PHYSICAL REVIEW A VOLUME 26, NUMBER 3 SEPTEMBER 1982

Inhomogeneity effects on wave properties in Elmo Bumpy Torus-Scale

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(Received 29 June 1981; revised manuscript received 14 December 1981)

In many plasma-magnetic-field configurations, the ion skin depth is comparable to magnetic field and density-gradient scale lengths and so inhomogeneity must play an important role for wave propagation in the ion-cyclotron range of frequencies. As an example, we consider ion-cyclotron heating in the Elmo Bumpy Torus-Scale (EBT-S) device, where the coupling of infinite and homogeneous fast and slow modes implies that fundamental ion-cyclotron heating can occur even if the waves launched are fast waves (which otherwise do not heat at the fundamental). We also show that singular, weakly damped, two-ion hybrid electrostatic solutions of the cold-plasma dispersion relation which include inhomogeneities are not consistent with the small ion-gyroradius basis for this dispersion relation. This suggests that two-ion hybrid-mode conversion requires an analysis which does not assume small ion-gyroradius and so, if conversion can occur at all in EBT-S, it must be different from that in idealized weakly inhomogeneous slab-geometry plasma models. For shorter-wavelength higher-frequency waves in EBT-S and for ion-cyclotron range of frequency waves in larger bumpy tori, like the projected EBT-P (P for proof of principle), inhomogeneity should have a less profound, although not necessarily negligible, impact on wave propagation.

I. INTRODUCTION

In an infinite and homogeneous plasma the fast- and slow-wave solutions of the cold-plasma dispersion relation have phase velocities which are comparable to the Alfven speed in the ion-cyclotron range of frequencies. Consequently, these modes have wave numbers which are characteristically comparable to the inverse ion skin depth ω_i/c, in size where ω_i and c are the ion plasma frequency and the speed of light, respectively.[1] Clearly, in plasmas with ion skin depth much smaller than the characteristic dimensions of the plasma, the infinite and homogeneous dispersion relation can provide useful guidance in the understanding of wave propagation and absorption in the ion-cyclotron range of frequencies.[2] However, for smaller low-density devices in which the ion skin depth is greater than or similar to a characteristic dimension, inhomogeneity effects on mode propagation become appreciable. Infinite and homogeneous plasma-wave theory alone cannot provide strong intuition about the wave properties in such plasmas.

The plasma in the Elmo Bumpy Torus-Scale (EBT-S) device at Oak Ridge has gradient scale lengths $\sim$20 cm, which are comparable to the ion skin depth for a typical density of approximately 1×10^{12} cm^{-3}.[3] Hence, inhomogeneity must appreciably influence wave properties for frequencies like the ion-cyclotron frequency although its impact is less important for shorter-wavelength, higher-frequency waves.

The purpose of this paper is to address certain aspects of wave propagation in a highly inhomogeneous plasma like EBT-S without having to solve the complicated coupled set of three partial differential equations which constitute the dispersion relation. In fact, an actual solution of the dispersion relation in Bumpy Torus geometry is a massive numerical undertaking well beyond the scope of this paper.[4] Nevertheless, the results of the present analysis can provide guidance to construction of the code and interpretation of code results once development is complete.

