Measurement of the K-shell ionization probability across a wide resonance: ^{88}Sr(p,p_0) at 6.06 MeV

J.-F. Chemin,* W. E. Meyerhof, R. Anholt, J. D. Molitoris, and Ch. Stoller†
Department of Physics, Stanford University, California 94305
(Received 19 April 1982)

We have measured the K-shell ionization probability across the 6.06-MeV resonance in ^{88}Sr(p,p_0) where the resonance width is large compared to the energy transferred to the electron. The results are found to agree quantitatively with the theory developed by Blair and Anholt. The effect of the time delay on the ionization probability, introduced by the nuclear scattering at the resonance energy, is discussed.

I. INTRODUCTION

Recent measurements have shown that the ionization probability at zero or nearly zero impact parameter may depend significantly on the proton energy if a nuclear elastic scattering resonance is traversed. Two collision systems have been investigated up to now. Blair *et al.* have reported an effect in the case of an s-wave resonance in $H^+ + {}^{58}$Ni elastic scattering at 3.15 MeV.[1] Chemin *et al.* investigated a d-wave resonance in $H^+ + {}^{88}$Sr elastic scattering at 5.06 MeV.[2] In both cases, the effect is interpreted as being due to the interference between the amplitudes for ionizing the K-shell electron on the way in and on the way out of the nuclear reaction. Both resonances were chosen so that the ratio of the 1s-electron binding energy U_K matches closely the width of the resonance Γ in order to maximize the energy dependence of the ionization probability. A quantum-mechanical formulation of the problem reproduced the experimental data in both cases.[1,2]

Duinker *et al.*[3] reported a large ($\sim 70\%$) change in the ionization probability across the s-wave resonance in $H^+ + {}^{12}$C elastic scattering at 461 keV. Here, the situation is completely different compared to the preceding resonances. The ratio of U_K (0.28 keV) to Γ(35 keV) should lead only to a very small effect on the ionization probability, according to the theoretical work of Blair *et al.*[4] on which the preceding interpretations were based. Other theoretical groups find the same result.[5,6] Since Duinker *et al.* suspected that a modification of the Blair theory might be needed, especially if $U_K \ll \Gamma$, we investigated the K-shell ionization probability P_K across the 6.06-MeV, s-wave, elastic scattering resonance in $H^+ + {}^{88}$Sr for which $U_K/\Gamma \simeq 0.24$. In Sec. II, we present the experimental results and a theoretical description of the data according to the above-mentioned theory of Blair and Anholt. In Sec. III, we derive an approximate expression for the ionization probability at the resonance energy, in order to make evident the actual dependence of P_K on the ratio U_K/Γ, or, equivalently, on the nuclear time delay introduced by the excitation of a resonance.

After completion of this work, a remeasurement of P_K across the 461-keV ^{12}C(p,p_0) resonance by Meyerhof *et al.*[7] has indicated no variation of P_K (within $\pm 20\%$) with the proton bombarding energy, contrary to the work of Ref. 3. The results of Ref. 3 are attributed to spurious effects.[7]

II. EXPERIMENTAL RESULTS

We used the particle x-ray coincidence technique to measure the Sr K-shell ionization probability at six different proton energies across the resonance between 5800 and 6250 keV. The particles were detected at 90° with respect to the beam direction. It was shown previously that this particular angle facilitates the data analysis, since the spin-flip contribution in the nuclear amplitude is zero.[1,2] The x rays were detected on each side of the beam by two thin NaI scintillators coupled to photomultipliers. The experimental setup and data acquisition system have been described previously.[2]

The energy dependence of the proton elastic scattering cross section $d\sigma/d\Omega$ is shown in Fig. 1(a). Experimental results have been normalized to the theoretical cross section outside of the resonance where Rutherford scattering dominates. The parameters of the resonance have been analyzed previously by Cosman *et al.*[8] who reported a total width $\Gamma = 70$ keV and a partial elastic proton width

$\Gamma_p = 46$ keV. With the use of the following relation to describe the scattering amplitude, we find that our experimental data are best reproduced with a partial width $\Gamma_p = 35$ keV:

$$f(\theta,E)=f_C+i(2K)^{-1}\sum_{lJ}(J+\tfrac{1}{2})\exp(2i\sigma_l)\times S_{lJ}P_l(\cos\theta)\;, \tag{1}$$

where f_C is the Coulomb scattering amplitude, $l=0$ and $J=\frac{1}{2}$ for this s-wave resonance, σ_l is the Coulomb phase shift, θ is the center-of-mass (c.m.) scattering angle, and K the c.m. momentum. Furthermore,

$$S_{lJ}=e^{2i\delta_l}\left[1-\frac{i\Gamma_p}{E-E_R-i\Gamma/2}\right]-1\;,$$

where δ_l is the complex background phase shift determined from an optical model and E and E_R are the bombarding and resonance energies, respectively.

