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Modulational instability of electromagnetic ion-cyclotron waves by ion acoustic waves in a collisionless plasma

Md. Salimullah and Farida Majid
Department of Physics, Jahangirnagar University, Dacca, Bangladesh
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Using the hydrodynamic model for a homogeneous plasma, we have investigated the modulational instability of an electromagnetic ion-cyclotron wave by a low-frequency electrostatic ion acoustic mode in a collisionless plasma. The low-frequency nonlinearity of the instability arises through the parallel ponderomotive force on the ions, and the high-frequency nonlinearity arises through the nonlinear current densities of the ions. For typical plasma parameters $n_0^0 = 10^{13}$ cm^{-3}, $\omega_{ci}/\omega_0 = 1.2$, $B_s = 10$ kG, $T_i = 1$ keV, $T_i = 0.05$ keV, and $\theta = 30°$, the growth rate of the instability turns out to be $\sim 10^4$ rad sec^{-1} for $|v_{0ix}|C_s| = 10^{-2}$, where v_{0ix} is the pump-induced drift velocity of the ions and C_s is the ion acoustic velocity in the plasma.

I. INTRODUCTION

The heating of ions in a tokamak by ion-cyclotron radio-frequency (ICRF) waves has attained considerable interest in recent years.[1-6] This is recognized as one of the most promising methods of heating a fusion plasma. Many experiments were performed using slow waves or fast waves in toroidal or linear machines. However, at powers (≥ 200 kW) usually employed in ICRF heating one would expect a variety of parametric processes. Liu and Tripathi[7] have demonstrated that the electrostatic ion-cyclotron waves are unstable for oscillating two-stream instabilities at such power levels in Alcator C and PLT-type devices.

In this paper, we have studied the parametric decay of an electromagnetic (em) ion-cyclotron wave into a low-frequency ion acoustic wave and two scattered em ion-cyclotron sidebands. The growth of the low-frequency electrostatic ion acoustic wave whose phase velocity is equal to the group velocity of the incident ion-cyclotron wave causes the modulation of the wave front of the incident em ion-cyclotron beam. We have used fluid equations for the nonlinear response of electrons and ions in the plasma. In the present investigation for the em ion-cyclotron pump wave the low-frequency nonlinearity arises predominantly through the ponderomotive force on ions, whereas the high-frequency nonlinearity arises through the nonlinear current densities for the scattered waves.

In Sec. II we have derived the nonlinear dispersion relation for the low-frequency ion acoustic wave in the presence of the ion-cyclotron pump wave and the scattered sidebands. The dispersion relation has been solved analytically, and the dependence of the growth rate of the low-frequency ion acoustic wave on various parameters of interest has been obtained in the same section. A brief discussion of the results is presented in Sec. III.

II. NONLINEAR DISPERSION RELATION AND THE GROWTH RATE

We consider the propagation of a left-hand-circularly-polarized (LHCP) electromagnetic ion-cyclotron wave in a homogeneous collisionless plasma in the direction of the static magnetic field $\vec{B}_s \| \hat{z}$,

$$\vec{E}_0 = \vec{E}_0' \exp[-i(\omega_0 t - k_0 z)],$$
$$E_{0x} = iE_{0y}, \tag{1}$$
$$k_0 \cong \frac{\omega_0}{c} \frac{\omega_{pi}}{\omega_{ci}(1-\omega_0/\omega_{ci})^{1/2}},$$

where ω_{pi} and ω_{ci} are the ion-plasma frequency and the ion-cyclotron frequency.

We assume the existence of a low-frequency density perturbation $(\omega, \vec{k})$ for electrons and ions due to the presence of an ion acoustic mode in the plasma. The density perturbations are taken to be electrostatic ($\vec{E} = -\vec{\nabla}\phi$). The drift velocities of the electrons and ions, and the oscillatory magnetic field of the incident em ion-cyclotron wave $(\omega_0, \vec{k}_0)$ interact with the density perturbations $(\omega, \vec{k})$ and produce two high-frequency sidebands $(\omega_{1,2} = \omega \mp \omega_0,\ \vec{k}_{1,2} = \vec{k} \mp \vec{k}_0)$. The sidebands in turn interact with the pump $(\omega_0, \vec{k}_0)$ to produce a low-frequency ponderomotive force which then drives the low-frequency perturbations. We thus consider the four-wave parametric decay of an em ion-cyclotron wave into two scattered em ion-cyclotron waves $(\omega_{1,2}, \vec{k}_{1,2})$ and the growth of the electrostatic density perturbation associated with the ion acoustic wave. The nonlinear growth of the density perturbation is the basic cause of the modulation of the incident beam.

