Spectral characteristics of signals in the optical Hanle effect

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The results of an earlier investigation by the authors [Phys. Rev. A 23, 2553 (1981)] are extended to study the spectral features of the signals in the optical Hanle effect with regard to the various directions of observations and the polarizations of the emitted and the exciting radiation. The asymmetries in the spectra are found to be critically dependent on each of these parameters.

The recently discovered optical analog[1] of the usual magnetic field Hanle effect, where a circularly polarized laser field rather than a magnetic field has been used to lift the degeneracy of the level, is important from the fundamental viewpoint as well as for applications. Since in case of the optical Hanle effect there is considerable selectivity of the Zeeman sublevels, depending on the polarization of the field used to lift degeneracy; it is important to carry out a detailed experimental and theoretical study of the 'signals produced in such an effect. One has so far[2,3] studied the dependence of the intensity in various directions on the intensities and the bandwidth of the exciting radiation. A natural question arises—what are the spectral characteristics of the optical Hanle signals? How do these depend on the direction of observation, strength of the two laser fields, polarization of the exciting radiation? It is also of interest to understand the differences, if any, between the spectral characteristics of the optical Hanle signals and the usual magnetic-field Hanle signals. This is the object of the present study. It must, however, be added that the spectra of radiation emitted by a three- or four-level system have been calculated by a number of workers.[4,5] The most relevant study involving Hanle transitions is that of Kornblith and Eberly,[5] who studied the dependence of the spectra in the magnetic-field Hanle problem on the direction of observation. But since the optical Hanle effect involves the lifting of the degeneracy of the excited and ground states in a particular manner, and since this process introduces important asymmetries in the problem, it is worthwhile to examine the spectrum of the emitted radiation with parameters appropriate to the optical Hanle case.

In what follows we keep our discussion as brief as possible and refer to Ref. 2 (hereafter I) for the derivation of the basic equations of motion for the atomic system. For the purpose of illustrating the main results we have chosen the $J=0$ to $J=1$ transition. The levels $|J=0,m=0\rangle \equiv |g\rangle$, $|J=1,m=+1\rangle \equiv |+\rangle$ are coupled by the circularly polarized radiation which Stark shifts each of these. The linearly polarized (exciting radiation), making an angle θ with the direction of the atomic beam, couples the level $|J=0,m=0\rangle$ to $|J=|1,m=\pm 1\rangle \equiv |\pm\rangle$. Simple algebraic transformations enable us to cast the density matrix equation I (2.20) into the following matrix equation:

$$\dot{\psi}=M\psi+\psi \ , \tag{1}$$

$$\psi_1=\rho_{++},\ \ \psi_2=-\rho_{+-}e^{2i\theta},\ \ \psi_3=-\rho_{+g}e^{-i(\theta+\omega_L t)} ,$$
$$\psi_4=\psi_2^*,\ \ \psi_5=\rho_{--},\ \ \psi_6=\rho_{-g}e^{-i(\theta-\omega_L t)},\ \ \psi_7=\psi_3^*,\ \ \psi_8=\psi_6^* \ , \tag{2}$$

$$\phi_2=\phi_5=2\gamma_1,\ \ \phi_3=\phi_6=\phi_7^*=\phi_8^*=i\alpha,\ \ \phi_2=\phi_4=0 \ , \tag{3}$$

$$M=\begin{bmatrix} -2(\gamma+\gamma_\uparrow) & 0 & -i\alpha^* & 0 & -2\gamma_\uparrow & 0 & i\alpha & 0 \\ 0 & -(2\gamma+i\delta) & -i\alpha^* & 0 & 0 & 0 & 0 & i\alpha \\ -2i\alpha & -i\alpha & -(2i\delta+\gamma+2\gamma_\uparrow) & 0 & -i\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & (i\delta-2\gamma) & 0 & -i\alpha^* & i\alpha & 0 \\ 0 & 0 & 0 & 0 & -2(\gamma+\gamma_\uparrow) & -i\alpha^* & 0 & i\alpha \\ -i\alpha & 0 & 0 & -i\alpha & -2i\alpha & -(i\delta+\gamma+2\gamma_\uparrow) & 0 & 0 \\ 2i\alpha^* & 0 & 0 & i\alpha^* & i\alpha^* & 0 & (2i\delta-\gamma-2\gamma_\uparrow) & 0 \\ i\alpha^* & i\alpha^* & 0 & 0 & 2i\alpha^* & 0 & 0 & (i\delta-\gamma-2\gamma_\uparrow) \end{bmatrix} . \quad (4)$$

