Purely growing magnetic modes in anisotropic contrastreaming plasmas

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Purely-growing magnetostatic modes are shown to exist in a plasma either with contrastreaming electrons or with temperature anisotropy. The relevance of our investigation to the anomalous heat transport in tokamak devices is discussed.

In their original paper,[1] Chu *et al.* discovered a new zero-frequency (aperiodic) magnetostatic mode. In the absence of free-energy sources, the latter is a purely damped magnetic perturbation in the plane perpendicular to the external magnetic field. That is, it has the polarization of an ordinary electromagnetic wave. In the presence of this mode, the electrons are able to drift across the external magnetic field by moving along the perturbation magnetic fields. It has been shown that even in an equilibrium plasma, in which the magnetic field fluctuates at the thermal level, Bohm-type electron diffusion can occur,[1,2] leading to anomalous heat conduction. In particular, when the plasma beta exceeds the electron-to-ion mass ratio, the electron diffusion caused by the magnetostatic modes can become comparable to the diffusion associated with the thermal-vortex modes.[3]

In this brief report, we show that the magnetostatic modes become purely growing in the presence of counterstreaming electrons or temperature anisotropy in a plasma. With an enhanced spectrum of magnetic fluctuations, the electron thermal conduction can be even more pronounced. This[1] and similar mechanisms[4] have often been invoked to explain anomalous transport in the numerical simulations,[2] as well as in tokamak devices.[4]

Consider an electron plasma embedded in a static homogeneous magnetic field $\vec{B}_0=\hat{z}B_0$. Electrons are contrastreaming with velocity $\pm v_0$ along the $\hat{z}$ direction, and the steady-state electron-distribution function F is taken to be drifted bi-Maxwellian. We model the electron collisions by means of a very simple Krook term in the linearized Boltzmann equation. In the following, we are interested in deriving a dispersion relation[5] for "ordinary" electromagnetic waves, i.e., $\vec{E}=\hat{z}E$, $\vec{B}=\hat{y}B$, $\vec{k}=\hat{x}k$. Furthermore, since the electrons stream along $\hat{z}$, the contribution of the current density gives rise to the dielectric function in the $\hat{z}$ direction, ϵ_{zz}, which is of interest to us. The dispersion relation for the ordinary mode is then found to be

$$\frac{c^2k^2}{\omega^2}=\epsilon_{zz} , \tag{1}$$

where for our purposes ($\omega<\Omega=eB_0/mc$)

$$\epsilon_{zz}=1-\frac{\omega_p^2}{\omega^2}+i\frac{\omega_p^2\nu\Lambda}{\omega^2(\omega+i\nu)}+\frac{\omega_p^2}{\omega^2}\frac{T}{T_\perp}(1-\Lambda) . \tag{2}$$

In (1) and (2), c is the velocity of light, ω and k are the frequency and wave number, ω_p and ν are the electron plasma and collision frequencies, respectively. Furthermore, in Eq. (2), $\Lambda=I_0(b)\exp(-b)$, $b=k^2\rho^2$, $\rho=v_e/\Omega$, v_e is the electron thermal velocity, and I_0 is the modified Bessel function of the zeroth order. Moreover, $T=T_\parallel+T_0$, $T_0=mv_0^2$ is the temperature associated with the parallel electron stream, $T_\parallel$ and $T_\perp$ are the parallel and perpendicular electron temperatures originating from the anisotropic distribution in a plasma. We have neglected the ion contribution in obtaining (2), since it is smaller by a mass ratio.

Let us focus our attention on the low-frequency magnetostatic branch. For this purpose, we take $\omega^2<<\omega_p^2, c^2k^2$. Thus, Eq. (1) becomes

$$\omega+i\nu=\frac{i\nu\Lambda}{1+Q-(1-\Lambda)T/T_\perp} , \tag{3}$$

where $Q=c^2k^2/\omega_p^2\equiv b/\beta$. In the absence of the electron stream ($T_0=0$), and when $T_\parallel=T_\perp$, (3) gives the spectrum of the magnetostatic mode including finite Larmor-radius corrections,

$$\omega=-\frac{i\nu Q}{Q+\Lambda} . \tag{4}$$

We note that for long-wavelength ($b<<1$) perturbations, Eq. (4) reduces to the well-known dispersion relation of the magnetostatic mode.[1] Equation (3) exhibits a purely growing mode. The growth rate is given by

$$\gamma=\frac{\nu[\Lambda-(Q+1-p)]}{1+Q-p} , \quad (5)$$

where $\omega=i\gamma$, γ is the growth rate, and $p=(1-\Lambda)T/T_\perp$. Clearly, unstable modes occur provided that

$$(1-\Lambda)^{-1}(1+Q-\Lambda)<\eta<(1+Q)(1-\Lambda)^{-1} , \quad (6)$$

where $\eta=T/T_\perp$.