In the presence of the electric and magnetic fields of the pump and the decay waves, the response of electrons and ions is governed by the equations of motion and continuity:

$$\frac{\partial n}{\partial t}+\vec{\nabla}\cdot(n\vec{v})=0, \tag{2}$$

$$\frac{\partial \vec{v}}{\partial t}+(\vec{v}\cdot\vec{\nabla})\vec{v}=-\frac{e\vec{E}}{m}-\frac{e}{mc}(\vec{v}\times\vec{B})-\frac{v_e^2}{n}\vec{\nabla}n, \tag{3}$$

$$\frac{\partial n_i}{\partial t}+\vec{\nabla}\cdot(n_i\vec{v}_i)=0, \tag{4}$$

$$\frac{\partial \vec{v}_i}{\partial t}+(\vec{v}_i\cdot\vec{\nabla})\vec{v}_i=\frac{e\vec{E}}{m_i}+\frac{e}{m_ic}(\vec{v}_i\times\vec{B}), \tag{5}$$

where $-e$, m, n, $\vec{v}$, and v_e are the electronic charge, mass, density, drift velocity, and the thermal speed, and the quantities with suffix i refer to ions. Here, c is the velocity of light in vacuum. We now express the various quantities as follows:

$$\begin{aligned}
&\vec{E}=\vec{E}_0(\omega_0,\vec{k}_0)+\vec{E}(\omega,\vec{k})+\vec{E}_1(\omega_1,\vec{k}_1)+\vec{E}_2(\omega_2,\vec{k}_2),\\
&\vec{B}=\hat{z}B_s+\vec{B}_0(\omega_0,\vec{k}_0)+\vec{B}_1(\omega_1,\vec{k}_1)+\vec{B}_2(\omega_2,\vec{k}_2),\\
&\vec{v}=\vec{v}_0(\omega_0,\vec{k}_0)+\vec{v}(\omega,\vec{k})+\vec{v}_1(\omega_1,\vec{k}_1)+\vec{v}_2(\omega_2,\vec{k}_2),\\
&\vec{v}_i=\vec{v}_{i0}(\omega_0,\vec{k}_0)+\vec{v}_i(\omega,\vec{k})+\vec{v}_{i1}(\omega_1,\vec{k}_1)+\vec{v}_{i2}(\omega_2,\vec{k}_2),\\
&n=n_0^0+n(\omega,\vec{k})+n_1(\omega_1,\vec{k}_1)+n_2(\omega_2,\vec{k}_2),\\
&n_i=n_0^0+n_i(\omega,\vec{k})+n_{i1}(\omega_1,\vec{k}_1)+n_{i2}(\omega_2,\vec{k}_2),\\
&\vec{B}_0=c\vec{k}_0\times\vec{E}_0/\omega_0,\\
&\vec{B}_1=c\vec{k}_1\times\vec{E}_1/\omega_1,\\
&\vec{B}_2=c\vec{k}_2\times\vec{E}_2/\omega_2,
\end{aligned} \tag{6}$$

where n_0^0 is the equilibrium density of electrons or ions.

Substituting Eqs. (6) in Eq. (2)–(5) we obtain the following linear response:

$$\begin{aligned}
&\vec{v}_0=\frac{e(\vec{E}_0\times\vec{\omega}_c+i\omega_0\vec{E}_0)}{m(\omega_c^2-\omega_0^2)},\\
&\vec{v}_{i0}=-\frac{e(-\vec{E}_0\times\vec{\omega}_{ci}+i\omega_0\vec{E}_0)}{m_i(\omega_{ci}^2-\omega_0^2)},\\
&\vec{v}_{1\perp}=\frac{e(\vec{E}_{1\perp}\times\vec{\omega}_c+i\omega_1\vec{E}_{1\perp})}{m(\omega_c^2-\omega_1^2)},\\
&\vec{v}_{2\perp}=\frac{e(\vec{E}_{2\perp}\times\vec{\omega}_c+i\omega_2\vec{E}_{2\perp})}{m(\omega_c^2-\omega_2^2)},\\
&\vec{v}_{i1\perp}=-\frac{e(-\vec{E}_{1\perp}\times\vec{\omega}_{ci}+i\omega_1\vec{E}_{1\perp})}{m_i(\omega_{ci}^2-\omega_1^2)},\\
&\vec{v}_{i2\perp}=-\frac{e(-\vec{E}_{2\perp}\times\vec{\omega}_{ci}+i\omega_2\vec{E}_{2\perp})}{m_i(\omega_{ci}^2-\omega_2^2)},\\
&v_{1z}=eE_{1z}/mi\omega_1(1-k_z^2v_e^2/\omega_1^2),\\
&v_{2z}=eE_{2z}/mi\omega_2(1-k_z^2v_e^2/\omega_2^2),\\
&v_{i1z}=-eE_{1z}/m_ii\omega_1,\\
&v_{i2z}=-eE_{2z}/m_ii\omega_2.
\end{aligned} \tag{7}$$

