PHYSICAL REVIEW A VOLUME 26, NUMBER 3 SEPTEMBER 1982

Linear and nonlinear hydrodynamics of low-friction adsorbed systems

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(Received 28 December 1981)

A model is proposed which interpolates between the diffusive picture for particles adsorbed on a surface and the full hydrodynamics of a compressible fluid. The model is the Navier-Stokes equations modified by a friction term. When the friction parameter σ is set equal to zero, the Navier-Stokes equations obtain, but for large σ, Fick's law and diffusive behavior for the density emerge. It is found that for σ small enough, the dynamic structure factor should display sound peaks in a certain wave-number range. It is also shown that the infrared divergences that signal the breakdown of hydrodynamics for two-dimensional fluids are regulated by the friction term. The possibility of observing the sound modes and the nonlinear corrections to the transport coefficients is discussed.

I. INTRODUCTION

It is well known that the hydrodynamical behavior of particles adsorbed on a solid surface is governed by particle diffusion. It is also by now well established[1,2] that conventional hydrodynamics breaks down for strictly two-dimensional fluids. Since if one were to turn off the interaction between the particles and the substrate one should obtain such a two-dimensional fluid, it would seem natural to seek models that interpolate between the two cases—between, say, fluctuating Navier-Stokes hydrodynamics and the hopping of particles on a substrate. Such a model would, in its simplest form, contain a friction coefficient[3] σ representing the interaction between the "fluid" and the substrate. Setting σ to zero would yield the equations of fluctuating hydrodynamics for a compressible fluid. For large σ the description would correspond to the hopping of particles from site to site, giving a diffusion equation for the density.

We construct and study in this paper such an interpolating model at both the linear and nonlinear levels. In the linearized model, we find a crossover from diffusive to sound-mode behavior for the density as the friction coefficient is decreased. In other words, if the friction coefficient of a system is small enough, we find that it should display sound modes for a certain range of wave numbers. This feature persists when nonlinearities are taken into account. The effects of the breakdown of hydrodynamics however do not show up in a very striking fashion, since the small-wave-number properties of the system are for the most part regulated severely by the friction parameter. Nevertheless, there are distinct departures from the linearized theory in the small friction limit. It is uncertain whether there are physical systems with friction coefficients small enough for these effects to be observable.

The breakdown of hydrodynamics for dimensionality $d \leq 2$ is associated with infrared problems which are due to the convective nonlinearities,[2] that is, the mode-coupling terms in the conservation equation for the current. Precisely at $d=2$, the physical viscosity is predicted to diverge logarithmically with the size of the system. Friction with the substrate (thought of as a rough surface) should regulate the infrared singularities. That this is the very mechanism that brings about diffusive behavior for the density is clear, since the friction term renders the current "short ranged"; the particles move only in bounded hops, which, in the large, is diffusion.

While our model cannot be quantitative on a microscopic scale, it should give a reasonable description of the long-time, large-scale properties of the system. We shall discuss our model in the next section. In Sec. III, we discuss features of the linearized version of the model. We shall show that for $\sigma \neq 0$, the dynamic structure factor displays vestiges of sound-mode behavior for k and σ small enough. This behavior, moreover, persists when the mode-coupling effects are included. In Sec. IV, we present a perturbative calculation (reliable for $d=2$ for this model) of corrections to the transport coefficients, and discuss the qualitative effects produced by these corrections. We examine the transport coefficients (the shear viscosity and the sound attenuation) (a) as functions of the friction parameter for small, fixed wave number and (b) as functions of wave number for small, fixed friction parameter, and show that there are indeed departures from the linearized theory, but that these effects are not extraordinarily strong. They tend to be regulated, in (a) by the nonzero wave number and in (b) by the nonzero friction coefficient. At strictly zero wave number, moreover, the frictional damping dominates everything else.

We close in Sec. V with some remarks on the wave number and parameter ranges in which the effects described here should be seen.