In obtaining (4) we have chosen the bandwidth parameter of the exciting field zero. We have, however, added a relaxation rate which, for example, may represent collisional relaxation $\gamma_\uparrow$ from the ground level to the excited levels.[6] This, for example, would lead to a term such as $2\gamma_\uparrow\rho_{gg}$ in the equation for ρ_{++}, which has been eliminated because of the trace condition $\mathrm{Tr}\rho=1$ and thus $2\gamma_\uparrow\rho_{gg}\rightarrow-2\gamma_\uparrow(\rho_{++}+\rho_{--}-1)$. γ now will be the total relaxation rate for the downward transitions. δ is the Stark shift parameter and α denotes the Rabi frequency associated with the transition. Note the characteristic feature of the optical Hanle signal in the matrix M, in the sense that states $|+\rangle$ and $|-\rangle$ have different δ values rather than equal and opposite values as in the magnetic-field Hanle effect. The spectrum of signals in various directions can be expressed in terms of certain atomic correlation functions in the usual fashion [cf. Refs. 4 and 5]. The spectrum of signals detected along y and x directions with polarizations along x and y directions and denoted, respectively, by S_y and S_x can be shown to be given by, apart from some proportionality constants that depend on the radial matrix elements, frequency ω, etc.,

$$S_{x(y)}(\omega)=\mathrm{Re}\{\hat{\Gamma}_{+gg+}(z)+\hat{\Gamma}_{-gg-}(z)\mp[e^{2i\theta}\hat{\Gamma}_{+gg-}(z)+e^{-2i\theta}\hat{\Gamma}_{-gg+}(z)]\}\,|_{z=i(\omega-\omega_L)}\,, \quad (5)$$

where $\hat{\Gamma}_{\alpha\beta\gamma\delta}(z)$ is the Laplace transform of the correlation function $\langle(|\alpha\rangle\langle\beta|)_t(|\gamma\rangle\langle\delta|)_0\rangle$, where suffix zero refers to the steady state of the system. $S_x(S_y)$ is obtained by choosing the minus (plus) sign before the square bracket in (5). All the correlation functions appearing in (5) can be obtained from the solution of (1) by using the quantum regression theorem. Expression (5) shows that the spectral features could depend in an important way on the direction of polarization of the exciting radiation as well as the direction of observation. The results of numerical calculations for some typical values of $\delta,\theta,\alpha,\gamma_\uparrow$ are shown in Figs. 1–3.

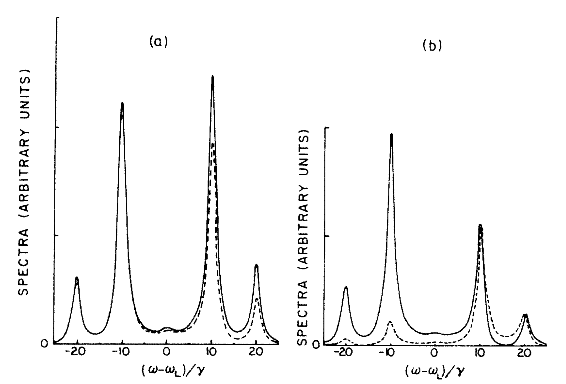

FIG. 1. Spectrum of the radiation in the optical Hanle effect for moderate field $\alpha/\gamma=1$, $\delta/\gamma=10$, $\gamma_\uparrow=0$ and for (a) $\theta=\pi/4$; (b) $\theta=\pi/2$. The solid curves represent S_x whereas dashed curves give S_y.

We have in these figures only plotted the incoherent part of the spectra since the coherent part is, in the weak-field limit and for $\gamma_\uparrow=0$ just related to the total intensity, and in the strong-field limit is rather small. Figure 1 gives the behavior of the spectra in moderate fields, showing thus the marked dependence on the direction of polarization[7] of the incident beam and the direction of observation. Note that the matrix elements $\langle+|\vec{d}\cdot\vec{E}|g\rangle$ and $\langle-|\vec{d}\cdot\vec{E}|g\rangle$ are in phase for

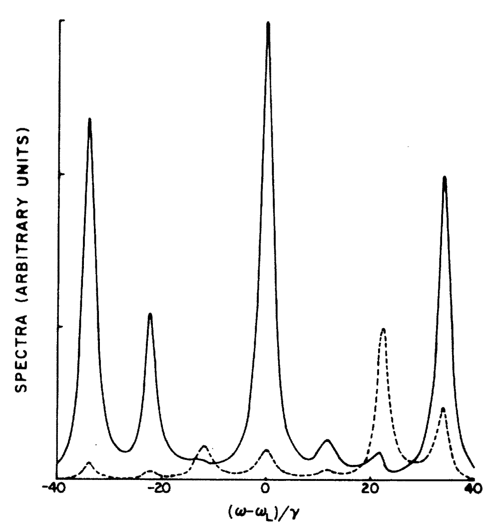

FIG 2. Same as in Fig. 1(b) except now the field is large $\alpha/\gamma=10$.