For long-wavelength ($b<<1$) modes, we have $\Lambda\approx 1-b$, $Q=k^2R^2$, $R=c/\omega_p$, and $p=k^2\rho^2\eta$. Correspondingly, the growth rate above threshold is

$$\frac{\gamma}{\nu}\approx\omega_p^2/[c^2k^2+\omega_p^2(1-b\eta)] .$$

On the other hand, if the condition[6]

$$\frac{T}{T_\perp}=\frac{c^2k^2+\omega_p^2}{\omega_p^2(1-\Lambda)} , \quad (7)$$

is satisfied, Eq. (1) gives ($\omega=i\gamma$), then

$$\gamma^2=\frac{\nu\Lambda\omega_p^2}{\gamma+\nu} , \quad (8)$$

which for $\gamma>>\nu$ yields $\gamma=(\nu\omega_p^2\Lambda)^{1/3}$.

It follows that the long-wavelength ($b<<1$) perturbations, which are long lived, grow at a very fast rate because now we have $\gamma=\nu(\omega_p/\nu)^{2/3}$. This, however, happens when $\eta=(1+k^2R^2)/k^2\rho^2$. The latter is a rather stringent condition in a low-β plasma with a weak temperature anisotropy or a weak streaming flow.

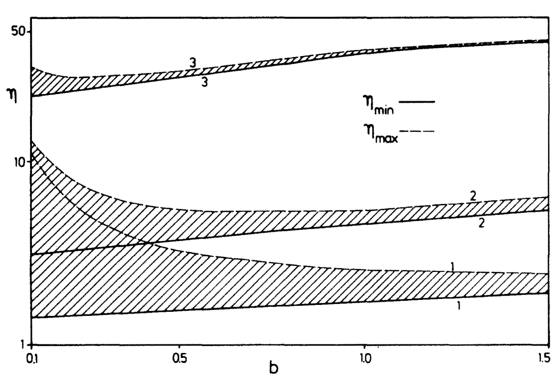

FIG. 1. Variations of $\eta_{\min}$ (solid curves) and $\eta_{\max}$ (dashed curves) vs b for $2\beta=5.0$, 1.0, and 0.1 for the curves 1, 2, and 3, respectively. For each curve, the area enclosed by $\eta_{\min}$ and $\eta_{\max}$ lines (shaded area) corresponds to the unstable region. Here, $\eta_{\min}$ ($\eta_{\max}$) corresponds to the left (right) side of the inequality (6).

Figure 1 is a plot of η vs b. Note that there exist critical η values for which the instability occurs. The hatched area is the unstable region. Figure 2 exhibits the variation of growth rate [Eq. (5)] with b. We see that the growth rate increases with the increase of b, reaching a maximum, and after that γ rapidly decreases for shorter wavelengths.

In summary, we have demonstrated that anisotropic contrastreaming plasmas support unstable magnetostatic modes. The physical mechanism of the resistive instability discussed here is similar to that of the Weibel instability.[7] The electrons streaming along the external magnetic field feel a Lorentz force due to the wave magnetic field which finally leads to the bunching of the electron current. The latter enhances the original wave magnetic field. Although this brief report does not attempt to derive a nonlinear theory which limits the wave growth, our speculation is that the final state of the present instability can lead to a strongly turbulent state consisting of enhanced magnetic fluctuations. However, if the saturated magnetic field energy density exceeds by 2 orders of magnitude the thermal level, then the electron diffusion (hence the thermal conductivity) can be increased[8] by an order of magnitude. Anomalous heat losses in tokamak devices,[4] and the cross-field electron diffusion in computer simulations[2] as well as in laser-produced plasmas,[9] might be due to the magnetic-mode turbulence driven by temperature anisotropy in a contrastreaming plasma.

Finally, we mention that the purely growing magnetostatic modes can nonlinearly interact among themselves. This interaction is described[10] by the Hasegawa-Mima equation.[11,12] Such in-

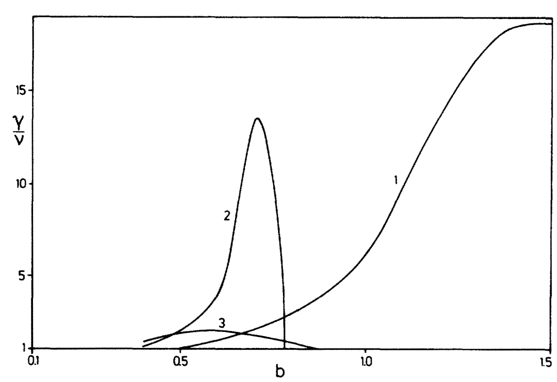

FIG. 2. Variation of growth rate γ/ν vs b for various values of β and η. For the curve 1, $2\beta=5.0$ and $\eta=2.5$. For the curve 2, $2\beta=10.0$ and $\eta=2.5$, and for the curve 3, $2\beta=1.0$ and $\eta=5.0$.

teresting phenomena as magnetic vortices, and spectrum cascading have recently been discussed in Ref. 10.

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