and electromagnetic Poynting vectors, respectively. The above considerations are illustrated in Fig. 3. It is especially realized that the two Poynting vectors are *collinear* for $\omega\tau=1$, and *anticollinear for*

$$\mathrm{Re}k_\perp^{\mathrm{res}}/\mathrm{Im}k_\perp^{\mathrm{res}} = [(\omega\tau)^2-1]/(4\omega\tau) .$$

It is a general feature in the near-local regime that *anticollinearity* occurs for $\omega\tau > 1$. For normal incidence, the two Poynting vectors are antiparallel if the equality (91) holds, otherwise they are parallel.

D. Ratio of the magnitudes of the plasma and the electromagnetic Poynting vectors

In the near-local regime the ratio of the material and the electromagnetic cycle-averaged Poynting vectors is independent of the depth below the surface. By making use of the relation $3\pi^2 N_0 = (m^* v_F)^3/\hbar^3$, where N_0 is the dc free-carrier density, and by introducing the screened plasma frequency $\omega_p = [N_0 e^2/(m^*\epsilon_0)]^{1/2}$, the ratio of the cycle-averaged Poynting vectors given in Eqs. (90) and (84) can be written

$$\frac{|\vec{S}^{f,\mathrm{TE}}(k_\parallel,\omega,z)|}{|\vec{S}_t^{e,\mathrm{TE}}(k_\parallel,\omega,z)|} = \frac{2}{5}\left[\frac{v_F}{c_0}\right]^2 (\omega_p\tau)^2 \frac{(\omega\tau)^2}{[1+(\omega\tau)^2]^2}\left[\frac{k_\parallel^2+[\mathrm{Re}k_\perp^{\mathrm{res}}+\frac{1-(\omega\tau)^2}{2\omega\tau}\mathrm{Im}k_\perp^{\mathrm{res}}]^2}{k_\parallel^2+(\mathrm{Re}k_\perp^{\mathrm{res}})^2}\right]^{1/2} . \tag{93}$$

The result of Eq. (93) is particularly simple in the case of *collinear* material and electromagnetic energy flows. Thus, $\omega\tau=1$ implies in the near-local regime

$$\frac{|\vec{S}^{f,\mathrm{TE}}|}{|\vec{S}_t^{e,\mathrm{TE}}|}=\frac{2}{5}\pi^2\left[\frac{l}{\lambda_p}\right]^2,\quad \omega\tau=1\ , \tag{94}$$

where $\lambda_p=2\pi c_0/\omega_p$ is the plasma wavelength and $l=v_F\tau$ is the mean free path of the conduction electrons. For the *anticollinear* case obtained for a frequency say $\omega=\omega_{ac}$ one finds

$$\frac{|\vec{S}^{f,\mathrm{TE}}|}{|\vec{S}_t^{e,\mathrm{TE}}|}=\frac{8\pi^2}{5}\left[\frac{l}{\lambda_p}\right]^2\frac{(\omega_{ac}\tau)^2}{[1+(\omega_{ac}\tau)^2]^2}\ . \tag{95}$$

Note that under the above assumptions the ratio in the anticollinear case always is smaller than in the collinear case.

VII. THE "PROBLEM" OF ENERGY CONSERVATION AT A SHARP BOUNDARY

It is obvious from the derivation presented in Sec. IV that

$$1+r^{\mathrm{TE}}(k_{\|},\omega)=t^{\mathrm{TE}}(k_{\|},\omega)(2\pi)^{-1}\int_{-\infty}^{\infty}\Xi_{yy}(k_{\|},k_{\perp},\omega)dk_{\perp}\ , \tag{96}$$

a relation which in the *linear theory* expresses the continuity of the electric field across the boundary, and which in turn leads to the energy conservation law for the TE part of the electromagnetic field at the surface, i.e.,