Substituting Eqs. (7) in Eq. (3), the z component of the low-frequency ponderomotive force on electrons turns out to be

$$\begin{aligned}
F_z&=-\frac{m}{2}\left((\vec{v}_0\cdot\vec{\nabla}_\perp)v_{1z}+(\vec{v}_0^*\cdot\vec{\nabla}_\perp)v_{2z}+\frac{e}{mc}[(\vec{v}_0\times\vec{B}_1)_z+(\vec{v}_1\times\vec{B}_0)_z+(\vec{v}_0^*\times\vec{B}_2)_z+(\vec{v}_2\times\vec{B}_0^*)_z]\right)\\
&=-\frac{m}{2}\left[i(\vec{k}_1\cdot\vec{v}_{0\perp})v_{1z}+i(\vec{k}_2\cdot\vec{v}_{0\perp}^*)v_{2z}+\frac{e}{m}\left(\frac{k_{1z}}{\omega_1}(\vec{v}_{0\perp}\cdot\vec{E}_{1\perp})-\frac{E_{1z}}{\omega_1}(\vec{v}_{0\perp}\cdot\vec{k}_{1\perp})+\frac{k_0}{\omega_0}(\vec{v}_{1\perp}\cdot\vec{E}_{0\perp})\right.\right.\\
&\qquad\left.\left.+\frac{k_{2z}}{\omega_2}(\vec{v}_{0\perp}^*\cdot\vec{E}_{2\perp})-\frac{E_{2z}}{\omega_2}(\vec{v}_{0\perp}^*\cdot\vec{k}_{2\perp})-\frac{k_0}{\omega_0}(\vec{v}_{2\perp}\cdot\vec{E}_{0\perp}^*)\right)\right].
\end{aligned} \tag{8}$$

On simplification

$$\begin{aligned}
F_z\cong&-\frac{e^2}{2m\omega_1\omega_c^2}\frac{(\vec{E}_{0\perp}\times\vec{\omega}_c\cdot\vec{k}_{1\perp})}{(1-k_z^2v_e^2/\omega_1^2)}\frac{k_z^2v_e^2}{\omega_1^2}E_{1z}-\frac{e^2}{2m\omega_2\omega_c^2}\frac{(\vec{E}_{0\perp}^*\times\vec{\omega}_c\cdot\vec{k}_{2\perp})}{(1-k_z^2v_e^2/\omega_2^2)}\frac{k_z^2v_e^2}{\omega_2^2}E_{2z}\\
&-\frac{e^2}{2m\omega_c^2}\left(\frac{k_{1z}}{\omega_1}-\frac{k_0}{\omega_0}\right)(\vec{E}_{0\perp}\times\vec{\omega}_c\cdot\vec{E}_{1\perp})-\frac{e^2}{2m\omega_c^2}\left(\frac{k_{2z}}{\omega_2}+\frac{k_0}{\omega_0}\right)(\vec{E}_{0\perp}^*\times\vec{\omega}_c\cdot\vec{E}_{2\perp}).
\end{aligned} \tag{9}$$

In a similar way, the z component of the ponderomotive force on ions turns out to be