It has already been demonstrated that parallel wave electric fields, although generally very much smaller than the perpendicular wave electric fields, must be considered in the analysis of wave propagation and absorption below and in the ion-cyclotron range of frequencies within inhomogeneous plasmas.[5–12] This paper has three other major conclusions regarding wave propagation in an inhomogeneous plasma. First, inhomogeneity results in the coupling of the fast- and slow-wave solutions of infinite and homogeneous plasma theory. Consequently, fast-wave excitation based upon weakly inhomogeneous plasma-wave theory should lead to ion-cyclotron heating even in a single-ion species plasma. However, this absorption is likely to be incomplete because the infinite and homogeneous fast-wave contribution to the correct inhomogeneous mode is generally only weakly affected by ion-cyclotron resonance. Simultaneous strong absorption at the ion-cyclotron resonance zone and penetration of wave fields beyond the zone are consistent with ion-cyclotron resonance heating experiments on the Wisconsin-supported toroidal octapole.[13] Second, the significant parallel gradients in background magnetic field strength present in EBT-S significantly modify the singular, electrostatic two-ion hybrid mode of infinite and homogeneous cold-plasma-wave theory.[2,8,14–16] Specifically, the electrostatic solution is not present in EBT-S because of the important contribution of inhomogeneity to the dispersion relation. This indicates, that in a two-ion species plasma with strong magnetic field variations such as found in EBT-S, the standard theoretical treatment of mode conversion from electromagnetic to electrostatic waves, based upon a small ion-gyroradius expansion of the dispersion relation, is inappropriate. In fact, previous theoretical studies show that small finite parallel wave number discourages mode conversion and so it follows that substantial variations in magnetic field strength along the magnetic field direction likely lead to direct particle heating without the mode conversion.[8,16] Third, wave properties in the ion-cyclotron range of frequencies are very different in EBT-S than they are likely to be in projected Bumpy Torus devices like EBT-P.[17] In Bumpy Torus devices larger and more dense than EBT-S, the ion skin depth (i.e., $\lesssim 7$ cm for density $\gtrsim 1\times 10^{13}$ cm^{-3}) becomes smaller while plasma scale lengths become longer. As a result, although inhomogeneity effects still can influence wave properties, infinite and homogeneous wave theory should provide better guidance for interpreting ion-cyclotron range of frequency heating results in these larger devices than in EBT-S. Other works, relating to eigenmode structure,[7,9,10,18] electron bounce motion[19] and magnetic field strength gradient effects on ion-cyclotron damping,[20] support the premise that wave propagation and heating processes do not scale from EBT-S to larger machines like EBT-P. This paper does not address the important question of whether energetic ion confinement and transport might scale in going from EBT-S to EBT-P.

The remainder of this paper is divided into two sections. Section II self-consistently includes dielectric derivatives in the dispersion relation to demonstrate the significance of inhomogeneity in the propagation of modes in the ion-cyclotron range of frequencies. Section III summarizes the highlights of the analysis and reviews the major conclusions.

II. WAVE DISPERSION IN A COLD PLASMA WITH INHOMOGENEOUS MAGNETIC FIELDS

For the present analysis, waves are assumed to have their dispersion properties governed by the cold-plasma dispersion relation[2]:

$$0=\begin{bmatrix} S-N_w^2 & -iD & N_uN_w \\ iD & S-N_u^2-N_w^2 & 0 \\ N_uN_w & 0 & P-N_u^2 \end{bmatrix}\begin{bmatrix} E_u \\ E_v \\ E_w \end{bmatrix} \tag{1}$$

with

$$S=1-\sum_s \frac{\omega_s^2}{\omega^2-\Omega_s^2},\quad D=\sum_s \frac{\Omega_s}{\omega}\frac{\omega_s^2}{\omega^2-\Omega_s^2},$$

$$P=1-\sum_s \frac{\omega_s^2}{\omega^2},\quad N_u=\frac{k_u c}{\omega},\quad N_w=\frac{k_w c}{\omega}. \tag{2}$$

In Eq. (2), c, ω_s, Ω_s, and ω denote the speed of light, the plasma frequency of species s, cyclotron frequency of species s, and the wave frequency, respectively. The u, v, and w directions are orthogonal with w oriented along the background magnetic field. The u-, v-, and w-directed electric fields are represented by E_u, E_v, and E_w, respectively. For simplicity, it is assumed that there is no v-directed wave variation and so the dispersion relation only depends on the u wave number (k_u) and the w wave number (k_w).

For the general inhomogeneous magnetic field case, the wave numbers represent differential operators which do not commute with the dielectrics. If the density is assumed to be constant, the three separate equations given by Eq. (1) can be combined into a single fourth-order partial differential equation in E_v. For consecutively lower dependencies on indices of refraction

$$\begin{aligned}0=\ &(PN_w^2+SN_u^2)(N_w^2+N_u^2)E_v+2D\{P[N_w(D^{-1})]N_w+[N_u(SD^{-1})]N_u\}(N_w^2+N_u^2)E_v\\ &-\{PS(2N_w^2+N_u^2)+(S^2-D^2)N_u^2-PD[N_w^2(D^{-1})](N_w^2+N_u^2)-D[N_u^2(SD^{-1})](N_u^2+N_w^2)\}E_v\\ &-2D\{P[N_w(SD^{-1})]N_w+[N_u(S^2D^{-1})]N_u-[N_u(D)]N_u\}E_v\\ &+\{P(S^2-D^2)-PD[N_w^2(SD^{-1})]-D[N_u^2(S^2D^{-1})]+D[N_u^2(D)]\}E_v\ .\end{aligned} \tag{3}$$

In Eq. (3), terms like $[N_w(D^{-1})]$ denote that the wave number operator acts only on the dielectric. Such terms do not enter the infinite and homogeneous dispersion relation. Also, contrary to infinite and homogeneous theory, there exist contributions to the dispersion relation which depend on first and third derivatives of E_v. These terms signify that direction of propagation affects the dispersion characteristics of waves.