II. THE MODEL

We would like a description of the hydrodynamics of adsorbed particles. We want therefore to average over the fluctuating degrees of freedom of the substrate. If the substrate degrees of freedom (phonons) are very fast compared to the hydrodynamic processes of interest for the adparticles, then we may average over them and summarize their effect and that of the adsorption sites in a simple friction coefficient. In the absence of a substrate the current obeys a conservation law. With a substrate present, there is a frictional "force" on the adparticles due to collisions with the substrate, which breaks the conservation law. It is this effect that is contained in the friction term. These arguments lead us to a model for "fluids" on a substrate which, formally, differs very little from ordinary hydrodynamics. The only departure lies in the damping term, which, in addition to the viscous damping, proportional to k^2 in momentum space (k being the wave vector) contains a purely dissipative term, independent of wave number. The equations are the continuity equation for the density

$$\frac{\partial n}{\partial t}+\vec{\nabla}\cdot(n\vec{v})=0 \tag{1}$$

and the momentum equation

$$\frac{\partial v_i}{\partial t}+(\sigma\delta_{ij}+\hat{\Omega}_{ij})v_j+(\delta_{ik}\nabla_j+\tfrac{1}{2}\delta_{jk}\nabla_i)(v_jv_k)+\frac{c^2}{n_0}\nabla_i n+\frac{c^2}{2n_0^2}\nabla_i(n^2)=\zeta_i(x,t)\ . \tag{2}$$

Here $n(xt)$ is the density, n_0 its average (quiescent) value, n is $n-n_0$, v_i is the fluid velocity field, c^2 is the speed of sound in the fluid (we have assumed a simple equation of state $p=nc^2$), and $\zeta_i(xt)$ is the random force,

$$\hat{\Omega}_{ij}=-[\nu_T(\delta_{ij}\nabla^2-\nabla_i\nabla_j)+\nu_L\nabla_i\nabla_j]\ , \tag{3}$$

where ν_T and ν_L are the shear and bulk viscosities, respectively. For fluctuations about equilibrium, ζ_i satisfies (when Fourier transformed)

$$\langle\zeta_i(k,\omega)\zeta_j(k',\omega')\rangle=2k_BT(2\pi)^{d+1}\delta^d(k+k')\delta(\omega+\omega')(\sigma\delta_{ij}+k^2\Omega_{ij})\ , \tag{4}$$

where

$$\Omega_{ij}=\nu_T\left[\delta_{ij}-\frac{k_ik_j}{k^2}\right]+\nu_L\frac{k_ik_j}{k^2}$$

$$\equiv\nu_T\mathcal{P}_{ij}(k)+\nu_L\mathcal{Q}_{ij}(k)\ . \tag{5}$$

We have for simplicity ignored heat diffusion in our discussion. The effects of the substrate are all contained in the friction coefficient σ. Setting σ to zero yields ordinary nonlinear hydrodynamics. It should be noted that the equations are assumed to hold only down to some length scale cutoff Λ^{-1}. In natural units where the shear viscosity sets the timescale ($t\to\nu_Tt$), the temperature sets the scale for the velocity field [$v_i\to(n_0/k_BT)^{1/2}v_i$], and density fluctuations are measured in units of n_0, and where the noise, transport coefficients, and sound speed are appropriately rescaled, the natural coupling parameter multiplying the nonlinear terms in (1) and (2) is

$$\lambda=\frac{1}{\nu_T}\left[\frac{k_BT}{mn_0}\right]^{1/2}\ . \tag{6}$$

It has dimensions $(\text{length})^{-1+d/2}$, and is dimensionless for $d=2$. This power-counting tells us that the upper marginal dimension for the problem is 2. For $d>2$, there are no divergences. For $d=2$, there are logarithmic divergences for $\sigma\to0$, but these can be calculated in ordinary perturbation theory. For $d<2$, an expansion[4] in $\epsilon=2-d$ must be used to supplement the perturbation theory to get accurate results on the nature of the singularities for $\sigma \to 0$. We shall concentrate on $d=2$, that being of relevance for adsorbed systems. We can therefore simply use perturbation theory.

We shall study the linearized problem first, then estimate perturbatively the renormalized transport coefficients for small σ, k, and ω. We shall not go into great calculational detail here, the methods being fairly standard.