$$\begin{aligned}
F_{zi}\cong&\frac{e^2(\vec{E}_{0\perp}\times\vec{\omega}_{ci}\cdot\vec{E}_{1\perp})}{2m_i}\left(\frac{k_{1z}}{\omega_1(\omega_{ci}^2-\omega_0^2)}-\frac{k_0}{\omega_0(\omega_{ci}^2-\omega_1^2)}\right)-\frac{ie^2(\vec{E}_{0\perp}\cdot\vec{E}_{1\perp})}{2m_i}\left(\frac{\omega_0k_{1z}}{\omega_1(\omega_{ci}^2-\omega_0^2)}+\frac{\omega_1k_0}{\omega_0(\omega_{ci}^2-\omega_1^2)}\right)\\
&+\frac{e^2(\vec{E}_{0\perp}^*\times\vec{\omega}_{ci}\cdot\vec{E}_{2\perp})}{2m_i}\left(\frac{k_{2z}}{\omega_2(\omega_{ci}^2-\omega_0^2)}-\frac{k_0}{\omega_0(\omega_{ci}^2-\omega_2^2)}\right)+\frac{ie^2(\vec{E}_{0\perp}^*\cdot\vec{E}_{2\perp})}{2m_i}\left(\frac{\omega_0k_{2z}}{\omega_2(\omega_{ci}^2-\omega_0^2)}-\frac{\omega_2k_0}{\omega_0(\omega_{ci}^2-\omega_2^2)}\right).
\end{aligned} \tag{10}$$

A comparison of Eqs. (9) and (10) in the limit $\omega_0 \to \omega_{ci}$ reveals that the pondermotive force on ions is much greater than that on electrons.

Hence, the low-frequency components of electron and ion density perturbations turn out to be

$$n \cong \frac{\chi_e k^2 \phi}{4\pi e}, \tag{11}$$

$$n_i \cong -\frac{\chi_i k^2}{4\pi e}\left(\phi + \frac{F_{zi}}{eik_z}\right), \tag{12}$$

where

$$\chi_e = -\frac{\omega_{pi}^2}{\omega^2}\frac{k_z^2}{k^2} \tag{13}$$

is the electronic susceptibility and χ_i is the ionic susceptibility of the plasma.

Using Eqs. (11) and (12) we obtain the nonlinear parts of the current densities due to the sidebands as follows:

$$\vec{J}_1^{nl}(\omega_1 \cdot \vec{k}_1) = -ne\vec{v}_{0\perp}^*/2 + n_i e\vec{v}_{0i\perp}^*/2 \tag{14}$$

$$\cong -\frac{\omega_{pi}^2}{8\pi}\phi\frac{k_z^2}{\omega^2}(-\vec{v}_{0\perp}^* + \vec{v}_{0i\perp}^*), \tag{15}$$

$$\vec{J}_2^{nl}(\omega_2, \vec{k}_2) \cong -\frac{\omega_{pi}^2}{8\pi}\phi\frac{k_z^2}{\omega^2}(-\vec{v}_{0\perp} + \vec{v}_{0i\perp}). \tag{16}$$

In Eqs. (15) and (16) we have neglected the nonlinear parts of n and n_i.

Now, substituting Eqs. (11), (12), (15), and (16) into Poisson's equation and the wave equations for the sidebands we obtain

$$\epsilon\phi = \vec{A}\cdot\vec{E}_{1\perp} + \vec{B}\cdot\vec{E}_{2\perp}, \tag{17}$$

$$\underline{D}_1 \cdot \vec{E}_1 = \vec{C}\phi, \tag{18}$$

$$\underline{D}_2 \cdot \vec{E}_2 = \vec{D}\phi, \tag{19}$$

where

$$\vec{A} = -\frac{\omega_{pi}^2 e k_z(\hat{x} - i\hat{y})E_{0x}}{2m_i\omega^2\omega_0\omega_1 k^2(\omega_{ci}^2 - \omega_0^2)}[\omega_{ci}(\omega_0 k_{1z} - \omega_1 k_0) + (\omega_0^2 k_{1z} + \omega_1^2 k_0],$$

$$\vec{B} = -\frac{\omega_{pi}^2 e k_z(\hat{x} + i\hat{y})E_{0x}^*}{2m_i\omega^2\omega_0\omega_2 k^2(\omega_{ci}^2 - \omega_0^2)}[\omega_{ci}(\omega_0 k_{2z} - \omega_2 k_0) + (\omega_0^2 k_{2z} - \omega_2^2 k_0)],$$

$$\vec{C} = -\frac{\omega_{pi}^2 e\omega_1 k_z^2 E_{0x}^*}{2c^2\omega^2}\left(-\frac{1}{m\omega_c} + \frac{1}{m_i(\omega_{ci} - \omega_0)}\right)(\hat{x} - i\hat{y}), \tag{20}$$

$$\vec{D} = -\frac{\omega_{pi}^2 e\omega_2 k_z^2 E_{0x}}{2c^2\omega^2}\left(-\frac{1}{m\omega_c} + \frac{1}{m_i(\omega_{ci} - \omega_0)}\right)(\hat{x} + i\hat{y}).$$