Figure 1(b) shows relative ionization probabilities P_K measured at several energies across the resonance. Experimentally, P_K is the number of Sr K x rays emitted per elastically scattered proton at 90°. The error bars reflect only the statistical uncertainty for each measurement. Typically, the statistical error was $\pm 9\%$ after an average twenty-four-hour accumulation time per point with a true-to-random coincidence ratio equal to 1.

From this result, we note that:

1) The measured ionization probabilities outside of the resonance are almost identical, as expected. According to ionization theory, P_K should vary by less than 2% over this range of projectile energies.

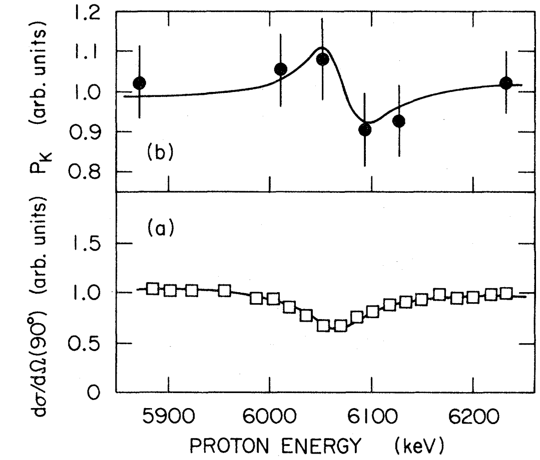

FIG. 1. (a) Differential elastic scattering cross section in arbitrary units vs the projectile energy for the reaction ^{88}Sr$(p,p_0)^{88}$Sr. Curve is calculated using relation (1). (b) Dependence of the relative Sr K-shell ionization probability on proton (laboratory) energy. Full curve is calculated according to relation (2) with the parameters given in the text.

2) In the vicinity of the resonance energy, the P_K values exhibit fluctuations which result from an interference effect between the ionization amplitudes on the way in and out of the nuclear reaction. The experimental inaccuracy prevents any definite conclusion concerning a real variation of P_K in this case, but indicates that this variation may be of the order of $\pm 10\%$.

This interpretation is supported by a comparison of the experimental results with theory. The ionization probability is written as[4]

$$P_K(\theta,E)=\int_0^\infty dE_f\sum_{\lambda\mu}|f(\theta,E)D^{\lambda}_{\mu 0}b_\lambda+(-1)^{-\lambda}f(\theta,E-\Delta E)\delta_{\mu 0}b^*_\lambda|^2/|f(\theta,E)|^2\;, \tag{2}$$

where $D^{\lambda}_{\mu 0}(\theta)$ is the rotational matrix element, $f(\theta,E)$ is the Coulomb and nuclear resonance scattering amplitude given by Eq. (1), $\Delta E = U_K + E_f$ is the energy lost by the projectile exciting the electron to a continuum state with energy E_f, $b_\lambda(E,E_f)$ is the amplitude for exciting the K-shell electron to a state with angular momentum λ and energy E_f on the way out of the collision, and b^*_λ is the amplitude for ionizing the electron on the way into the collision. Relative values of b_λ have been calculated from a semiclassical theory at the resonance energy.[2] In Fig. 1(b), the theoretical values of the ionization probability $P_K(E)$ have been corrected to account for the small ($<2\%$) overall energy dependence calculated according to the semiclassical theory. The solid line in Fig. 1(b) is the final result of the calculations. The experimental results are also plotted in relative values. A normalization was made by setting the mean value of P_K at the energies $E_P = 5870$ keV and 6232 keV equal to unity.

