Alfvén-wave generation in a beam-plasma system

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This paper considers nonlinear decay of a beam-driven plasma mode into an Alfvén wave plus a circularly polarized electromagnetic wave near the electron-plasma frequency. Explicit expressions for the growth rate and threshold are obtained analytically. Relevance of our work to space as well as laboratory plasmas is discussed.

It is well known that the two-stream instability[1] in a beam-plasma system creates an unstable beam mode. The maximum growth rate γ_l occurs for $v_b = \omega_0/k_0$, and is given by[1]

$$\gamma_l = \frac{3^{1/2}}{2}\left[\frac{n_b}{2n_0}\right]^{1/3}\omega_{pe} , \tag{1}$$

where $n_b(n_0)$ is the beam (average plasma) density, v_b is the beam velocity, k_0 is the wave vector of the most unstable mode, and ω_{pe} is the electron-plasma frequency.

Such nonlinear processes as the particle trapping halts the growth of the beam mode. At saturation, the wave electric field is found to be[1]

$$E_0 = \frac{m_e k_0 v_b^2}{16e}\left[\frac{n_b}{n_0}\right]^{2/3} , \tag{2}$$

where e and m_e are the charge and mass of the electron, respectively. A coherent plasma mode is subjected to numerous kinds of parametric instabilities[2] in an unmagnetized plasma. The presence of a guide magnetic field $B_0\hat{z}$ allows one to investigate a great variety of nonlinear interactions. For example, recent laboratory[3] and computer[4] experiments have conclusively shown that an electron-plasma mode can decay into an ordinary electromagnetic wave near ω_{pe}, and into an ion wave. This process, which occurs in a multidimensional situation, has theoretically been analyzed by us.[5] Our findings are in agreement with the experimental observations.[3]

In his original paper,[6] Hasegawa pointed out the possibility of a new kind of decay interaction. In this novel process, a large-amplitude beam mode can nonlinearly decay into a high-frequency left-handed circularly polarized electromagnetic wave and a low-frequency right-handed electron whistler. All the waves are assumed to propagate along $B_0\hat{z}$. This can be viewed as Raman interaction in a magnetoplasma. A general formulation of coupling coefficients is contained in Ref. 7. However, the latter work does not give explicit results for the growth rate and threshold.

Here, we extend the work of Hasegawa by including ion motion. Thus, our low-frequency decay channels consist of Alfvén or ion-cyclotron modes. Using a very simple approach, we derive a general dispersion relation including proper dampings. Explicit expressions for the growth rate and threshold are derived.

Consider a cold plasma embedded in an external magnetic field in the presence of a beam mode whose properties follow (1) and (2). Nonlinear interaction of a finite-amplitude plasma wave with high- and low-frequency circularly polarized waves is governed by

$$\partial_t n_j + \nabla\cdot n_j \vec{v}_j = 0 , \tag{3}$$

$$\partial_t \vec{v}_j + \vec{v}_j\cdot\nabla\vec{v}_j = \frac{e_j}{m_j}[\vec{E} + \vec{v}_j\times(\vec{B}_0+\vec{B})] - \nu\vec{v}_j , \tag{4}$$

$$c^2\nabla\nabla\cdot\vec{E} - c^2\nabla^2\vec{E} + \partial_t^2\vec{E} - 4\pi e\partial_t(n_e\vec{v}_e - n_i\vec{v}_i) = 0 , \tag{5}$$

where n_j, m_j, $\vec{v}_j$ are the particle density, the mass, and the fluid velocity of species $j = e, i$; $\vec{E}$ and $\vec{B}$ are the self-consistent electric and magnetic fields associated with the wave, and ν is the electron-ion collision frequency.

Like Hasegawa, we are interested in a one-dimensional problem in which all the waves are collinear with $B_0\hat{z}$. In the field of the pump wave (ω_0, k_0) with the electric field

$$\vec{E}_0 = \hat{z}E_0\exp(ik_0 z - i\omega_0 t) + H , \tag{6}$$

the electron quiver velocity $v_0\hat{z}$, and the electron density perturbation $\tilde{n}_p$ are related by

$$\frac{\tilde{n}_p}{n_0}\equiv\frac{k_0}{\omega_0}v_0\equiv\mu \ , \tag{7}$$

where H stands for the complex conjugate, and $v_0=eE_0/im_e\omega_0$.