It is instructive to rewrite Eq. (3) in a form which clearly separates terms which arise only in the homogeneous limit from those which are nonzero only when plasma or magnetic field gradients are present. In particular,

$$\begin{aligned}0=&\{(PN_w^2+SN_u^2)(N_w^2+N_u^2)-[PS(2N_w^2+N_u^2)+(S^2-D^2)N_u^2+P(S^2-D^2)]\}E_v\\&+D\{2P[N_w(D^{-1})]N_w(N_w^2+N_u^2)+2[N_u(SD^{-1})]N_u(N_w^2+N_u^2)+P[N_w^2(D^{-1})](N_w^2+N_u^2)\\&\quad+[N_u^2(SD^{-1})](N_w^2+N_u^2)-2P[N_w(SD^{-1})]N_w-2[N_u(S^2D^{-1})]N_u+2[N_u(D)]N_u\\&\quad-P[N_w^2(SD^{-1})]-[N_u^2(S^2D^{-1})]+[N_u^2(D)]\}E_v\ .\end{aligned} \tag{4}$$

The expression included within the first set of curly brackets on the right-hand side of Eq. (4) is present in the dispersion relation appropriate to a homogeneous cold plasma. The second set of curly brackets includes terms which depend on the derivatives of the dielectrics and so contribute to the dispersion relation only if the plasma is inhomogeneous. The form of Eq. (4) illustrates the significant point that the mode solutions appropriate to general oblique propagation in a cold homogeneous plasma are coupled through the explicit contributions associated with the derivatives of the dielectrics.

An analysis of mode propagation based on weakly inhomogeneous plasma theory is not appropriate to situations where the scale length of dielectric variation is comparable to the characteristic wavelength of modes in a homogeneous plasma.[21,22] In general, for ion-cyclotron range of frequency waves, this means that a WKB-type approach cannot be reasonably invoked if the dielectrics vary appreciably within a distance of an ion skin depth.

A. Coupling of infinite and homogeneous fast and slow mode

In infinite and homogeneous cold-plasma theory with one-ion species, the wave corresponding to the fast-wave root does not heat ions at the fundamental of the ion-cyclotron frequency because of its pure right-hand circular electric field polarization.[2] Based on the discussion of Sec. II, to demonstrate that plasma inhomogeneities couple the infinite and homogeneous fast and slow waves, it is only necessary to show that the inhomogeneous contributions to the dispersion relation, Eq. (4), are at least as large as the contributions which persist in the homogeneous limit. In particular, for propagation oblique to the background magnetic field,

$$1\lesssim |S|^{-1}|DN_w^2(D^{-1})| \tag{5}$$

is a sufficient condition to show that strong coupling can occur in the limit of $|\omega-\Omega_i|/\Omega_i \ll 1$ for Ω_i the ion gyrofrequency.

Since magnetic field lines in EBT-S are, in general, not coincident with surfaces of constant magnetic field strength, the ion-gyrofrequency spatial variation along magnetic field lines near the gyroresonance point is taken to have the form

$$\Omega_i \simeq \omega(1+w/L_1+w^2/2L_2^2)\ , \tag{6}$$

where L is a constant. It follows from Eq. (2) that Eq. (5) can be rewritten as

$$\begin{aligned}1&\lesssim\left|\frac{\omega^2-\Omega_i^2}{\omega_i^2}\right|\left[\frac{c}{\omega L_1}\right]^2\left|\frac{2(\omega^4-3\omega^2\Omega_i^2+6\Omega_i^4)}{(\omega^2-\Omega_i^2)^2}+\left[\frac{L_1}{L_2}\right]^2\frac{3\Omega_i^2-\omega^2}{\omega^2-\Omega_i^2}-\frac{2(3\Omega_i^2-\omega^2)^2}{(\omega^2-\Omega_i^2)^2}\right|\\&\underset{\omega\to\Omega_i}{=}2\left[\frac{c}{\omega_i L_1}\right]^2(3+L_1^2/L_2^2)\ ,\end{aligned} \tag{7}$$

which is independent of magnetic field strength.