$$\underline{D}_{1,2} = k_{1,2}^2\underline{I} - \vec{k}_{1,2}\vec{k}_{1,2} - \omega_{1,2}^2\underline{\epsilon}_{1,2}/c^2,$$

and $\underline{I}$ is the unit dyadic. The linear dielectric tensors for the sidebands are given by[8]

$$\epsilon_{1,2} \cong \begin{pmatrix} 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega_{1,2}^2} & \frac{i\omega_{pi}^2}{\omega_{ci}^2 - \omega_{1,2}^2}\frac{\omega_{ci}}{\omega_{1,2}} & 0 \\ \frac{-i\omega_{pi}^2}{\omega_{ci}^2 - \omega_{1,2}^2}\frac{\omega_{ci}}{\omega_{1,2}} & 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega_{1,2}^2} & 0 \\ 0 & 0 & 1 - \frac{\omega_{pi}^2}{\omega_{1,2}^2}\left(1 - \frac{m_i}{m}\right) \end{pmatrix}. \tag{21}$$

Without loss of generality, we now assume that $\vec{k}$, $\vec{k}_1$, and $\vec{k}_2$ are confined in the xz plane (i.e., $k_y, k_{1y}, k_{2y} = 0$). The nonlinear dispersion relation for the low-frequency perturbation is obtained straightaway by eliminating ϕ, $\vec{E}_1$, and $\vec{E}_2$ from Eqs. (17)–(19) as follows:

$$\epsilon = \frac{\mu_1}{|\underline{D}_1|} + \frac{\mu_2}{|\underline{D}_2|}, \tag{22}$$

where

$$\mu_1 \cong -\frac{\omega_{pi}^4 e^2 k_1^2 k_z(E_{0x}E_{0x}^*)}{4m_i\omega^2 c^2 k^2(\omega_{ci}^2 - \omega_0^2)}\frac{k_z^2}{\omega^2}\left(k_1^2 + \frac{\omega_{pi}^2}{c^2}\frac{m_i}{m}\right)\left(\frac{\omega_{ci}}{\omega_0}(\omega_0 k_{1z} - \omega_1 k_0) + \omega_0 k_z\right)\left(\frac{1}{m\omega_c} - \frac{1}{m_i(\omega_{ci} - \omega_0)}\right), \tag{23}$$

$$\mu_2 \cong -\frac{\omega_{pi}^4 e^2 k_2^2 k_z(E_{0x}E_{0x}^*)}{4m_i\omega^2 c^2 k^2(\omega_{ci}^2 - \omega_0^2)}\frac{k_z^2}{\omega^2}\left(k_2^2 + \frac{\omega_{pi}^2}{c^2}\frac{m_i}{m}\right)\left(\frac{\omega_{ci}}{\omega_0}(\omega_0 k_{2z} - \omega_2 k_0) + \omega_0 k_z\right)\left(\frac{1}{m\omega_c} - \frac{1}{m_i(\omega_{ci} - \omega_0)}\right), \tag{24}$$

$$|\underline{D}_{1,2}| \cong \frac{\omega_{pi}^2}{c^2}\frac{m_i}{m}\left(\frac{\omega_{1,2}}{c}\right)^4\left(\frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega_{1,2}^2}\right)^2\left(1 + \frac{\omega_{1,2}^2}{\omega_{ci}^2}\right). \tag{25}$$

In Eq. (17) $\epsilon = \epsilon(\omega, \vec{k})$ is the linear dielectric function of the low frequency electrostatic ion acoustic mode and is given by

$$\epsilon = 1+\frac{1}{k^2\lambda_D^2}\left[1+\frac{\omega}{k_z v_e}\sum_n Z\left(\frac{\omega-n\omega_c}{k_z v_e}\right)I_n(b)\exp(-b)\right]+\frac{1}{k^2\lambda_D^2}\frac{T_e}{T_i}\left[1+\frac{\omega}{k_z v_{ei}}\sum_n Z\left(\frac{\omega-n\omega_c}{k_z v_{ei}}\right)I_n(b_i)\exp(-b_i)\right], \quad (26)$$

where $\lambda_D = v_e/(2^{1/2}\omega_p)$ is the electron Debye length, Z is the plasma dispersion function, I_n the modified Bessel function of second kind, v_e and v_{ei} the electron and ion thermal speeds, and

$$b=k_\perp^2\rho_e^2,\quad b_i=k_\perp^2\rho_i^2,\quad \rho_e^2=v_e^2/2\omega_c^2,\quad \rho_i^2=v_{ei}^2/2\omega_{ci}^2\,. \quad (27)$$