Nonlinear interaction of an electron-plasma mode with a low-frequency circularly polarized electromagnetic wave (ω_2,k_2) gives rise to a high-frequency circularly polarized wave (ω_1,k_1). The helicity is thus conserved. On the other hand, the momentum and energy conservations require that $\omega_1=\omega_0-\omega_2$ and $k_1=k_0-k_2$.

For circularly polarized waves, we introduce the following complex representation for the electric field:

$$\vec{E}_1=\hat{r}E_1\exp(ik_1z-i\omega_1t)+H \ , \tag{8}$$

where for the left-(right-) handed wave we have $\hat{r}=\hat{x}+i\hat{y}(\hat{r}=\hat{x}-i\hat{y})$, with $\hat{x}$ and $\hat{y}$ being the unit vectors along the x and y directions, respectively.

Writing

$$n_e=n_0+\tilde{n}_p \ , \quad n_i=n_0 \ ,$$
$$\vec{v}_e=v_0\hat{z}+\vec{v}_e^{(1)}+\vec{v}_e^{(2)} \ ,$$
$$\vec{v}_i=\vec{v}_i^{(1)}+\vec{v}_i^{(2)} \ ,$$
$$\vec{E}=\vec{E}_0+\vec{E}_1+\vec{E}_2 \ ,$$

and noting the fact that density perturbations associated with the electromagnetic waves under consideration are zero, we obtain from (5) after Fourier transformation

$$D_1\vec{E}_1=4\pi n_0ie\omega_1\left[\vec{v}_e^{(2)}-\vec{v}_i^{(2)}+\frac{\tilde{n}_p}{n_0}\vec{v}_{el}^{(1)}\right] \ , \tag{9}$$

where

$$D_1=c^2k_1^2-\omega_1^2+\omega_{pe}^2\omega_1/(\omega_1+i\nu+\Omega_e) \ ,$$
$$\omega_1>>\Omega_i \ , \quad \Omega_j=|e_jB_0/m_jc| \ .$$

In deriving (9), which is the nonlinear wave equation for the high-frequency left-handed circularly polarized wave, we have used the linear velocity

$$\vec{v}_{eh}^{(1)}=-\frac{ie\vec{E}_1}{m_2(\omega_1+\Omega_e+i\nu)} \ , \tag{10}$$

a solution of (4). Furthermore, $\vec{v}_{el}^{(1)}$ is the linear electron velocity in the electric field $\vec{E}_2$ of the low-frequency wave, and is given by

$$\vec{v}_{el}^{(1)}=-\frac{ie\vec{E}_2}{m_e(\omega_2\pm\Omega_e+i\nu)} \ , \tag{11}$$

where plus (minus) sign in front of Ω_e corresponds to left (right) circularly polarized wave.

The nonlinear velocity perturbations $\vec{v}_j^{(2)}$ are produced by beating the electron-induced velocity in the beam mode with the $\vec{v}_{el}^{(1)}$, and the corresponding low-frequency magnetic field $\vec{B}_l=(ck_2/\omega_2)\hat{z}\times\vec{E}_2$. In particular, $\vec{v}_e^{(2)}$ is obtained by solving the equation

$$\partial_t\vec{v}_e^{(2)}+\Omega_e\vec{v}_2^{(1)}\times\hat{z}$$
$$=-v_0\left[\partial_z\vec{v}_{el}^{(1)}-\frac{e}{m_e}\frac{k_2}{\omega_2}\vec{E}_2\right]$$
$$\simeq\pm\frac{e\vec{E}_2}{m_e\omega_2}\frac{\Omega_e}{(\omega_2\pm\Omega_e)}k_2v_0 \ . \tag{12}$$