For parameters characteristic of a hydrogen ion-cyclotron heating experiment in EBT-S (i.e., $L_1 \approx L_2 \approx 20-40$ cm, and ion density $n_i \lesssim 2\times 10^{12}$ cm^{-3}), both the right-hand and left-hand sides of Eq. (7) are of comparable magnitude. Hence, infinite and homogeneous fast and slow waves must be strongly coupled. The composite inhomogeneous mode is generally elliptically polarized and so becomes subject to appreciable ion-cyclotron absorption. However, it also follows that absorption at fundamental ion-cyclotron layers is likely to be incomplete because the infinite and homogeneous fast-wave component of the inhomogeneous mode is generally only weakly affected at these locations. In general, the degree of coupling and heating requires the detailed evaluation of the exact wave equation in the relevant geometry.

B. Electrostatic waves in EBT-S

So far it has been demonstrated that inhomogeneity leads to coupling of infinite and homogeneous fast and slow modes. It is now shown that singular solutions to Eq. (3) are inconsistent with the small ion-gyroradius approximation assumed by the dispersion relation.

In infinite and homogeneous plasma theory, electrostatic waves with frequencies comparable to the ion-cyclotron frequency are possible for parameters typical of magnetic fusion-oriented devices if more than one-ion species is present within the plasma.[2,8,14–16] In particular, for a two-ion species plasma the wave frequency of the electrostatic mode lies between the ion gyrofrequencies of the two-ion species. Interest in these electrostatic waves arises in the application of wave heating to fusion-oriented devices because they are relatively short-wavelength phenomena which can be driven by mode coupling to long-wavelength fast waves in weakly inhomogeneous plasma theory.[8,14–16]

In the cold-plasma approximation waves with asymptotically electrostatic character are singular solutions to the dispersion relation when inverse perpendicular wave numbers are much longer than the ion gyroradius. Hence, for electrostatic waves not to exist within the cold-plasma approximation for EBT-S, it is sufficient to show that in Eq. (3)

$$|(PN_w^2+SN_u^2)F_v| \lesssim |DP[N_w^2(D^{-1})]F_v| \quad (8)$$

when

$$|F_v^{-1}\rho_i^2k_w^2F_v| \ , \quad |F_v^{-1}\rho_i^2k_u^2F_v| \ll 1 \ ,$$
$$F_v \equiv (N_w^2+N_u^2)E_v \ . \quad (9)$$

In Eq. (9), ρ_i is a typical ion gyroradius. Equations (8) and (9) represent a single sufficient condition for electrostatic waves not to be present, because other coefficients of third and lower derivative terms acting on E_v contribute, along with the term singled out in Eq. (8), to the dispersion relation.

If the z-directed ambient magnetic field strength variation has the form

$$\Omega_s = \Omega_{so}(1+w/L_1+w^2/2L_2^2) \ , \quad (10)$$

with Ω_{so} a constant, then Eq. (8) can be rewritten as

$$D^2|(N_w^2+SP^{-1}N_u^2)F_v| \lesssim |\{D[N_w^2(D)]-2[N_u(D)]^2\}F_v| \quad (11)$$

with

$$N_w^2D = \left[\frac{c}{\omega L_2}\right]^2 \sum_i \frac{\omega_i^2\omega(3\Omega_i^2-\omega^2)}{\Omega_i(\omega^2-\Omega_i^2)^2} + 2\left[\frac{c}{\omega L_1}\right]^2 \sum_i \frac{\omega_i^2\omega}{\Omega_i} \frac{(\omega^4-3\Omega_i^2\omega^2+6\Omega_i^4)}{(\omega^2-\Omega_i^2)^3} \ ,$$
$$N_wD = \left[\frac{c}{\omega L_1}\right] \sum_i \frac{\omega_i^2\omega}{\Omega_i(\omega^2-\Omega_i^2)^2}(3\Omega_i^2-\omega^2) \ . \quad (12)$$

In Eq. (12), subscript i denotes ion species.

Because weakly damped electrostatic waves and mode conversion in the ion-cyclotron range of frequencies are present in a weakly inhomogeneous plasma for $|N_w^2F_v| \ll |N_u^2F_v|$, Eqs. (9) and (11) are examined in the limit of $N_w^2F_v=0$.[8,14–16] However, it should be noted that the right-hand side of Eq. (8) is independent of angle of propagation and so magnetic inhomogeneity always has some effect on wave propagation for any angle of propagation.