Now, with the approximations that

$$\omega \ll \omega_{ci},\quad \omega\ll k_z v_e,\quad \omega \gg k_z v_{ei}\,,$$

the linear dielectric function becomes

$$\epsilon \cong 1+\frac{2\omega_p^2}{k^2v_e^2}\left(1+\frac{i\pi^{1/2}\omega}{k_z v_e}\right)+\frac{2\omega_{pi}^2}{k^2v_{ei}^2}\left\{1+I_n(b_i)\exp(-b_i)\left[-1-\frac{k_z^2v_{ei}^2}{2\omega^2}-\frac{i\pi^{1/2}\omega}{k_z v_{ei}}\exp\left(-\frac{\omega^2}{k_z^2v_{ei}^2}\right)\right]\right\}. \quad (28)$$

The real part of ϵ is then

$$\epsilon_r \cong \frac{\omega_{pi}^2}{k^2C_s^2}-\frac{\omega_{pi}^2}{\omega^2}\frac{k_z^2}{k^2}, \quad (29)$$

and the imaginary part is given by

$$\epsilon_i \cong \left[\frac{\omega_{pi}^2}{k^2C_s^2}\frac{\omega\pi^{1/2}}{k_z^2v_e^2}+\frac{2\omega_{pi}^2}{k^2v_{ei}^2}\frac{\omega}{k_z v_{ei}}\exp\left(\frac{-\omega^2}{k_z^2v_{ei}^2}\right)\right]$$

$$\cong i\pi^{1/2}\frac{\omega_{pi}^2}{k^2C_s^2}\left[\frac{C_s}{v_e}+\left(\frac{T_e}{T_i}\right)^{3/2}2^{1/2}\exp\left(\frac{-T_e}{2T_i}\right)\right]. \quad (30)$$

Near the resonance we can have the following expansions for ϵ, $|\underline{D}_1|$, and $|\underline{D}_2|$:

$$\omega=\omega_r+i\gamma\,,$$

$$\epsilon\cong(\gamma+\gamma_L)\frac{\partial\epsilon_r}{\partial\omega_r}\,,$$

$$|\underline{D}_1|\cong(\gamma+\gamma_{L1})\frac{\partial|\underline{D}_1|_r}{\partial\omega_1}\,,\quad (31)$$

$$|\underline{D}_2|\cong(\gamma+\gamma_{L2})\frac{\partial|\underline{D}_2|_r}{\partial\omega_2}\,,$$

where γ_L, γ_{L1} and γ_{L2} are the linear damping rates of the ion acoustic mode, lower and upper sideband, respectively:

$$\gamma_L=\epsilon_i\Big/\left(\frac{\partial\epsilon_r}{\partial\omega_r}\right)$$

$$=\frac{\omega\pi^{1/2}}{2}\left[\frac{C_s}{v_e}+\left(\frac{T_e}{T_i}\right)^{3/2}2^{1/2}\exp\left(\frac{-T_e}{2T_i}\right)\right], \quad (32)$$

$$\gamma_{L1,L2}=\frac{\nu(m/m_i)}{3(1+\nu^2/\omega_0^2)}\left(\frac{c}{\omega_0}\frac{\omega_{ci}}{\omega_{pi}}\right)^4\frac{(1-\omega_0^2/\omega_{ci}^2)^2}{(1+\omega_0^2/\omega_{ci}^2)^2}\left[\left(k_{1,2}^2\cos^2\theta-\frac{\omega_0^2}{c^2}\frac{\omega_{pi}^2}{\omega_{ci}^2-\omega_0^2}\right)\times\left(k_{1,2}^2-\frac{\omega_0^2}{c^2}\frac{\omega_{pi}^2}{\omega_{ci}^2-\omega_0^2}\right)+\left(\frac{\omega_0}{c}\right)^4\left(\frac{\omega_0}{\omega_{ci}}\right)^2\left(\frac{\omega_{pi}^2}{\omega_{ci}^2-\omega_0^2}\right)^2-(k_{1,2}^2\sin\theta\cos\theta)^2\right]. \quad (33)$$