III. LINEARIZED THEORY

If the nonlinear terms are neglected, the equations of motion read

$$\dot{n} + \vec{\nabla}\cdot\vec{v} = 0 \ , \tag{7a}$$

$$\dot{v}_i + (\sigma\delta_{ij} + \hat{\Omega}_{ij})v_j + c^2\nabla_i n = \zeta_i \ . \tag{7b}$$

We can then see that for times much longer than σ^{-1}, and for long wavelengths (or large scales), 7(b) becomes, averaging over the noise in some appropriate nonequilibrium ensemble

$$\sigma\langle v_i \rangle = -c^2\nabla_i\langle n \rangle$$

or

$$\langle v_i \rangle = -\frac{c^2}{\sigma}\nabla_i\langle n \rangle \ , \tag{8}$$

which is Fick's law.[5] This, inserted into 7(a), gives

$$\frac{\partial}{\partial t}\langle \delta n \rangle = \frac{c^2}{\sigma}\nabla^2\langle \delta n \rangle \ , \tag{9a}$$

which is the diffusion equation for the density, with a diffusion coefficient

$$D_0 = c^2/\sigma \ . \tag{9b}$$

If, however, σ were zero, we could not take the limit $t \gg \sigma^{-1}$. The long-time, large-scale limit then easily gives the wave equation for the density

$$\frac{\partial^2}{\partial t^2}\langle \delta n \rangle = c^2\nabla^2\langle \delta n \rangle \ . \tag{10}$$

That is, sound modes are the dominant hydrodynamic behavior.

In more detail, if we define L_i and T_i to be the longitudinal and transverse components of the velocity, and $\mathscr{P}_{ij}$ and $\mathscr{Q}_{ij}$ defined by (5) are the transverse and longitudinal projectors, then an easy calculation shows

$$C_{nn}(k,\omega) \equiv \frac{\langle \delta n(k,\omega)\delta n(-k,-\omega)\rangle}{(2\pi)^{d+1}\delta^{d+1}(0)} = \frac{2k^2(\sigma+\nu_L k^2)}{(\omega^2-c^2k^2)^2+\omega^2(\sigma+\nu_L k^2)^2} \ , \tag{11a}$$

$$C_{ij}^{L}(k,\omega) \equiv \frac{\langle L_i(k,\omega)L_j(-k,-\omega)\rangle}{(2\pi)^{d+1}\delta^{d+1}(0)} = \frac{2\omega^2(\sigma+\nu_L k^2)\mathscr{Q}_{ij}(k)}{(\omega^2-c^2k^2)^2+\omega^2(\sigma+\nu_L k^2)^2} \ , \tag{11b}$$

$$C_{ij}^{T}(k,\omega) \equiv \frac{\langle T_i(k,\omega)T_j(-k,-\omega)\rangle}{(2\pi)^{d+1}\delta^{d+1}(0)} = \frac{2(\sigma+\nu_T k^2)\mathscr{P}_{ij}(k)}{\omega^2+(\sigma+\nu_T k^2)^2} \ , \tag{11c}$$

$$C_{nL_i}(k,\omega) \equiv \frac{\langle \delta n(k,\omega)L_i(-k,-\omega)\rangle}{(2\pi)^{d+1}\delta^{d+1}(0)} = \frac{2\omega k_i(\sigma+\nu_L k^2)}{(\omega^2-c^2k^2)^2+\omega^2(\sigma+\nu_L k^2)^2} \ . \tag{11d}$$

These are the only nonzero two-point correlation functions to this order.

Let us fix our attention on the density-density correlation function, which might be measured, for instance, in inelastic neutron-scattering experiments. We shall study its behavior as σ is changed from zero to large values. First, define

$$\gamma = \frac{1}{ck}(\sigma+\nu_L k^2) \ , \tag{12a}$$

$$\xi = \omega/ck \ . \tag{12b}$$

Then

$$C_{nn}(k,\omega) = \frac{1}{c^3 k}\frac{\gamma}{(\xi^2-1)^2+\xi^2\gamma^2} \ . \tag{13}$$

