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Anomalous diffusion due to the magnetostatic mode in an electron-hole plasma

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It is noted that, in a manner similar to that of the electrostatic-convective-cell mode, the zero-frequency magnetostatic mode in an electron-hole plasma can cause anomalous particle diffusion across the external magnetic field.

Okuda, Dawson, and Hooke[1] have interpreted an enhanced Bohm-like diffusion[2] across the applied magnetic field in germanium plasmas near thermal equilibrium in terms of vortex diffussion[3–5] due to thermally excited plasma convection. The enhanced diffusion, occurring at high magnetic fields, arises from the collective $\vec{E} \times \vec{B}_0$ (where $\vec{E}$ is the electrostatic field of the mode and $\vec{B}_0 = B_0\hat{z}$ is the applied magnetic field) drifts of the electrons and holes. This phenomenon is physically analogous to the electrostatic convective cell diffusion in a low-density, highly magnetized electron-ion plasma, since the $\vec{E} \times \vec{B}_0$ drift is independent of the mass of the species.

Besides the convective cells, the plasma can also support a magnetostatic mode at zero frequency.[6] This mode is similar to the electrostatic zero-frequency convective cell mode in many respects. In particular, the cross-field test-electron diffusion due to the $v_z\vec{B}/B_0$ drift (where $\vec{B}$ is the perturbation magnetic field and v_z is the parallel velocity of the test electron) in a thermal plasma also exhibits Bohm-like behavior. The main difference, on the other hand, is that here the ion diffusion rate is much smaller than that of the electrons, since the ions have a heavier mass. The net-particle diffusion is, however, ambipolar, in order to maintain charge neutrality. Consequently, the magnetostatic mode in an electron-ion plasma only gives rise to enhanced electron shear viscosity and heat conductivity, but not particle diffusion.

From the above discussion, it is evident that in an electron-hole plasma, where the electrons and holes have nearly equal mass, the magnetostatic mode can also give rise to net-particle diffusion. In order to avoid nonessential details, we shall consider the particularly simple case[1] in which the electrons and holes have equal mass (m) and charge ($\pm q$), as well as equal collision frequency (ν) with the lattice. We shall also assume a two-dimensional-mode structure in the plane perpendicular to the applied field.

The equations governing the magnetostatic mode are

$$\partial_t v_{zj} = -(q_j/mc)\partial_t A - \nu v_{zj}, \tag{1}$$

$$c\nabla^2 A = -4\pi n_0 q(v_{iz} - v_{ez}), \tag{2}$$

where $q_j = \pm q$, and the perturbation magnetic field $\vec{B} = -\hat{z} \times \nabla A$ is perpendicular to $\vec{B}_0$.

From (1) and (2), one obtains

$$\omega = i\nu c^2k^2/(c^2k^2 + 2\omega_p^2), \tag{3}$$

where $\omega_p^2 = 4\pi n_0 q^2/\mathrm{m}$.

Following Ref. 6, one can easily obtain the test-particle (electron or hole) diffusion coefficient

$$D_m = \frac{T_{\parallel}}{B_0}\left[\frac{2}{mL_{\parallel}}\ln\frac{L_{\perp}\omega_p}{\pi c}\right]^{1/2}, \tag{4}$$

where $T_{\parallel}$ is the parallel temperature of the electrons and holes, $L_{\parallel}$ and $L_{\perp}$ are the parallel and perpendicular linear dimensions of the system. In obtaining (4), the fluctuation-dissipation theorem and the orbit diffusion theory[7] have been used to calculate the magnetic field correlation, which is needed because the test particle executes $v_z\vec{B}/B_0$ drift,[6] where v_z is the test-particle velocity parallel to $\vec{B}_0$. Equation (4) is valid if the fluctuation-induced orbit decorrelation dominates over the collisional damping, or

$$\frac{\omega_p^2 T}{c^2 B_0}\left[\frac{1}{mL_{\parallel}}\right]^{1/2} >> \nu,$$

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which is satisfied only for a hot-electron—hole plasma of small parallel extent.[8] On the other hand, above-thermal magnetic fluctuations can also cause the fluctuation-induced orbit diffusion to dominate.[6,8]

The diffusion coefficient given in (4) is valid for both electrons and holes, so that it is also ambipolar, leading to net-particle diffusion.[9] Thus, like the convective diffusion, the magnetostatic mode diffusion is also Bohm-like and independent of the collision frequency.

The ratio of magnetostatic mode diffusion coefficient to two-dimensional convective cell diffusion coefficient (D_c) is

$$\frac{D_m}{D_c}=\frac{T}{mc^2}\left[1+2\frac{\omega_p^2}{\Omega^2}\right]^{1/2}, \tag{5}$$

where $\Omega = qB_0/\mathrm{mc}$, and we have assumed that the temperature is isotropic. Thus, the magnetostatic mode diffusion is stronger when $2\omega_p^2 \gtrsim \Omega^2$ and $8\pi n_0 T/B_0^2 > 1$.

In conclusion, we have demonstrated by means of a simple model that two-dimensional magnetostatic mode electron and hole diffusion can be as important as the electrostatic convective cell diffusion. This result may have significant effect on related phenomena, such as interaction of high-frequency waves with the magnetostatic mode,[10] interaction between the magnetostatic mode and the convective cell mode,[11] etc. These problems, as well as a more realistic formulation of the present problem, are, however, beyond the scope of this note.

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[1]H. Okuda, J. M. Dawson, and W. M. Hooke, Phys. Rev. Lett. 29, 1658 (1972).

[2]M. N. Gurnee, W. M. Hooke, G. J. Goldsmith, and M. H. Brennan, Phys. Rev. A 5, 158 (1972).

[3]J. B. Taylor and B. McNamara, Phys. Fluids 14, 1492 (1971).

[4]J. M. Dawson, H. Okuda, and R. N. Carlile, Phys. Rev. Lett. 27, 491 (1971).

[5]H. Okuda and J. M. Dawson, Phys. Fluids 16, 408 (1973).

[6]C. Chu, M. S. Chu, and T. Ohkawa, Phys. Rev. Lett. 41, 653 (1978).

[7]C. T. Dum and T. H. Dupree, Phys. Fluids 13, 2064 (1970).

[8]H. Okuda, W. W. Lee, and A. T. Lin, Phys. Fluids 22, 1899 (1979).

[9]If the mass and temperature are not exactly the same, but similar, for the electrons and holes, the diffusion coefficients of the two species will differ somewhat from (4), the net diffusion will then follow the specie with smaller T/m.

[10]P. K. Shukla, H. U. Rahman, M. Y. Yu, and R. K. Varma, Phys. Rev. A (in press).

[11]P. K. Shukla, M. Y. Yu, and K. H. Spatschek, Phys. Rev. A 23, 3247 (1981).