$\theta=\pi/2$ whereas they are out of phase for $\theta=\pi/4$. The five peaks in the spectra can be understood by examining the behavior of the eigenvalues of the coherent part of the interactions:

$$H=\begin{bmatrix} 2\delta & 0 & \alpha \\ 0 & \delta & \alpha \\ \alpha & \alpha & 0 \end{bmatrix} . \tag{6}$$

Since α has been taken to be much smaller than δ, the approximate eigenvalues of (6) are $2\delta+\alpha^2/2\delta$, $\delta+\alpha^2/\delta$, $-3\alpha^2/2\delta$, and the approximate eigenfunctions are

$$\begin{aligned} |\phi_1\rangle &= |+\rangle-\frac{\alpha}{2\delta}|g\rangle , \\ |\phi_2\rangle &= |-\rangle-\frac{\alpha}{\delta}|g\rangle , \\ |\phi_3\rangle &= |g\rangle+\frac{\alpha}{2\delta}|+\rangle+\frac{\alpha}{\delta}|-\rangle . \end{aligned} \tag{7}$$

The seven peaks arise due to transitions between these new states, the two of which cannot be resolved as $\alpha^2/\delta << \gamma$. Figure 2 gives the spectra when the Stark shift is of the same order as the Rabi frequency associated with the transition. In this case all the seven peaks, as predicted by the eigenvalues of (6), are seen to be clearely resolved. A much smaller value of S_y as compared to S_x is connected with the fact that the total intensity of the signal S_x is much greater than that for S_y as is

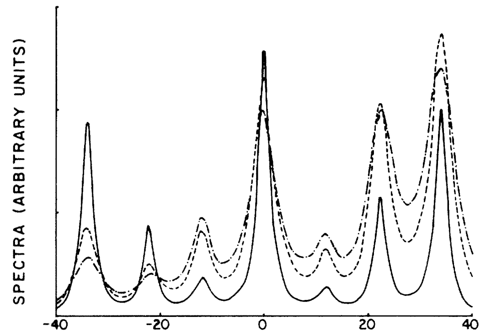

FIG 3. Effect of the relaxation rate $\gamma_\dagger$ on the spectra S_x for $\alpha/\gamma=10$, $\delta/\gamma=10$ and for $\gamma_\dagger/\gamma=0$ (solid curve), 0.5 (dashed curve), and 1.0 (dot-dashed curve).

easily verified from Eqs. I (3.10)–I (3.12). Finally Fig. 3 gives the dependence of the spectra on the incoherent relaxation rate[8] $\gamma_\dagger$, which is present in certain types of experimental situations.[3] The asymmetry of the spectra has again a strong dependence on $\gamma_\dagger$. The behavior of the various peaks in Fig. 3, as $\gamma_\dagger$ is changed, can be understood by doing a calculation on the density matrix equation in the basis in which the matrix (6) is diagonal [cf. Refs. 4(a) and 4(b)]. Thus to conclude we have demonstrated how sensitively the spectral features of the signals in optical Hanle effect depend on the direction of polarization of the incident radiation, its strength, the direction of observation, and the incoherent relaxation rates.

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[6]In this paper we only calculate the features of the spectra in the resonant region. A more general treatment of the relaxation parameter $\gamma_\dagger$ is needed if one wants to examine the spectral features in the far wings of the line [cf. K. Burnett, J. Cooper, R. J. Ballagh, and E. W. Smith, Phys. Rev. A 22, 2005 (1980); A. Ben-Reuven, *ibid.* 22, 2572 (1980)].

[7]Recently, C. Delsart, J.-C. Keller, and V. P. Kaftandjian [J. Phys. (Paris) 42, 529 (1981)] have presented a detailed experimental study of the intensity of fluorescence for various polarizations (θ) of the incident beam.

[8]It may be added that in the weak-field limit $\alpha << \gamma,\delta$, there would be an incoherent contribution due to $\gamma_\dagger\neq 0$. This incoherent contribution shows well resolved peaks at $(\omega-\omega_L)=0,\pm\delta,\pm 2\delta$.