Massless neutrinos and the stellar stopping power via the $V-A$ and Weinberg interactions

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A formula for the stopping power of matter for neutrinos and antineutrinos, interacting with electrons via the weak interaction in the context of both the $V-A$ theory and the Weinberg theory is derived in the high-Q approximation and applied to study the solar-neutrino anomaly. The result shows that neutrinos extending in energy from 1 to 10 MeV produced inside the sun escape it essentially without any loss of energy. In agreement with previous investigators, Rustgi, Leung, Turner, and Brandt, it is found that the neutrinos should retain the energy given them at their creation essentially forever and permeate the universe uniformly rather than cluster around galaxies.

In a recent paper by Rustgi *et al.*,[1] a formula for the stopping power of matter for massive neutrinos interacting with electrons via the neutrino magnetic moment had been derived. The result was then applied to study the solar-neutrino anomaly and the stellar stopping power. It was found that the neutrinos produced in the sun lose insignificant amounts of energy as they travel through the sun and intergalactic space.[1–4] This led Rustgi *et al.* to conclude that the neutrinos should retain the energy given to them at their creation essentially forever. Because of the somewhat hypothetical nature of the interaction used in Ref. 1, the problem is reexamined in this note with more realistic e-ν_e interactions. The $V-A$ (Ref. 5) and Weinberg interactions[6] are used.

The differential cross section for the neutrino-electron scattering process in the context of both the conventional $V-A$ theory and Weinberg theory, and following the convention of Reines, Gurr, and Sobel,[7] may be written in a unified manner as

$$\frac{d\sigma}{dQ}=\frac{G^2m_e}{2\pi}\left[(C_V-C_A)^2+(C_V+C_A)^2\left[1-\frac{Q}{E_\nu}\right]^2+\frac{m_eQ}{E_\nu^2}(C_A^2-C_V^2)\right], \tag{1}$$

where E_ν is the energy of the incident neutrino and $Q=E_\nu-E_\nu'$ is the energy transferred to the electron. The electron is treated as free and initially at rest, which is appropriate for the high-Q approximation to be used below. The coupling constants are given as

$$\begin{aligned}&G^2m_{e/2\pi}=4.1\times10^{-45}\text{cm}^2/\text{MeV},\\&C_V=-C_A=1,\ \text{ for the } V-A \text{ theory},\\&\left.\begin{aligned}&C_V=\tfrac{1}{2}+2x,\ \ x=\frac{e^2}{g^2}=\sin^2\theta_w\\&C_A=-\tfrac{1}{2}.\end{aligned}\right\}\text{for Weinberg's theory.}\end{aligned} \tag{2}$$

Equation (1) gives the correct result in the limit of the conventional $V-A$ theory.[5]

Following Bethe[8] and Fano[9] and using Eq. (1), the stopping power for neutrinos in the high-Q limit is found to be

$$-\frac{dE}{dS}=\frac{NZG^2m_e}{2\pi}\left[(C_V^2+C_A^2)(Q_m^2-\eta^2)+\left[\frac{m_e}{E_\nu^2}(C_A^2-C_V^2)-\frac{2}{E_\nu}(C_V+C_A)^2\right]\left[\frac{Q_m^3-\eta^3}{3}\right]+\frac{(C_V+C_A)^2}{E_\nu^2}\left[\frac{Q_m^4-\eta^4}{4}\right]\right], \tag{3}$$

where NZ is the number of electrons per volume in the target, $Q_m=2E_\nu^2/(m_e+2E_\nu)$ is the maximum energy transferred to the electron, and η is comparable to the atomic ionization energy which is of the order eV. Thus, for high-energy neutrinos ($\gtrsim 1$ MeV) and high Q_m ($\sim$ MeV), η can be ignored in actual numerical calculations.

According to (2), (3) can be divided into two cases: for the $V-A$ theory,

$$\left[-\frac{dE}{dS}\right]_{VA}=\frac{2NZG^2m_e}{\pi}(Q_m^2-\eta^2), \tag{4}$$

for Weinberg's theory,

$$\left[-\frac{dE}{dS}\right]_{\rm W}=\frac{NZG^2m_e}{2\pi}\left[(8x^2+4x+1)\left[\frac{Q_m^2-\eta^2}{2}\right]-\frac{2x}{E_\nu}\left[\frac{m_e}{E_\nu}(1+2x)+4x\right]\left[\frac{Q_m^3-\eta^3}{3}\right]+\frac{x^2}{E_\nu^2}(Q_m^4-\eta^4)\right]. \tag{5}$$

Taking the value of x to be 0.29 ± 0.05 from Reines, Gurr, and Sobel,[4] (5) becomes

$$\left[-\frac{dE}{dS}\right]_{\rm W}=\frac{NZG^2m_e}{2\pi}\left[1.4(Q_m^2-\eta^2)-\frac{0.58}{3E_\nu}\left[\frac{1.58m_e}{E_\nu}+1.16\right](Q_m^3-\eta^3)+\frac{0.08}{E_\nu^2}(Q_m^4-\eta^4)\right]. \tag{6}$$