$$\vec{e}_z\cdot\vec{S}_i^{e,\mathrm{TE}}(k_{\|},\omega)=\vec{e}_z\cdot\vec{S}_r^{e,\mathrm{TE}}(k_{\|},\omega)+\vec{e}_z\cdot\vec{S}_t^{e,\mathrm{TE}}(k_{\|},\omega,z=0)\ , \tag{97}$$

in the *linearized* approximation. A similar conclusion of energy conservation for the linear part of the electromagnetic TM field does, of course, hold. In the near-local regime where spatial dispersion effects are included in lowest order, one arrives, by approximating the linearized transmitted field by the single mode of Eq. (84) and the linear amplitude reflection coefficient by Eq. (81), at the conclusion in Eq. (97). Thus, the separate energy-conservation law for the linear part of the electromagnetic field at the surface is not violated in the near-local regime.

It appears from Eq. (90) that the cycle-averaged free-carrier Poynting vector, in general, is different from zero at the surface ($z=0$). In vacuum $|\vec{S}^{f,\mathrm{TE}}|$ is, of course, zero. Thus, the correct energy-conservation law which must include the energy transported by the electromagnetic field and the plasma is violated if one just considers the *linear* part of the electromagnetic energy flow together with the *nonlinear* flow of energy in the plasma system.

One could imagine that energy conservation would be restored if *nonlinearities,* i.e., second harmonics and higher harmonics, were included in the electromagnetic field. In a forthcoming paper we shall study the important question of harmonic generation in metals and show that *even if one includes the energy flow carried by second and higher harmonics in the electromagnetic field (and in the plasma) the energy-conservation law will not be restored.* As an example, it is easy to see that, in general, the dominating part of the nonlinear electromagnetic energy flux is carried *away* from the surface by the second harmonics in the reflected and transmitted fields, so that when the material energy flow is away from the surface the energy conservation law is still violated.

By denoting the fundamental and the second harmonic modes by (1) and (2), respectively, it will be shown in the forthcoming paper that the Fourier amplitude of the transmitted second harmonic electromagnetic field will be given by

$$\vec{E}_{t(2)}(k_{\|},k_{\perp}^{(2)},2\omega)=\overleftrightarrow{\Xi}_{(2)}(k_{\|},k_{\perp}^{(2)},2\omega)\cdot\Big[\vec{g}_{(2)}(k_{\|},2\omega,z\to0+,\vec{E}_i)-i\mu_0(2\omega)\int_{-\infty}^{\infty}\overleftrightarrow{\Sigma}(k_{\|},k_{\perp}^{(1)},k_{\perp}^{(2)},\omega):\vec{E}_{t(1)}(k_{\|},k_{\perp}^{(1)},\omega)\vec{E}_{t(1)}(k_{\|},k_{\perp}^{(2)}-k_{\perp}^{(1)},\omega)dk_{\perp}^{(1)}\Big]\ , \tag{98}$$

where the nonlinear third-order response function $\overleftrightarrow{\Sigma}(k_{\|},k_{\perp}^{(1)},k_{\perp}^{(2)},\omega)$ describing second harmonic generation takes the explicit form

$$\overleftrightarrow{\Sigma}(k_{\|},k_{\perp}^{(1)},k_{\perp}^{(2)},\omega)=-\frac{e^3\tau^2(m^*)^2}{\pi h^3}\int\int\int_{-\infty}^{\infty}\frac{\vec{v}\overleftrightarrow{R}(k_{\|},k_{\perp}^{(1)},\omega,\vec{v})d^3v}{1+i(\vec{k}^{(2)}\cdot\vec{v}-2\omega)\tau}\,, \tag{99}$$

with

$$\overleftrightarrow{R}(k_{\|},k_{\perp}^{(1)},\omega,\vec{v})=\frac{\dfrac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}\overleftrightarrow{1}+m^*\dfrac{\partial^2 f_0(\mathscr{E})}{\partial\mathscr{E}^2}\vec{v}\vec{v}}{1+i(\vec{k}^{(1)}\cdot\vec{v}-\omega)\tau}-i\tau\frac{\dfrac{\partial f_0(\mathscr{E})}{\partial\mathscr{E}}\vec{k}^{(1)}\vec{v}}{[1+i(\vec{k}^{(1)}\cdot\vec{v}-\omega)\tau]^2}\,. \tag{100}$$

The explicit expression for the second harmonic field gradient at the surface, $\vec{g}_{(2)}(k_{\|},2\omega,z\to 0+,\vec{E}_i)$ will not be given here.