If we let $\vec{v}_e^{(2)}=v_e^{(2)}(\hat{x}-i\hat{y})$, Fourier transformation of (12) then yields

$$\vec{v}_e^{(2)}=\frac{ie\vec{E}_2^*k_2v_0}{\omega_2m_e(\Omega_e+\omega_1)(\omega_2-\Omega_e)} \ , \tag{13}$$

where the low-frequency wave is taken to be a right-handed polarized. As a matter of fact, for the Stokes component of the satellite, the helicity conservation requires that the low-frequency wave to be of opposite polarization. Combining (7), (9), (11), and (13), we readily obtain after matching phasor

$$D_1\vec{E}_1=\frac{\omega_{pe}^2\omega_1}{(\omega_2-\Omega_e)}\left[1-\frac{k_2}{k_0}\frac{\omega_0\Omega_e}{\omega_2(\Omega_e+\omega_1)}\right]\mu E_2^* \ . \tag{14}$$

In the above, contribution of the ion term to the nonlinear coupling coefficient is small and is, therefore, neglected.

Next, we obtain a wave equation for $\vec{E}_2$ in terms of the electric fields of the pump and $\vec{E}_1$. Here, we include ion dynamics because $\Omega_e>\omega_2\sim\Omega_i$ or $\omega_2<<\Omega_e$, Ω_i. It is easy to show that

$$D_2\vec{E}_2=\frac{\omega_2\omega_{pe}^2}{(\omega_1+\Omega_e)}\left[1-\frac{\omega_0}{\omega_1}\frac{k_1}{k_0}\frac{\Omega_e}{(-\omega_2+\Omega_e)}\right]$$
$$\times\langle\mu E_1^*\rangle \ , \tag{15}$$

where the angular brackets denote averaging over the high-frequency oscillations. We have defined

$$D_2=c^2k_2^2-\omega_2^2+\frac{\omega_{pe}^2\omega_2\bar{\omega}_2}{(\bar{\omega}\mp\Omega_e)(\bar{\omega}\pm\Omega_i)} \ , \tag{16}$$

where $\bar{\omega}_2=\omega_2+i\nu$ and $\pm\omega_{pe}^2\Omega_i \mp \Omega_e\omega_{pi}^2=0$ by virtue of equilibrium charge neutrality. If we combine (14) and (15), we obtain a nonlinear dispersion equation

$$D_1^* D_2=\alpha\,|\mu|^2 , \tag{17}$$

where

$$\alpha=\frac{\omega_1\omega_2\omega_{pe}^4}{(\omega_2-\Omega_e)(\omega_1+\Omega_e)}\left[1-\frac{k_2}{k_0}\frac{\omega_0}{\omega_2}\frac{\Omega_e}{(\Omega_e+\omega_1)}\right]\times\left[1-\frac{k_1}{k_0}\frac{\omega_0}{\omega_1}\frac{\Omega_e}{(-\omega_2+\Omega_e)}\right] , \tag{18}$$

and the superscript asterisk denotes the complex conjugate.

For a problem involving an Alfvén wave as the low-frequency decay branch, (18) becomes for $|k_1|\sim|k_2|$, $\omega_1\sim\omega_{pe}>\Omega_e$, $\omega_2<<\Omega_i,\Omega_e$,

$$\alpha\approx-\omega_{pe}^4\left|\frac{k_1}{k_0}\right|^2 . \tag{19}$$

Letting $\omega_1=\omega_1+i\gamma_0$, $\omega_2=\omega_2+i\gamma_0$, we use the Taylor expansion (17) around $D_1\equiv 0$, and $D_2\equiv 0$. Thus, we have

$$D_1^*=i\frac{\partial D_1}{\partial\omega_1}(\gamma_0+\nu_1) , \tag{20a}$$

and

$$D_2=i\frac{\partial D_2}{\partial\omega_2}(\gamma_0+\nu_2) , \tag{20b}$$

where γ_0 is the growth rate. For $\omega_2<<\Omega_e$, Ω_i, it can be verified that

$$\frac{\partial D_1}{\partial\omega_1}=-2\omega_1\left[1-\frac{\Omega_e\omega_{pe}^2}{2\omega_1(\omega_1+\Omega_e)^2}\right] , \tag{21a}$$

$$\frac{\partial D_2}{\partial\omega_2}=-2\omega_2(1+\omega_{pe}^2/\Omega_i\Omega_e) . \tag{21b}$$

The damping rates for our purposes are

$$\nu_1=\frac{\nu\omega_{pe}^2\omega_1}{\omega_{pe}^2\Omega_e+2\omega_1(\omega_1+\Omega_e)^2} , \tag{22a}$$

$$\nu_2=\frac{\nu}{2\Omega_e}k_2 v_A , \tag{22b}$$

where v_A is the Alfvén speed.