The corresponding results for antineutrino-electron scattering are obtained by replacing C_A by $-C_A$ in Eq. (1); the results are

$$\left[-\frac{dE}{dS}\right]_{VA}=\frac{NZG^2m_e}{2\pi}\left[2(Q_m^2-\eta^2)-\frac{8}{3E_\nu}(Q_m^3-\eta^3)+\frac{1}{E_\nu^2}(Q_m^4-\eta^4)\right], \tag{7}$$

$$\begin{aligned}\left[-\frac{dE}{dS}\right]_{\rm W}&=\frac{NZG^2m_e}{2\pi}\left[(8x^2+4x+1)\left[\frac{Q_m^2-\eta^2}{2}\right]-\frac{2(1+2x)}{E_\nu}\left[\frac{m_e x}{E_\nu}+1+2x\right]\right.\\&\qquad\left.\times\left[\frac{Q_m^3-\eta^3}{3}\right]+\frac{(1+2x)^2}{E_\nu^2}\left[\frac{Q_m^4-\eta^4}{4}\right]\right]\\&=\frac{NZG^2m_e}{2\pi}\left[1.4(Q_m^2-\eta^2)-\frac{3.16}{3E_\nu}\left[\frac{0.29m_e}{E_\nu}+1.58\right](Q_m^3-\eta^3)+\frac{0.62}{E_\nu^2}(Q_m^4-\eta^4)\right].\end{aligned} \tag{8}$$

To apply the above to the case of solar neutrinos, the solar electron density is taken to be $N=1.2\times10^{25}$ cm^{-3},[10] $Z=1$, and η is approximately given by the plasma energy of the solar electrons, $\eta\sim\hbar(4\pi Ne^2/m)^{1/2}\sim100$ eV. With the value of $G^2m_e/2\pi$ in (2) and to a good approximation setting $\eta=0$, Eqs. (4) and (6)–(8) become (for neutrinos)

$$\left[-\frac{dE}{dS}\right]_{VA}=9.8\times10^{-20}Q_m^2, \tag{9}$$

$$\left[-\frac{dE}{dS}\right]_W = 4.9\times10^{-10}\left[1.4Q_m^2-\frac{1}{3E_\nu}\left[\frac{0.47}{E_\nu}+0.67\right]Q_m^3+\frac{0.08}{E_\nu^2}Q_m^4\right], \tag{10}$$

and (for antineutrinos)

$$\left[-\frac{dE}{dS}\right]_{VA} = 4.9\times10^{-20}\left[2Q_m^2-\frac{8}{3E_\nu}Q_m^3+\frac{1}{E_\nu^2}Q_m^4\right], \tag{11}$$

$$\left[-\frac{dE}{dS}\right]_W = 4.9\times10^{-20}\left[1.4Q_m^2-\frac{1}{3E_\nu}\left[\frac{0.47}{E_\nu}+5.0\right]Q_m^3+\frac{0.62}{E_\nu^2}Q_m^4\right]. \tag{12}$$

Equations (9)–(12) are in units of MeV/cm. Furthermore, if we assume that all these stopping powers are constant all along the range of the neutrino, then the total loss of energy $(-\Delta E)$ for a neutrino created at the center and escaping at the surface of the sun can be calculated just by multiplying the stopping power with the radius of the sun $\sim 7\times10^{10}$ cm.

The numerical results of Eqs. (9) and (10) for neutrinos are given in Table I. For comparison, it may be mentioned that the energy loss of neutrinos is somewhat larger than that of antineutrinos of the same energy. We also see that the Weinberg theory, in general, gives a smaller value for the energy loss than the conventional $V-A$ theory. These results will not be changed in any significant way if the more recent value of $\sin^2\theta_w = 0.224 \pm 0.020$ is used.[11] In any case, the energy loss of ν_e created inside the sun is completely insignificant compared to their original energy, and hence, these neutrinos will essentially escape into space with undiminished energy, a conclusion in agreement with the previous investigators.[1,10]

We will now study the cosmological implications of our result. A comparison with Rustgi *et al.*,[1] indicates that the energy lost by 1-MeV neutrinos via weak interactions in transversing through the sun which is a typical star, is $\sim 10^{-9}$ MeV, while it was estimated to be $\sim 10^{-7}$ MeV for the massive neutrinos.[1] For 10-MeV neutrinos it is found to be $\sim 10^{-6}$ MeV. A 10-MeV neutrino therefore must strike $\sim 10^4$ stars to acquire thermal energies ($\sim 10^{-2}$ MeV). This number will be 100 times bigger for 1-MeV neutrinos. It has been estimated in Ref. 1 that the average number of stars traversed by a neutrino in traversing the known universe is $\sim 10^{-12}$, far below the number ($\sim 10^4 - 10^6$) estimated here. The thermalization distance of the stellar neutrinos must therefore be many orders of magnitude—perhaps 16–18—greater than the radius of the known universe.

We therefore conclude in agreement with Ref. 1 that the energy of the stellar neutrinos remains essentially undiminished as they traverse through the known universe. It may be mentioned that the formation of the neutrino-bound system will not be possible for these neutrinos, as has been claimed for massive neutrinos by Sato and Takahara,[12] since the neutrinos that we consider are massless. They should be distributed uniformly throughout the space.

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TABLE I. Stopping power and energy loss for solar neutrinos.

E_ν (MeV)	Q_m (MeV)	Eq. (9) (MeV/cm)	Eq. (10) (MeV/cm)	$(-\Delta E)_{VA}$ (MeV)	$(-\Delta E)_W$ (MeV)
1	0.80	6.3×10^{-20}	3.6×10^{-20}	4.4×10^{-9}	2.5×10^{-9}
2	1.77	30.7×10^{-20}	18.3×10^{-20}	21.3×10^{-9}	12.7×10^{-9}
3	2.77	75.2×10^{-20}	45.6×10^{-20}	52.2×10^{-9}	31.7×10^{-9}
5	4.76	221.8×10^{-20}	136.4×10^{-20}	155.2×10^{-9}	95.5×10^{-9}
8	7.75	588.6×10^{-20}	365.1×10^{-20}	408.4×10^{-9}	255.6×10^{-9}
10	9.75	931.6×10^{-20}	579.0×10^{-20}	646.4×10^{-9}	401.8×10^{-9}

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