The comparison between the experimental data and the theoretical results shows good agreement. This demonstrates the ability of the theoretical formula to reproduce the variation of $P_K(E)$ across an s-wave resonance, even if U_K/Γ is smaller than unity. Since this theory also describes the data in a case of a d-wave resonance fairly well, its formulation appears to be well confirmed. Consequently, the large discrepancy between theoretical and experimental results found by Duinker *et al.*[3] in the case of the s-wave resonance at 461 keV in $H^+ + {}^{12}C$, where the ratio $U_K/\Gamma \simeq 0.008$, must be due to other causes. This has been confirmed meanwhile.[7]

III. INTERPRETATION OF RESULTS

We have shown[2] that the theory of Blair and Anholt[4] reproduces qualitatively and quantatively the experimental values of P_K in the two cases $U_K/\Gamma \simeq 1$ and $U_K/\Gamma < 1$. Nevertheless, the maximum variation of the ionization probability in the ${}^{88}Sr(p,p_0){}^{88}Sr$ system at

$$5.06 \text{ MeV } (U_K/\Gamma = 0.90)$$

and at

$$6.06 \text{ MeV } (U_K/\Gamma = 0.24)$$

is about the same, $\sim 20\%$ in the first case[2] and $\sim 10\%$ in the second case. In the experiment by Blair *et al.*[1] [${}^{58}Ni$ $(p,p_0){}^{58}Ni$ at 3.15 MeV], the effect is $\sim 50\%$ in a case where $U_K/\Gamma = 1.5$. A comparison between the values of the maximum excursions of $P_K(E)$ and the values of U_K/Γ does not lead to any simple relation.

In the following, we propose an approximate relation between the ionization values measured at the resonance energy and far from the resonance. The aim of this relation is not to replace the full calculation using Eq. (2), but to make evident the parameters which play a significant role in the magnitude of the $P_K(E)$ excursions.

We make the following approximations in Eq. (2): (1) The resonance is an s-wave resonance; (2) the background phase shift can be neglected, $\delta_0 = 0$; (3) in the atomic amplitude, we retain only the monopole interaction term ($\lambda = 0$); (4) we assume that the energy lost by the proton in the ionization process ΔE is equal to the binding energy of the electron U_K; (5) we set the phase of the atomic amplitude equal to zero. The analysis of the data taken near 5060 keV in ${}^{88}Sr(p,p_0)$ showed that this phase does not play an important role near $E = E_R$.[2] We define the following parameters:

$$y = 2U_K/\Gamma \ , \quad G = \eta \csc^2 \tfrac{1}{2}\theta \ ,$$

$$\gamma = \eta \ln(\csc^2 \tfrac{1}{2}\theta) \ , \quad g = \Gamma_p/(\Gamma G) \ ,$$

where $\eta = Z_1 Z_2 e^2/\hbar v$, Z_1, Z_2 are projectile, target atomic numbers, and v is the projectile velocity.

We describe the variation of the ionization probability across a scattering resonance using the ratio R defined by

$$R = P_K(\theta, E = E_R)/P_K(\theta, E \gg E_R) \ .$$

Through the use of the parameters given above, R can be written as

$$R = 1 - \frac{yg}{(1+y^2)} \frac{2\cos\gamma + y(3g - 2\sin\gamma)}{1 + 4g(g - \sin\gamma)} \ . \qquad (3)$$

One can see from this relation that there is no simple dependence of R on the parameter y alone, although $R = 1$ if $y = 0$. On the contrary, R is a function mainly of two parameters, y and g. The parameter y represents the degree of matching between the K-shell binding energy and the resonance width. The parameter g reflects the ratio between the fraction of the nuclear scattering reemitted in the elastic channel and the Coulomb scattering probability (proportional to the parameter G). The parameter g can also be interpreted as reflecting the fraction of particles in the elastic channel delayed by a mean lifetime $\hbar/\Gamma$ compared to the undelayed fraction of particles scattered by the Coulomb potential.

