Comments

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Comment on "Radiative collapse of a relativistic electron-positron plasma to ultrahigh densities"

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The ultradense plasma appearing as a result of the radiative collapse and the radiative cooling of two counterstreaming relativistic electron and positron beams is considered on the basis of quantum Fermi statistics. There are two different limiting cases (quasiclassical and ultraquantum) depending on the value of the relativistic factor γ. Winterberg's estimate for the density, based on Heisenberg's uncertainty principle, corresponds to the ultraquantum limit. In the quasiclassical case the compression is limited by the Pauli-Fermi principle, and the density of the plasma increases more sharply with the current (as $I^{3/2}$ for $I << I_A/\beta\gamma$ and I^3 for $I >> I_A/\beta\gamma$, $I_A = 17$ kA being the Alfvèn current).

In his paper[1] Winterberg proposed to create a plasma of ultrahigh density by means of the radiative collapse of two counterstreaming beams of relativistic electrons and positrons. This proposal can turn out to be essential for the future development of physics. Successes of high-current electronics give hope that the achievement of 10- and possibly 100-kA current values of positrons in storage rings is not unfeasible. The realization of such a project would allow one to obtain ultradense matter, ultrahigh electric and magnetic fields, and ultrahigh power bursts of coherent x-ray and γ-ray radiation in the laboratory. For this reason the detailed analysis of the physical nature of the plasma equilibrium in the collapsed state based on the theory of equilibrium[2,3] and radiation[4,5] of intense streams of charged particles, confined by electromagnetic forces of collective interaction, is of considerable interest.

Winterberg's estimate[1] for the density has been based on the assumption that all the charges occupy the lowest energetic state (the ground state). The radius of the region occupied by the charges in the ground state was evaluated with the aid of Heisenberg's uncertainty principle. The exact analysis of the plasma structure for this ultraquantum limit would have required jointly solving Dirac equations for electron and positron wave functions with Maxwell equations for the field.

We consider the opposite (quasiclassical) limit, when the radial motion of charges corresponds to a large number of occupied energy levels. In this case the plasma radius in the state of the maximal compression is determined by the Pauli principle for electrons and positrons. This plasma structure is similar to that of a "linear atom"[6] appearing in high-current diodes as a result of superpinching.[7] The main difference is the presence of positrons instead of ions.

We assume that the kinetic energy of the streaming motion of charges $(\gamma-1)mc^2$ is large compared to the energy dispersion: $(\gamma-1)mc^2 >> T,E_F$. The energy dispersion of charges is defined by the range of occupied energy states (by the Fermi energy E_F in the low-temperature limit $T << E_F$ or by the temperature T in the case $T >> E_F$). Under these conditions thermodynamic equlibrium is achieved separately in each of the two subsystems (electrons and positrons) much more quickly than the relaxation of the plasma as a whole takes place. The collapsed electron-positron plasma is of physical interest for times which are large compared to the relaxation times inside separate subsystems, but small compared to the time of the electron-positron annihilation. Over these times electron and positron distribution functions in the moving frames of reference K'_α have the form of equilibrium Fermi functions

$$f'_\alpha=\{\,1+\exp[(E'_\alpha-\Gamma_\alpha)/T_\alpha]\,\}^{-1}\,. \tag{1}$$

The index α means e and p for electrons and positrons, respectively; E'_α, Γ_α, and T_α are the total energy, the chemical potential and the temperature of the α-type charges. Using the relativistic invariance of distribution functions,[8] we find f_α in the laboratory frame just by expressing E'_α in (1) through the energy E_α and the generalized momentum $\vec{\mathrm{P}}_\alpha$ with the aid of the Lorentz transformation formula,

$$E'_\alpha=\gamma_\alpha(E_\alpha-\vec{\mathrm{P}}\cdot\vec{\mathrm{V}}_{0\alpha})$$
$$=\gamma_\alpha\left[e_\alpha\phi+\epsilon_\alpha\left[\vec{\mathrm{P}}_\alpha-\frac{e_\alpha}{c}\vec{\mathrm{A}}\right]-\vec{\mathrm{P}}_\alpha\cdot\vec{\mathrm{V}}_{0\alpha}\right]\,.$$

Here $\epsilon_\alpha(p)=(m^2c^4+p^2c^2)^{1/2}$; $\vec{\mathrm{V}}_{0\alpha}$ is the velocity of the frame K'_α against the laboratory frame K; ϕ and $\vec{\mathrm{A}}$ are the potentials of the electromagnetic field created by electrons and positrons. At the maximum contraction this field is much greater than external fields, which may be neglected in the plasma structure analysis. With the same degree of accuracy the curvature of the current canal may also be neglected.

As a result of radiative cooling the temperature of the charges decreases rapidly and becomes much lower than the Fermi energy: $T_\alpha << E_F$. So one is allowed to substitute $T_\alpha=0$ in (1), keeping in mind however that the temperature of charges may still be high from other points of view. This approximation has been used by Landau[9] in the analysis of the equilibrium structure of nonrelativistic stars.