It turns out that "violation" of the law of energy conservation has to do with the fact that, somewhat unphysically, a *sharp* boundary is assumed. This conclusion is in agreement with that given by Bishop and Maradudin[6] in connection with their investigation of the generation of optical lattice waves by light. In a subsequent paper we shall study the penetration of light into a solid-state plasma having a smooth surface profile and show that no violation of the law of energy conservation occurs when the spatially transient behavior of the material field in the surface layer is taken into account.

Since the conversion of energy from the incident light beam to the free-carrier system is of order $1{:}10^3$ or less in metals and heavily doped semiconductors[20] the amount of energy which a sharp surface model cannot account for is a very small fraction of the total energy. Furthermore, since the penetration depth of the electromagnetic field is $\sim 10^3-10^4$ Å or more,[20] it is expected that the present model fails only in a thin ($\lesssim 10^2$ Å) surface layer. No continuous build up (or removal) of energy in (or from) surface modes occurs in the cw steady state when a smooth surface profile is used. A comparison of the energy flows obtained by means of smooth and sharp surface models in the surface layer will be considered elsewhere.

APPENDIX A: CALCULATION OF $\overleftrightarrow{\sigma}(k_{\|},k_{\perp},\omega)$ ON THE BASIS OF DEGENERATE FERMI-DIRAC STATISTICS

To derive the explicit expressions for the components of the conductivity tensor spherical coordinates $v_x=v\sin\theta\cos\phi$, $v_y=v\sin\theta\sin\phi$, and $v_z=v\cos\theta$ are introduced. Then by combining Eqs. (24), (26), and (27), the integration over the numerical velocity can be carried out. Doing this one gets

$$\overleftrightarrow{\sigma}(k_{\|},k_{\perp},\omega)=\frac{2e^2\tau v_F^3(m^*)^2}{h^3}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\frac{\overleftrightarrow{D}(\theta,\phi)d\phi\,d\theta}{G(\theta)\cos\phi+H(\theta)}\,, \tag{A1}$$

where

$$G(\theta)=ik_{\|}v_F\tau\sin\theta\,, \tag{A2}$$

$$H(\theta)=1-i\omega\tau+ik_{\perp}v_F\tau\cos\theta\,, \tag{A3}$$

and

$$\overleftrightarrow{D}(\theta,\phi)=\begin{bmatrix}\sin^2\theta\cos^2\phi & 0 & \sin\theta\cos\theta\cos\phi\\ 0 & \sin^2\theta\sin^2\phi & 0\\ \sin\theta\cos\theta\cos\phi & 0 & \cos^2\theta\end{bmatrix}. \tag{A4}$$

Next, the integrations over ϕ can be performed. The integrations in σ_{yy} are done by substituting $u=\tan(\phi/2)$ and decomposing the integrands. Thus,

$$\int_0^{2\pi}\frac{\cos^2\phi\,d\phi}{G(\theta)\cos\phi+H(\theta)}=\frac{2\pi H}{G^2}\left[\frac{H}{(H^2-G^2)^{1/2}}-1\right], \tag{A5}$$

and

$$\int_0^{2\pi} \frac{\sin^2\phi \, d\phi}{G(\theta)\cos\phi + H(\theta)} = \frac{2\pi H}{G^2} \left[1 - \frac{(H^2-G^2)^{1/2}}{H} \right] . \tag{A6}$$