Introducing (20) into (17), and making use of (19) and (21), we obtain the growth rate much above threshold

$$\gamma_0=\left[\frac{\Omega_i}{2\omega_2}\right]^{1/2}\gamma_H , \tag{23}$$

where $\omega_2=k_2 v_A$, $v_A<<c$, and

$$\gamma_H=2^{-1/2}\omega_{pe}(\Omega_e/\omega_{pe})^{1/2}\,|k_1\mu/k_0|$$

is the growth rate of Hasegawa for a problem involving electron whistler. Threshold is given by

$$|\mu|\geq\frac{\omega_2}{\Omega_i}\left[\frac{m_e}{2m_i}\frac{\omega_{pe}}{\Omega_e}\right]^{1/2}|\mu_H| , \tag{24}$$

where $|\mu_H|=|k_0/k_1|\,\nu/\omega_{pe}$ is the corresponding threshold for Hasegawa's problem.

Equations (23) and (24) show that, for a given pump, the Alfvén waves grow with a faster rate at low pump power. Note that our growth rate also competes with that of Nishikawa's oscillating two-stream instability.[2]

As an illustration, we apply our results to a man-made laboratory plasma. Accordingly, we take the following parameters: $n_0=5\times10^{11}$ cm^{-3}, $B_0=20$ G, $n_b/n_0=1\%$, $v_b\approx10^9$ cm/sec, $T_e=10T_i=3$ eV. For these values, we have $E_0^2/16\pi n_0 T_e=4\times10^{-4}$, and the maximum growth rate is found to be 10^4 sec^{-1}.

In the magnetosphere, there exist numerous data showing the presence of an electron-beam and beam-driven plasma mode. The phenomena of micropulsations,[8] which involve ultra-low-frequency magnetic pulsation, can be attributed to the nonlinear decay interaction described here. Unfortunately, at present we do not have any direct experimental data which show simultaneous existence of waves near ω_{pe}, and the Alfvén waves. However, on-going beam experiments[9] in space have promised more results. A meaningful comparison can be made at a later stage. The idea of this brief report, however, was to call attention of the fact that electromagnetic waves near ω_{pe} and the Alfvén waves may indeed originate in a beam-plasma system even in a one-dimensional simple model. Further, we note that decay into a low-frequency electrostatic ion wave is prohibited within the framework of the present model.

We have also considered decay into an ion-cyclotron wave. The growth rate

$$\gamma_i\simeq10^{-1/2}\gamma_H , \tag{25}$$

is generally smaller in comparison to that found earlier. Threshold for this case is found to be

$$|\mu|\geq\left[\frac{10\nu_{Ic}\omega_{pe}}{\nu\Omega_e}\right]^{1/2}|\mu_H| . \tag{26}$$

Here, ν_{Ic} is the damping rate of the ion-cyclotron mode.

On the other hand, the pump can also decay into a high-frequency right-handed circularly polarized wave, plus a left-handed Alfvén. The order of magnitude of that growth rate and threshold is about the same as given by (23) and (24).

It is of interest to note that when the anti-Stokes satellites are involved in a specific problem, it can then be possible to allow the low-frequency wave to be of the same polarization. This problem, which is rather basic from the three-wave interaction point of view, has been formulated elsewhere.[7] Explicit results can, however, be obtained after choosing particular modes and performing some algebra so that the coupling coefficients are simplified. If both the satellites are treated on the same footing, and the low-frequency wave is nonresonant, one encounters then the problem of wave filamentation. This deserves further investigation.

In conclusion, we have shown that even in a one-dimensional situation, there exists the possibility of nonlinear decay phenomena in a beam-plasma system. Here, the beam mode is found to decay into an electromagnetic wave near ω_{pe}, an Alfvén, or an ion-cyclotron wave. If the latter is left-handed polarized, there might result strong ion heating owing to cyclotron interaction. Alternatively, low-frequency waves can be correlated with the magnetic pulsations.

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