For $\gamma^2 < 2$, C_{nn} has maxima at $\xi = \pm(1-\frac{1}{2}\gamma^2)^{1/2}$, for fixed k, i.e., at

$$\omega = \pm ck\left[1-\frac{(\sigma+\nu_L k^2)^2}{2c^2k^2}\right]^{1/2} \ .$$

These are clearly remnants of sound-mode behavior, and are displayed in Fig. 1(a). As is increased, the peaks move in until they coalesce into a single peak at $\xi=0$, for $\gamma^2=2$, as seen in Fig. 1(b). The correlation function then becomes

$$C_{nn}(k,\omega)=\frac{\sqrt{2}}{c^3k}\frac{1}{\xi^4+1}\ , \tag{14}$$

which is not a Lorentzian. For large γ ($\gamma \gg 2$), the ξ^2 term begins to overwhelm the ξ^4 term, and a diffusive peak (width αk^2) develops at $\xi=0$ [Fig. 1(c)]. The condition $\gamma^2<2$ corresponds, in terms of the original parameters, to

$$\sigma<\sqrt{2}ck-\nu_L k^2\equiv\sigma_0(k)\ . \tag{15}$$

Since for a given system, σ is fixed, this is really a condition on k. That is, the sound modes can be seen only for

$$k_-<k<k_+\ , \tag{16a}$$

where

$$k_\pm=\frac{\sqrt{2}c\pm(2c^2-4\sigma\nu_L)^{1/2}}{2\nu_L}\ . \tag{16b}$$

For σ large enough, i.e., $\sigma>\sigma_c\equiv c^2/2\nu_L$ they cannot be seen at all, since the discriminant in (16b) is negative. Thus, not all systems are expected to display these residual sound modes.

The qualitative features of the linearized theory are summarized in Fig. 2. It should be noted that nonlinearities will affect this picture only slightly, since, as we shall see, they alter ν_L only by some additive terms of the form $\ln(\sigma+\nu_L k^2)$. To determine whether the diffusion-sound crossover can be observed in experiments we must examine the condition $\sigma<\sigma_c\equiv c^2/2\nu_L$, or

$$D_0>2\nu_L \tag{17}$$

to see if it can be realized. Consider a system with a fairly large diffusion coefficient ($D_0=3\times10^{-3}$ cm^2/s) discussed by Banavar *et al.*,[4] namely, tungsten on tungsten (W-W). Using low-density kinetic-theory expressions[6]

$$c^2=\frac{k_BT}{m}\ , \tag{18a}$$

$$\nu_L=\frac{c}{2\sqrt{\pi}dn_0}\ , \tag{18b}$$

where d is the diameter of the adatoms, assumed to be about 4 Å, we find (in cm^2/s)

$$\nu_L=\frac{1.7\times10^{-4}}{n_0d^2}\ .$$

Thus (17) is satisfied only if $n_0d^2>0.1$. Increasing the density would help only a little. For large enough densities, the viscosity *increases* [i.e., finite-density corrections[6] to (18) become important] and hence the sound regime shrinks. A system with a much larger diffusion coefficient (0.03 to 0.3 cm^2/s), on the other hand, would make the diffusion-sound crossover much easier to see. It does appear, though, that the W-W system is a candidate.