The integrations in σ_{zz} and σ_{xz} are carried out by making use of the formula

$$\int_0^{2\pi} \frac{d\phi}{G(\theta)\cos\phi + H(\theta)} = \frac{2\pi}{(H^2-G^2)^{1/2}} , \tag{A7}$$

and by integration by parts. The final results of Eqs. (28)–(31) are obtained by making the substitution $x=\cos\theta$, and by noting that

$$(H^2-G^2)^{1/2} = 2(1-i\omega\tau)(c_1x^2+c_2x+c_3)^{1/2} . \tag{A8}$$

APPENDIX B: CALCULATION OF $\vec{W}(k_{\|},k_{\perp},\omega)$ FOR A FULLY DEGENERATE PLASMA

By introducing spherical coordinates, as described in Appendix A, the integration over the numerical velocity in Eq. (72) can immediately be carried out. Doing this one obtains

$$W_x^{yy}(k_{\|},k_{\perp},\omega) = -\frac{v_F^4}{m^*} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\sin^4\theta \sin^2\phi \cos\phi \, d\phi \, d\theta}{G(\theta)\cos\phi + H(\theta)} , \tag{B1}$$

and

$$W_z^{yy}(k_{\|},k_{\perp},\omega) = -\frac{v_F^4}{m^*} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\sin^3\theta \cos\theta \sin^2\phi \, d\phi \, d\theta}{G(\theta)\cos\phi + H(\theta)} . \tag{B2}$$

By means of the substitution $u=\tan(\phi/2)$ one gets

$$\int_0^{2\pi} \frac{\cos^3\phi \, d\phi}{G(\theta)\cos\phi + H(\theta)} = \frac{\pi}{G} \left[1 + 2\left[\frac{H}{G}\right]^2 \right] - \frac{2\pi}{(H^2-G^2)^{1/2}} \left[\frac{H}{G}\right]^3 . \tag{B3}$$

By combining Eqs. (A6), (A7), and (B1)–(B3), by introducing the substitution $x=\cos\theta$, and by utilizing Eqs. (32)–(36) and (A8) one obtains the expressions for W_x^{yy} and W_z^{yy} given in Eqs. (85) and (86).

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[1]O. Keller, Phys. Rev. B 13, 4612 (1976).

[2]S. Fukuda, T. Karasaki, T. Shiosaki, and A. Kawabata, Phys. Rev. B 20, 4109 (1979).

[3]O. Keller, J. Opt. Soc. Am. 68, 42 (1978).

[4]O. Keller, Phys. Rev. B 19, 2175 (1979).

[5]L. R. Ram Mohan, E. Kartheuser, and S. Rodriguez, Phys. Rev. B 20, 3233 (1979).

[6]M. F. Bishop and A. A. Maradudin, Phys. Rev. B 14, 3384 (1976).

[7]O. Keller, Phys. Rev. B 22, 4853 (1980).

[8]G. E. H. Reuter and E. H. Sondheimer, Proc. R. Soc. London Ser. A 195, 336 (1948).

[9]F. E. Hutchison and W. N. Hansen, Phys. Rev. B 20, 4069 (1979).

[10]R. Loudon, J. Phys. A 3, 233 (1970).

[11]O. Keller, Phys. Rev. B 19, 6315 (1979).

[12]G. S. Agarwal, D. N. Pattanayak, and E. Wolf, Phys. Rev. Lett. 27, 1022 (1971).

[13]G. S. Agarwal, D. N. Pattanayak, and E. Wolf, Opt. Commun. 4, 255 (1971).

[14]G. S. Agarwal, D. N. Pattanayak, and E. Wolf, Opt. Commun. 4, 260 (1971).

[15]R. Zeyher, J. L. Birman, and W. Brenig, Phys. Rev. B 6, 4613 (1972).

[16]J. L. Birman and J. J. Sein, Phys. Rev. B 6, 2482 (1972).

[17]J. Lindhard, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 28, No. 8, 1 (1954).

[18]M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1970).

[19]V. L. Ginzburg, *Propagation of Electromagnetic Waves in Plasmas* (Pergamon, Oxford, 1970).

[20]O. Keller (unpublished).