It is interesting to plot the function $(R-1)/g$ vs y for the resonant systems which have been investigated: $H^+ + {}^{88}Sr$(5.060 and 6.060 MeV) (Ref. 2); $H^+ + {}^{58}Ni$(3.151 MeV) (Ref. 1); and $H^+ + {}^{12}C$(461 keV).[7] The resonance at 5.060 MeV in ${}^{88}Sr$ is a d-wave resonance. This can be accounted for in relation (3) by changing the parameters g to

$$g_l = (J + \tfrac{1}{2}) P_l(\cos\theta) \Gamma_p / \Gamma G \ ,$$

where $P_l(\cos\theta)$ is the Legendre polynominal of or-

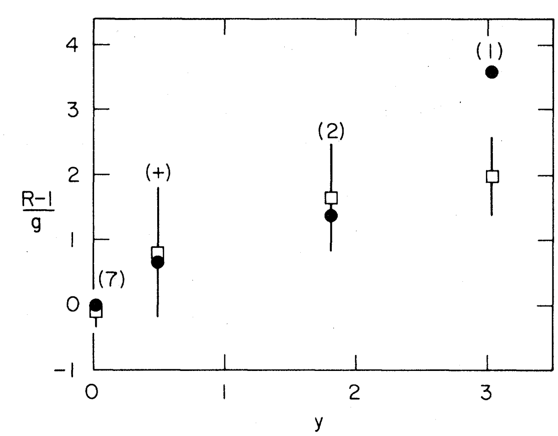

FIG. 2. Plot of a parameter describing the excursion of the ionization probability near a nuclear resonance, $(R-1)/g$, against the parameter $y \equiv U_K/\Gamma$. Open symbols represent the experimental values. Error bars correspond to the statistical uncertainty of the measurement. Solid symbols are calculated values from relation (3). Number above each point corresponds to the reference of the work, the cross to the present data.

der l, and γ to

$$\gamma_l = \ln(\csc^2 \tfrac{1}{2}\theta) - 2(\sigma_l - \sigma_0) \ .$$

The results of relation (3) are given by solid symbols in Fig. 2. The experimental results, taken from Refs. 1, 2, 7, and from the present data, are represented by open squares.

In spite of the drastic approximations made in deriving Eq. (3), the results appear in rather good agreement with the experimental data points. The discrepancy found for ^{58}Ni is mainly due to the fact that relation (3) does not account for the finite beam energy spread which reduces $R-1$ if the resonance is too narrow. In the representation of Eq. (3), the dependence of the P_K excursion on the lifetime factor y is clearly shown, as well as the requirement of a strong *nuclear* elastic fraction which is the only contribution in the elastic scattering cross section leading to a variation in the ionization probability. As an example, the larger ratio of U_K/Γ for the Sr 5.06-MeV resonance is partially counter balanced at the 6.06-MeV resonance by a smaller Coulomb scattering. In the Ni case, the larger Coulomb scattering is compensated by the presence of only one decay channel ($\Gamma_p/\Gamma = 1$).

IV. CONCLUSION

We have shown that the formulation developed by Blair and Anholt[4] describes successfully the ionization probability across a nuclear elastic scattering resonance in a case where the resonance width is larger than K-shell electron binding energy. The value of the ionization probability at the resonance energy has been shown to depend not only on the resonance width but also on the relative fraction of the nuclear elastic scattering compared to the Coulomb scattering.

ACKNOWLEDGMENTS

We appreciate helpful discussions with P. A. Amundsen. This work was supported in part by the National Science Foundation under Grant No. PHY80-15348.

---

*Permanent address: Centre d'Etudes Nucléaires, Université de Bordeaux I, Le Haut-Vigneau, 33170 Gradignan, France.

†Present address: Laboratorium für Kernphysik, Eidgenössische Technische Hochschule, 8093 Zürich, Switzerland.

[1]J. S. Blair, P. Dyer, K. A. Snover, and T. A. Trainor, Phys. Rev. Lett. 41, 1712 (1978).

[2]J. F. Chemin, R. Anholt, Ch. Stoller, W. E. Meyerhof, and P. A. Amundsen, Phys. Rev. A 24, 1218 (1981).

[3]W. Duinker, J. Van Eck, and A. Niehaus, Phys. Rev. Lett. 45, 2102 (1980).

[4]J. S. Blair and R. Anholt, Phys. Rev. A 25, 907 (1982).

[5]J. M. Feagin and L. Kocbach, J. Phys. B 14, 4349 (1981).

[6]K. W. McVoy and H. A. Weidenmüller, Phys. Rev. A 25, 1462 (1982).

[7]W. E. Meyerhof, G. Astner, D. Hofmann, K.-O. Groeneveld, and J. F. Chemin (unpublished).

[8]E. R. Cosman, H. A. Enge, and A. Sperduto, Phys. Lett. 22, 195 (1966).