Neglecting the linear damping rates of the high-frequency sidebands we obtain

$$\gamma(\gamma+\gamma_L)\cong\gamma_0^2$$

$$=-\left(1\Big/\frac{\partial\epsilon_r}{\partial\omega_r}\right)\left(\mu_1\Big/\frac{\partial|\underline{D}_1|_r}{\partial\omega_1}+\mu_2\Big/\frac{\partial|\underline{D}_2|_r}{\partial\omega_2}\right). \quad (34)$$

From Eq. (29)

$$\frac{\partial\epsilon_r}{\partial\omega_r}\cong 2\,\frac{\omega_{pi}^2}{\omega^3}\frac{k_z^2}{k^2}\,. \quad (35)$$

From Eq. (25)

$$\frac{\partial|\underline{D}_{1,2}|_r}{\partial\omega_{1,2}}\cong\frac{2\omega_{1,2}^3}{\omega_{ci}^4}\frac{\omega_{pi}^6}{c^6}\frac{m_i}{m}\frac{(2+3\omega_{1,2}^2/\omega_{ci}^2-\omega_{1,2}^4/\omega_{ci}^4)}{(1-\omega_{1,2}^2/\omega_{ci}^2)^3}\,. \quad (36)$$

The linear dispersion relations for the scattered sidebands are given by

$$k_{1,2}^2=\frac{\omega_{1,2}^2\omega_{pi}^2}{c^2}\frac{\{1+\cos^2\theta+[\sin^4\theta+(4\omega_{1,2}^2/\omega_{ci}^2)\cos^2\theta]^{1/2}\}}{2(\omega_{ci}^2-\omega_{1,2}^2)\cos^2\theta}\,, \quad (37)$$

where θ is angle between $\vec{k}_0$ and $\vec{k}_1$ or $\vec{k}_2$. For simplicity we have taken that both the scattered ion-cyclotron waves make the same scattering angle θ.

When the resonance conditions

$$\omega=\omega_0\pm\omega_{1,2}\,,\qquad \vec{k}=\vec{k}_0\pm\vec{k}_{1,2} \quad (38)$$

are satisfied, then the angular frequency of the low-frequency ion acoustic wave is given by

$$\frac{\omega}{k_0 C_s} = \left\{ 1 \mp \left(\frac{1}{2(1+\omega_0/\omega_{ci})} \right)^{1/2} \times \left[1 + \cos^2\theta + \left(\sin^4\theta + \frac{4\omega_0^2}{\omega_{ci}^2}\cos^2\theta \right)^{1/2} \right]^{1/2} \right\}, \tag{39}$$

where − sign is for $\theta < \pi/2$ and + sign is for $\theta > \pi/2$. Now, the condition that the phase velocity of the ion acoustic wave will be equal to the group velocity of the incident ion-cyclotron wave requires that ω must be taken from the following equation:

$$\frac{\omega}{\omega_0} = \left\{ 1 \mp \left(\frac{1}{2(1+\omega_0/\omega_{ci})} \right)^{1/2} \times \left[1 + \cos^2\theta + \left(\sin^4\theta + \frac{4\omega_0^2}{\omega_{ci}^2}\cos^2\theta \right)^{1/2} \right]^{1/2} \right\}. \tag{40}$$

Using Eqs. (35) and (36) the expression for the unperturbed growth rate of the instability reduces to

$$\gamma_0^2 = \frac{1}{16}\left|\frac{v_{0ix}}{C_s}\right|^2 \frac{\omega}{k_z}\frac{m}{m_i}\frac{\omega_{ci}^4}{\omega_{pi}^4} c^4 \left(\frac{m_i}{m}\frac{1}{\omega_c} - \frac{1}{\omega_{ci}(1-\omega_0/\omega_{ci})} \right) \times \left(\frac{k_1^2}{\omega_1^3} \frac{[k_1^2 + (\omega_{pi}^2/c^2)(m_i/m)](1-\omega_1^2/\omega_{ci}^2)^3 [(\omega_{ci}/\omega_0)(\omega_0 k_{1z} - \omega_1 k_0) + \omega_0 k_z]}{(2 + 3\,\omega_1^2/\omega_{ci}^2 - \omega_1^4/\omega_{ci}^4)} + \frac{k_2^2}{\omega_2^3} \frac{[k_2^2 + (\omega_{pi}^2/c^2)(m_i/m)](1-\omega_2^2/\omega_{ci}^2)^3 [(\omega_{ci}/\omega_0)(\omega_0 k_{2z} - \omega_2 k_0) + \omega_0 k_z]}{(2 + 3\,\omega_2^2/\omega_{ci}^2 - \omega_2^4/\omega_{ci}^4)} \right), \tag{41}$$