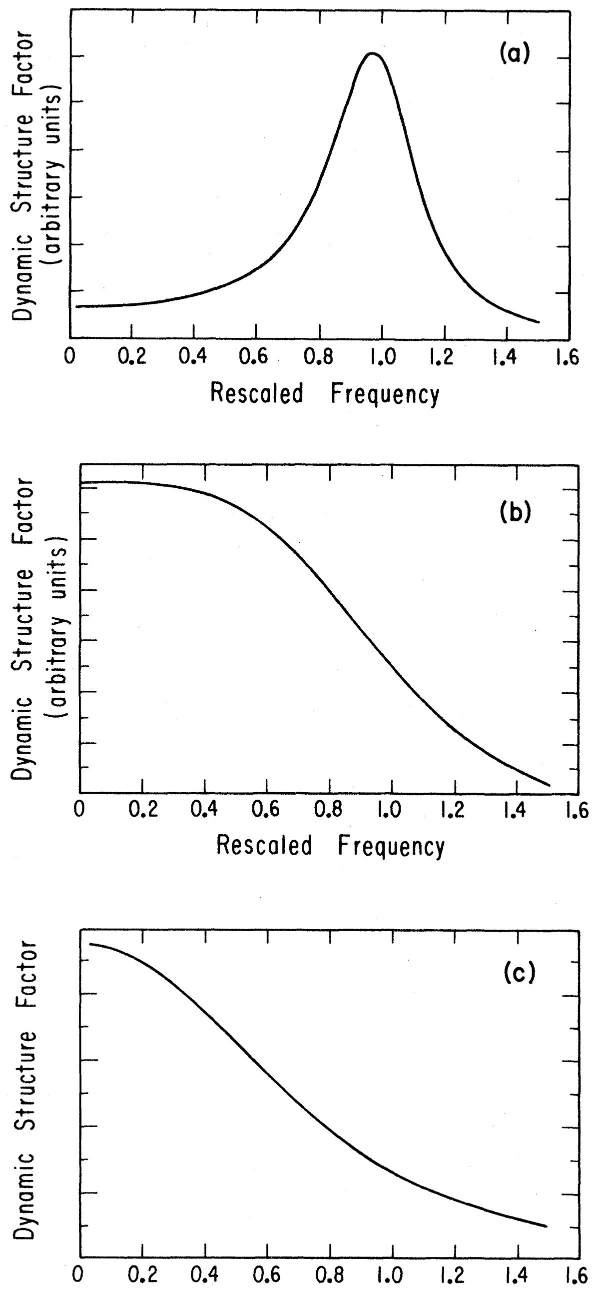

FIG. 1. Evolution of the dynamic structure factor as the friction parameter is varied. (a) $\sigma/ck=0.35$. (b) $\sigma/ck=1.5$. (c) $\sigma/ck=2.0$.

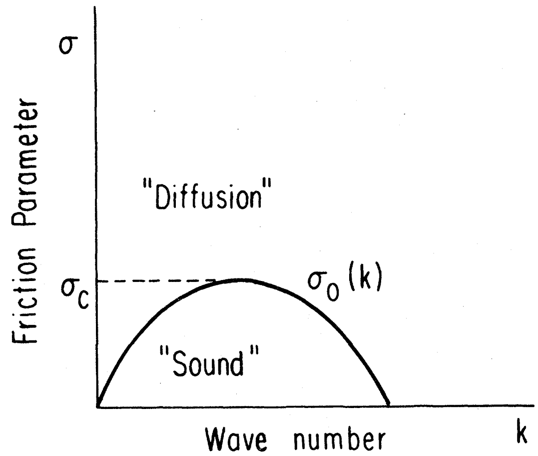

FIG. 2. Qualitative features of the linearized theory: the diffusion-sound crossover.

IV. MODE-COUPLING EFFECTS

The presence of nonlinear (mode-coupling) terms in our equations means that the observed transport coefficients are no longer the bare parameters seen in the linearized theory, but are "renormalized." They are now wave-number and frequency dependent, and, as we shall see, their dependence on σ is also different from that in the linearized theory. We shall study, to second order in perturbation theory, the renormalized coefficients for diffusion and sound attenuation, which are extracted from the renormalized correlation functions by means of the Green-Kubo relations.[7] We find it convenient, in the perturbation theory, to use a memory function approach,[7] which entails working with Laplace-transformed correlation functions defined by

$$\bar{C}_{\alpha\beta}(k,z) \equiv i\int_0^\infty dt\, e^{izt} C_{\alpha\beta}(k,t) \equiv i\int_0^\infty dt\, e^{izt}\langle \phi_\alpha(k,0)\phi_\beta(-k,t)\rangle \;, \tag{19}$$

where (ϕ_α) are any of the fields (n,L_i,T_i) in the problem. In terms of $\bar{C}_{\alpha\beta}(k,z)$, the Green-Kubo relations are

$$D=\{\lim_{k\to 0}[ik^2\lim_{z\to 0}\bar{C}_{nn}(k,z)]\}^{-1} \tag{20a}$$

for the diffusion coefficient, and

$$\Gamma=-\lim_{z\to ck}[\mathrm{Im}\bar{C}_{nn}(k,z)]^{-1} \tag{20b}$$

for the sound-attenuation coefficient. Using these relations in the linearized theory gives

$$D=D_0=c^2/\sigma \;, \tag{21a}$$

$$\Gamma=\sigma+\nu_L k^2 \;, \tag{21b}$$

as expected.