To quantitatively evaluate Eq. (11) under the constraint of Eq. (9), we consider a hydrogen-deuterium plasma with different values of hydrogen density (N_1) and deuterium density (N_2) and a total ion density of 2×10^{12} cm^{-3}. The magnetic field strength at $w=0$ is taken to be 7×10^3 G and an ion temperature $T_i=100$ eV is assumed. In infinite homogeneous plasma theory, weakly damped electrostatic waves occur in the limit of $S\rightarrow 0$, with S defined in Eq. (4). Hence, as an additional conservative act, the numerical value of S in Eq. (11) is given the value

$$S=S' \equiv (\omega_1^2+\omega_2^2)/|\Omega_1^2-\Omega_2^2| \quad (13)$$

with subscripts 1 and 2 denoting hydrogen and deuterium quantities, respectively.

Results of the numerical evaluation of Eq. (11) are tabulated in Table I for the special case $L_2 = L_1/(2)^{1/2}$. The important feature of the table is the clearly greater-than-one values for $|F_v^{-1}\rho_1^2 k_u^2 F_v|$ for a variety of different hydrogen and deuterium densities. The quantity is especially large when one of the ions is clearly a minority species. The implication of this result is that the cold-plasma electrostatic approximation and weakly inhomogeneous mode conversion theory are not valid in the ion-cyclotron range for a strongly inhomogeneous plasma like EBT-S. It is also worth noting that if the conservative assumptions of $|S| = |S'|$ and $T_i = 100$ eV are relaxed such that $|S| < |S'|$ and $T_i > 100$ eV are permitted, then the values of $|F_v^{-1}\rho_1^2 k_u^2 F_v|$ are larger than those tabulated.

III. SUMMARY AND DISCUSSION

This paper has theoretically addressed the important role which inhomogeneity plays in electromagnetic mode propagation. It has been shown that the derivatives of the electromagnetic dielectric elements must be included self consistently in analyzing the propagation of waves in the ion-cyclotron range of frequencies for relatively low-density machines like EBT-S which have large continuous magnetic field variations. Two examples have been examined which indicate that conclusions about wave propagation and heating based upon infinite and homogeneous or weakly inhomogeneous WKB plasma-wave theory are not reliable for devices like EBT-S. First, it has been shown that ion skin depth c/ω_i is comparable to the magnetic field inhomogeneity scale lengths. Consequently, fast-wave excitation based on weakly inhomogeneous wave theory should result in the coupling of the infinite and homogeneous fast and slow waves in the strongly inhomogeneous EBT-S plasma. The result should be fundamental ion-cyclotron frequency in a single-ion species plasma. Similarly, slow-wave excitation based on weakly inhomogeneous plasma theory should result in wave penetration beyond the ion-cyclotron resonance layer because of coupling between the infinite and homogeneous slow and fast waves in the strongly inhomogeneous EBT-S plasma. Second, it has been demonstrated that the inhomogeneous cold-plasma dispersion relation does not consistently include singular electrostatic solutions for EBT-S parameters and $k_u\rho_i < 1$. Moreover, the large values of $|F_v^{-1}\rho_1^2 k_u^2 F_v|$ in Table I indicate that in a two-ion species plasma values of $k_u\rho_i > 1$ are required for electrostatic waves with frequencies between the two-ion gyrofrequencies. Hence, to ascertain the possible mode conversion process of electromagnetic to electrostatic waves in a machine like EBT-S with substantial magnetic field gradients parallel to the magnetic field direction, it is inappropriate to assume $k_u\rho_i$ small, as in previous mode conversion analyses, but rather it is necessary to include arbitrary $k_u\rho_i$. In fact, previous theoretical studies show that small finite parallel wave number precludes mode conversion and so it follows that strong parallel magnetic field strength variations, such as found in EBT-S, likely lead to direct particle heating rather than mode conversion.[8,16]

Although effects of magnetic field gradients on wave properties near the fundamental ion-cyclotron resonances were primarily examined, it should be pointed out that the gradients can influence wave properties for $\Omega \approx 2\Omega_i$, albeit to lesser extent than for $\omega \approx \Omega_i$. In particular, for $\omega \approx 2\Omega_i$, Eq. (5) indicates appreciable modification of wave properties for relatively low-density operation (e.g., $n_i \lesssim 2\times 10^{12}$ cm^{-3}) and relatively sharp gradients $L_1 \approx 20$ cm. For higher-frequency, shorter-wavelength modes inhomogeneity effects are increasingly less significant in EBT-S.