where

$$v_{0ix} = -ieE_{0x}/m_i\omega_{ci}(1-\omega_0/\omega_{ci}). \tag{42}$$

In the presence of the linear damping of the ion acoustic wave, the growth rate of the instability is obtained from the following relation:

$$\gamma = [(\gamma_L^2 + 4\gamma_0^2)^{1/2} - \gamma_L]/2. \tag{43}$$

To have a numerical appreciation of the results we have calculated the growth rates of the instability for the following plasma parameters: $\omega_{ci}/\omega_0 \cong 1.2$, $n_0^0 \cong 10^{13}$ cm^{-3}, $B_s \cong 10$ kG, $T_e \cong 1$ keV, $T_i \cong 0.05$ keV. Figure 1 shows the variation of the growth rates γ and γ_0 with the pump-induced velocity $|v_{0ix}/C_s|$. The growth rates increase with $|v_{0ix}/C_s|$. The perturbed growth rate γ has been calculated for $T_e/T_i = 20$. For higher ratio of T_e/T_i, the linear damping rate γ_L decreases and γ is then nearly equal to the unperturbed growth rate γ_0. It may be mentioned here that the growth

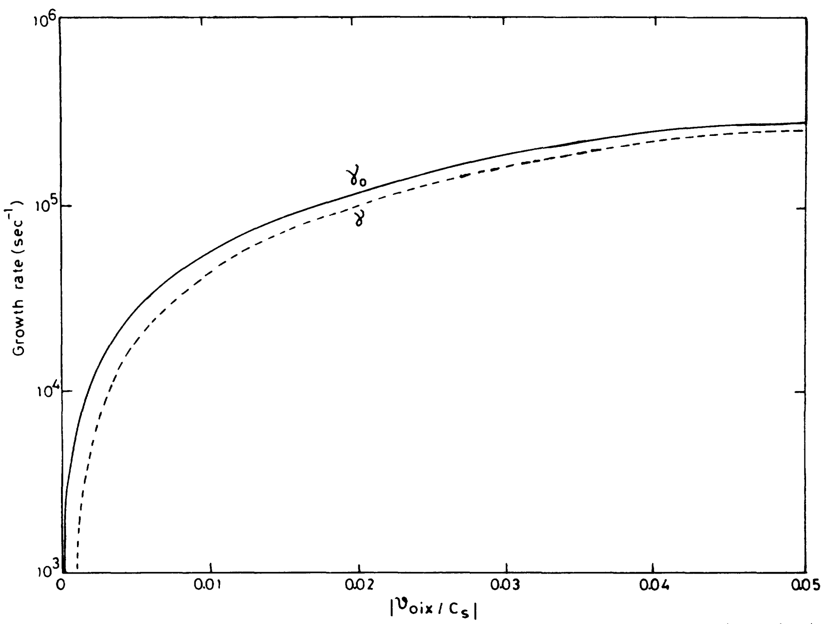

FIG. 1. Variation of the growth rates γ_0 and γ as a function of the pump-induced velocity $|v_{0ix}/C_s|$ for the following set of plasma parameters: $n_0^0 = 10^{13}$ cm^{-3}, $\omega_{ci}/\omega_0 = 1.2$, $B_s = 10$ kG, $T_e = 1$ keV, $T_i = 0.05$ keV, $\theta = 30°$.

rate takes a very high value when the pump induced drift velocity of ions is nearly equal to the ion acoustic velocity in the plasma.

III. DISCUSSION

A high-power em ion-cyclotron beam propagating in the direction of the external static magnetic field in a plasma couples parametrically with the ion acoustic mode of the plasma. The incident em ion-cyclotron wave gets modulated by the growth of the ion acoustic wave when the group velocity of the incident wave is equal to the phase velocity of the ion acoustic wave in the plasma. It may be noted that the ponderomotive nonlinearity is much greater for the motion of ions than that for the motion of electrons for waves in the ion-cyclotron range of frequencies. For typical plasma parameters: $n_0^0 = 10^{13}$ cm^{-3}, $\omega_{ci}/\omega_0 = 1.2$, $B_s = 10$ kG, $T_e = 1$ keV, $T_i = 0.05$ keV, $\theta = 30°$, $|v_{0ix}/C_s| = 10^{-2}$, the growth rate of the instability turns out to be $\sim 10^4$ rad sec^{-1}.

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