The memory function $M_{\alpha\beta}(z)$ is defined by a Dyson equation

$$[z\delta_{\alpha\beta}-M_{\alpha\beta}(z)]\bar{C}_{\beta\mu}(z)=\chi_{\alpha\mu} \;, \tag{22}$$

where $\chi_{\alpha\mu}$ is the full static susceptibility, $M_{\alpha\beta}$ can be interpreted as a matrix of physical transport coefficients, and self-energy corrections to $\bar{C}_{\alpha\beta}(k,z)$ appear precisely as contributions to $M_{\alpha\beta}(z)$. $M_{\alpha\beta}$ can be evaluated in perturbation theory using methods described by Mazenko, Nolan, and Freedman,[8] and the renormalized correlation functions thus determined. This method has the advantage that it separates the static and dynamic effects, since the full (renormalized) static correlation function is used as an input to the dynamic calculation. In our case, however, the statics are trivial.

We evaluated the corrections $\Delta M_{\alpha\beta}$ to the memory function to second order (i.e., we calculated the lowest-order self-energy graphs) and used the Green-Kubo relations on the resulting correlation functions to extract the renormalized transport coefficients.

Inverting Eq. (22) and using the Green-Kubo relations shows that the leading corrections are

$$\Delta\Gamma=i(\Delta M_{nn}+\Delta M_{L_iL_i})_{\omega=ck} \tag{23a}$$

for the sound attenuation and

$$\Delta D=\frac{i}{k^2}\Delta M_{nn} \tag{23b}$$

for the diffusion coefficient. In the limit $\omega \ll \nu_T\Lambda^2$, $k\ll\Lambda$, $\sigma\ll\nu_T\Lambda^2$ we find

$$D^{\mathrm{ren}}=D_0+\Delta D\simeq\frac{c^2}{\sigma}+\frac{k_BT}{8\pi m n_0\nu_L}\ln\left[\frac{\nu_L^2\Lambda^4}{(\sigma+\nu_Lk^2)^2+\omega^2}\right] , \tag{24a}$$

$$\Gamma_L^{\mathrm{ren}}=\Gamma_L^{(0)}+\Delta\Gamma_L\simeq(\sigma+\nu_Lk^2)+\frac{k_BT}{8\pi m n_0}k^2\left[\frac{9}{4\nu_T}\ln\left[\frac{c^2\Lambda^2}{4\sigma^2+c^2k^2}\right]+\frac{21}{4\nu_L}\ln\left[\frac{c^2\Lambda^2}{\sigma^2+c^2k^2}\right]+\frac{2\sigma}{c^2}\ln\left[\frac{4c^4\Lambda^4}{\sigma^2c^2k^2}\right]\right] , \tag{24b}$$

$$\Gamma_T^{\rm ren} \simeq (\sigma+\nu_T k^2) + \frac{k_B T}{4\pi m n_0} k^2 \left[\frac{3}{4\nu_T} \ln \left[\frac{\nu_T \Lambda^2}{\sigma+\nu_T k^2} \right] + \frac{\sigma}{c^2} \ln \left[\frac{c^2 \Lambda^2}{(\sigma+\nu_T k^2)^2} \right] \right] . \tag{24c}$$

Notice that $\Gamma_L^{\rm ren}$ contains a logarithm that is *not* regulated by σ, which is quite surprising. It is clear that as σ goes to zero, the logarithmic corrections to the diffusion coefficient are overwhelmed by the bare c^2/σ term. We therefore turn to the sound attenuation and the damping of the transverse mode (which is not strictly a viscosity, since it now contains a wave-number-independent part) to see what nonlinear effects show up in the small friction limit.

Let us look first at the transverse damping as a function of σ. Assuming a small, fixed wave number k, we define rescaled (dimensionless) quantities using $\nu_T k^2$ as an inverse time scale:

$$\gamma_T \equiv \Gamma_T/(\nu_T k^2), \quad x \equiv \sigma/(\nu_T k^2) . \tag{25a}$$

Also define

$$a \equiv \frac{k_B T}{16\pi m n_0 \nu_T^2}, \quad b \equiv \frac{k^2 k_B T}{2\pi n_0 m c^2} . \tag{25b}$$

Then

$$\gamma_T = x + (a+bx)\ln\frac{1}{1+x} + G , \tag{26}$$

G being a constant (independent of x).