Effects like bounce electron motion[19] and finite ion transit time through cyclotron resonance zones[20] have not been included in the present formalism. Suffice it to say that these processes further modify the dispersion characteristics of waves away from the predictions of infinite and homogeneous cold-plasma theory. It follows that the results of this paper, particularly those related to elliptic polarization of modes at the fundamental ion-cyclotron resonance, are enhanced by the mentioned processes.

In conclusion, wave propagation properties in the ion-cyclotron range of frequencies has been examined for a strongly inhomogeneous plasma like

TABLE I. Evaluation of $|F_v^{-1}\rho_1^2 k_u^2 F_v|$ under the assumptions and parameters indicated in the text and for different hydrogen and deuterium densities.

$n_1(10^{12}$ cm$^{-3})$	$n_2(10^{12}$ cm$^{-3})$	$\|F_v^{-1}\rho_1^2 k_u^2 F_v\|$
1.9	0.1	2.2×10^2
1.5	0.5	1×10^1
1.0	1.0	5
0.5	1.5	8
0.1	1.9	1.4×10^2

EBT-S. Although realistic quantitative estimates of wave polarizations and heating must await the development of accurate eigenmode codes for realistic Bumpy Torus geometry, the results of this paper can help in the development of such codes and in the interpretation of numerical results once they are developed. In projected denser and larger Bumpy Torus devices like EBT-P, the inhomogeneous effects discussed at length in this paper should play a less significant, although not necessarily, negligible role.[9] Homogeneous wave theory should provide some increased intuition for ion-cyclotron range of frequency heating experiments in EBT-P.

ACKNOWLEDGMENTS

This paper has benefitted from discussions with Dr. M. Cotsaftis, Dr. N. T. Gladd, Dr. S. Hamasaki, Dr. N. A. Krall, Dr. R. Kashuba, Dr. J. Mullen, Dr. F. W. Perkins, Dr. J. C. Sprott, and Dr. D. G. Swanson.

[1]T. H. Stix, Nucl. Fusion 15, 737 (1975).
[2]T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962).
[3]G. R. Haste, Bull. Am. Phys. Soc. 25, 921 (1980).
[4]J. H. Mullen (private communication).
[5]W. P. Allis, S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas* (MIT, Cambridge, Mass., 1963).
[6]J. C. Hosea and R. M. Sinclair, Phys. Fluids 13, 701 (1970).
[7]F. J. Paoloni, Phys. Fluids 18, 640 (1975).
[8]F. W. Perkins, Nucl. Fusion 17, 1197 (1977).
[9]F. J. Paoloni, Nucl. Fusion 18, 359 (1978).
[10]W. N-C. Sy and M. Cotsaftis, Plasma Phys. 21, 985 (1979).
[11]J. M. Kindel, A. T. Lin, J. M. Dawson, and R. M. Martinez, Phys. Fluids 24, 498 (1981).
[12]D. W. Ross, G. L. Chen, and S. M. Mahajan, in *Annual Controlled Fusion Theory Conference* (University of Texas, Austin, Texas, 1981), Paper 2A4.
[13]J. D. Barter and J. C. Sprott, Plasma Phys. 19, 945 (1977).
[14]J. Jacquinot, B. D. McVey, and J. E. Scharer, Phys. Rev. Lett. 39, 88 (1977).
[15]J. E. Scharer, B. D. McVey, and T. K. Mav, Nucl. Fusion 17, 2 (1977).
[16]D. G. Swanson, Nucl. Fusion 20, 949 (1980).
[17]T. J. Menne, W. B. Ard, R. E. Juhala, R. J. Kashuba, J. H. Mullen, J. C. Sprott, G. A. Gerdin, P. L. Colestock, W. M. Hooke, J. C. Hosea, H. H. Klein, N. A. Krall, J. B. McBride, J. L. Sperling, K. A. Connors, R. L. Hickok, Jr., and J. T. Woo, Bull. Am. Phys. Soc. 24, 1051 (1979).
[18]T. L. Owens, J. H. Mullen, and P. L. Huddleston, Bull. Am. Phys. Soc. 25, 904 (1980).
[19]J. Sperling, Phys. Fluids (in press).
[20]F. W. Perkins, Bull. Am. Phys. Soc. 25, 904 (1980).
[21]L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968), Chap. 8.
[22]T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962), Chap. 3.