Introducing "potentials" U_α of forces acting on the charges of the α type,

$$U_\alpha=e_\alpha(\phi-\vec{\mathrm{V}}_{0\alpha}\vec{\mathrm{A}}/c)-\Gamma_\alpha/\gamma_\alpha\,,$$

and choosing additive constants $U_{0\alpha}+U_\alpha(0)<0$ so that $n_\alpha\to 0$ when $U_\alpha\to 0$, we find after the integration over the kinetic momentum $\vec{\mathrm{p}}=\vec{\mathrm{P}}-e_\alpha\vec{\mathrm{A}}/c$:

$$n_\alpha=\frac{2}{(2\pi\hbar)^3}\int f_\alpha d^3p$$
$$=\frac{\gamma_\alpha}{3\pi^2}\left[\frac{m_\alpha c}{\hbar}\right]^3F(\gamma_\alpha U_\alpha/m_\alpha c^2),\quad \alpha=e,p\,. \tag{2}$$

Here $F(x)=[(1-x)^2-1]^{3/2}$ for $x<0$ and $F(x)=0$ for $x>0$. The charges occupy all the quantum states in the energy range $U_{0\alpha}<E<0$, so that the Fermi energy is equal to $|U_{0\alpha}|$. The space distribution of U_α is determined by equations

$$\nabla^2U_\alpha=-\frac{4}{3\pi}\left[\frac{c}{\hbar}\right]^3e_\alpha\times\sum_\beta e_\beta m_\beta^3\gamma_\beta(1-\vec{\beta}_\alpha\vec{\beta}_\beta)F(\gamma_\beta U_\beta/m_\beta c^2),\quad \alpha=e,p \tag{3}$$

derived easily from the equations of magneto- and electrostatics

$$\nabla^2\vec{\mathrm{A}}=-4\pi\sum_\alpha e_\alpha n_\alpha\vec{\beta}_\alpha,\quad \nabla^2\phi=-4\pi\sum_\alpha e_\alpha n_\alpha,$$
$$\vec{\beta}_\alpha=\vec{\mathrm{V}}_{0\alpha}/c\quad \gamma_\alpha=(1-\beta_\alpha^2)^{-1/2}$$

with account of (2).

For counterstreaming electrons and positrons with equal current values $\gamma_e=\gamma_p\equiv\gamma$, $N_e=N_p\equiv N$ the system is symmetric against the exchange $e\leftrightarrow p$ and the radial distributions of electrons and positrons are just the same: $n_e=n_p\equiv n$ $U_e=U_p\equiv U$. In this case the set of Eq. (3) degenerates into the single equation, which can be written in the form

$$\frac{1}{\xi}\frac{d}{d\xi}\xi\frac{d\phi}{d\xi}=F(\phi)$$

with the aid of dimensionless variables $\phi=\gamma U/mc^2$, $\xi=r/r_0$, and

$$r_0^2=3\pi\hbar^3/8e^2m^2c\gamma^2\beta^2\,.$$

TABLE I. The function $\lambda(\phi_0)$ found numerically.

ψ_0	λ
0	0
0.2	0.2075
0.4	0.406
0.6	0.597
0.8	0.785
1	0.966
1.2	1.14
1.4	1.32
1.8	1.66
2	1.825
3	2.64
5	4.21
10	8.01
20	15.5
50	37.7
100	74.7

The boundary conditions are $\phi(0)=-\phi_0$, $\phi'(0)=0$.

The only dimensionless parameter ϕ_0 (the dimensionless Fermi energy) is connected with the number of particles per unit length of the current canal by the normalization relation

$$N=I/2eV_0=2\pi\int_0^\infty n(r)r\,dr \ ,$$

which can be written in the form

$$I=(I_A/2\beta\gamma)\lambda(\gamma E_F/mc^2).$$

Here $\lambda(\phi_0)=\lim_{\xi\to\infty}d\phi/d\ln\xi$ and $I_A=mc^3/e=17$ kA is the Alfvèn current. In Table I we present the function $\lambda(\phi_0)$ found numerically.

If both γ and $\beta\sim 1$ and $I\sim I_A$ the degenerate Fermi gases of the charges are relativistic: $\phi_0\sim 1$, i.e., $E_F\sim mc^2$. In the limits of small and large ϕ_0 the function $\lambda(\phi_0)$ is a linear one: $\lambda=1.06\phi_0$, $\phi_0<<1$; $\lambda=0.74\phi_0$, $\phi_0>>1$, Hence the Fermi energy is a linear function of the current

$$E_F\approx mc^2\beta I\Big/I_A \ .$$

The maximum value of the density near the axis is proportional to $I^{3/2}$ in the nonrelativistic case

$$n_{\max}=0.25(mc/\hbar)^3\gamma^{5/2}\beta^{3/2}(I/I_A)^{3/2}\ , \quad I<<I_A/\beta\gamma \tag{4}$$

and increases as I^3 in the ultrarelativistic case

$$n_{\max}=0.67(mc/\hbar)^3\gamma^4(\beta I/I_A)^3\ , \quad I>>I_A/\beta\gamma\ . \tag{5}$$

It can be shown that if the currents of electrons and positrons are not equal, the maximum value of the density is determined by the smaller one.

Noticing that in the region $\beta\sim 1$ and $I\sim I_A/\gamma$ the radius r_0 is of the order of $(\hbar/mc)\alpha^{-1/2}\gamma^{-1}$ and that the momentum of the transverse motion $p_r\sim mc$, one can reduce the quasiclassicality condition $r_0p_r>>\hbar$ to the form

$$\alpha\gamma^2<<1\ ,$$

where $\alpha=e^2/\hbar c=\frac{1}{137}$.

Together with the Winterberg's estimate [Eq. (18) in Ref. 1], valid for the high-energy region $\gamma^2>>\alpha^{-1}$, our formulas (4) and (5), valid for $\gamma^2<<\alpha^{-1}$, give the complete picture of the current dependence of the maximum density of matter achievable in the process of radiative collapse of a relativistic electron-positron plasma.

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