Assuming that we are working at low densities, we may use (18) to determine a and b in terms of microscopic parameters:

$$a = 4\pi n_0 d^2, \quad b = \frac{k^2}{2\pi n_0} . \tag{27}$$

The low-density assumption means that $a/4\pi$ must be small, and a hydrodynamic picture holds only if b is small. Relaxing the low-density condition does not help much. The viscosities decrease for a while, with increasing density, but then start to increase, making the nonlinear corrections smaller. So we shall discuss only the dilute case.

In order to see where the corrections become important, we must determine for what value of x they are of the same magnitude as the linear term. Suppose $n_0 d^2 \simeq 0.1$, which is fairly dilute, then $a \simeq 1$. The question then is, when are x and $\ln(1+x)$ comparable? This, of course, occurs for $x \ll 1$, i.e., for $\sigma \ll \nu_L k^2$. Using Eq. (18b) for ν_L, we find this requires

$$\sigma \ll \frac{ck^2}{2\sqrt{\pi} d n_0} ,$$

or

$$D_0 \gg 2\sqrt{\pi}\frac{c d n_0}{k^2} .$$

Let us apply this to the particular case of tungsten adsorbed on tungsten discussed in Sec. III. Using kinetic theory values for c^2 and assuming $d = 4$ Å we find that the condition on D_0 implies

$$3\times 10^{-3} \gg \frac{10^{-3} n_0}{k^2} \tag{28}$$

to realize which k^2 would have to be large compared with n_0, taking us out of the hydrodynamic regime. In short, if the effect is to be noticeable, it would require a system where the diffusion coefficient can be varied between say 0.2 and 3 $\mathrm{cm^2/s}$ (in order to make a comparison).

We turn next to the transverse damping as a function of k^2. Defining dimensionless variables $\Omega_T \equiv \Gamma_T/\sigma$, $x \equiv \nu_T k^2/\sigma$, and using kinetic theory values for the parameters, we obtain for small σ, discarding the $\sigma \ln(\sigma+\nu_T k^2)$ term in Eq. (24c),

$$\Omega_T \approx 1 + x + \epsilon \ln\frac{1}{1+x} , \tag{29a}$$

$$\epsilon \equiv \tfrac{3}{4} n_0 d^2 . \tag{29b}$$

If ϵ is 0.1, then, using values of d, σ, and ν for the W-W system, we find that the corrections become comparable to the linear term only for $k^2/n_0 \sim 10^4$, which is far from hydrodynamic. Even if ϵ were as large as 0.5, we would need $k^2/n_0 \sim 10$. Moreover, as ϵ increases, finite density effect become important, decreasing the corrections significantly. In other words, the transverse damping as a function of k^2, in the hydrodynamic regime changes very little when nonlinear effects are taken into account. One would have to decrease σ by at least a couple of orders of magnitude before the nonlinear effects could be seen.

The sound attenuation at first appears more promising since it contains an unregulated logarithm for $\sigma \neq 0$. This quantity, however, cannot be considered an infrared divergence since for any nonzero σ, the sound-mode regime terminates before zero wave number. The smaller σ is, the closer one can get to $k = 0$ and still stay in the sound regime. But the correction is itself proportional to σ, so that going to very small σ is not advantageous either. The behavior of the sound damping, both as a function of k for fixed σ and as a function of σ for fixed k, is therefore essentially the same as that of the damping of the transverse mode.

V. CONCLUSION

We have developed a model that interpolates between the diffusive behavior of adsorbed systems and the full hydrodynamics of a compressible fluid, as the friction parameter of the substrate is decreased. The model predicts distinct departures from a simple lattice-gas picture, at both the linear and the nonlinear levels, provided the friction parameter of the substrate is small enough. The linearized theory predicts the occurrence of sound modes in the dynamic structure factor, for a certain wave-number range. The nonlinear effects give rise to logarithmic corrections to the transport coefficients, altering their small-wave-number behavior. The corrections are of course the shadow of the breakdown of conventional hydrodynamics for $d \leq 2$.

In particular, if systems could be found with diffusion coefficients as large as 0.3 to 3 cm^2/s, i.e., with exceedingly low friction coefficients, the logarithmic dependence of the transport coefficients on the friction parameter could be verified.

ACKNOWLEDGMENTS

This work was supported by a National Science Foundation Grant No. DMR 80-20609 and the Materials Research Laboratory of the National Science Foundation at the University of Chicago, Grant No. NSF-MRL 79-24007.

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