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Stopping power of protons and alpha particles in H_2, He, N_2, O_2, CH_4, and air

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Recent developments in the theory of the energy loss of charged particles in the gases H_2, He, N_2, O_2, and CH_4, permit a calculation of the stopping power with only one free parameter—that needed in the description of the cubic correction term for the projectile charge. Experimental data for the stopping power of these gases for protons and alpha particles have been compared with calculated values. Once a value of the free parameter is assumed, it appears that no further improvements in the theory are required to explain the data for projectile energies above 0.5 MeV/u. The error of the theory is probably about $\pm 2\%$ for these energies.

I. INTRODUCTION

The stopping power of matter for charged particles is a subject of great importance in numerous areas of fundamental and applied physics. The Bethe theory can be used for the calculation of stopping power over a wide energy interval for fast charged particles. (See, e.g., Ref. 1 or 2.) This theory, based on the first Born approximation for atomic collision events, provides a stopping power dependent on the square of the projectile charge (ze). Deviations from the z^2 dependence are especially large at low, but they occur to some extent even for high, particle velocities.[3] A z^3 correction term must therefore be added to the Bethe formula.[3,4] This term has been labeled the "Barkas-effect" term.[5] Moreover, a correction term for even powers of z devised by Bloch[6] has recently been revived,[5] and a Bethe-Bloch formula corrected with a z^3 term has been used in the analysis of experimental data. Theoretical stopping-power calculations are further complicated by the presence in the Bethe-Bloch formula of shell corrections, of a density-effect correction for highly relativistic projectiles, and of the mean excitation energy, a parameter characteristic of the target atom. Moreover, at very low projectile velocities the projectile charge must be replaced by a velocity-dependent reduced charge, $z^* < z$.

Corrections for the reduction of stopping power given by the asymptotic Bethe formula, due to the binding of atomic electrons, were calculated for K and L shells some time ago.[7,8] Recently, values of the mean excitation energy I have been calculated for several materials with considerable accuracy.[9–14] It therefore seems appropriate to analyze current experimental data from the point of view of Bethe theory with the corrections noted above. The approach utilized herein differs from that of Andersen and Ziegler,[15] who used a semiempirical approach to treat all the corrections. The purpose of the present study is to provide a comparison of theory with experimental data for protons and alpha particles in the gases H_2, He, N_2, O_2, and CH_4, for each of which the mean excitation energy has been accurately calculated from optical or theoretical dipole oscillator strengths.[9–14] The sole free parameter employed in the present analysis is that associated with the projectile-z^3 correction formalism.[4] On the basis of the comparison an attempt will be made to furnish an appraisal of current theory and all available experiments.

II. THEORY

A. The stopping power

The stopping power S in units of MeV cm^2/g, of a target material with atomic weight A and atomic number Z for a projectile of charge ze traveling at velocity $v = \beta c$ is calculated from the expression,[2]

$$S=\frac{0.307\,08}{\beta^2}z^2\frac{Z}{A}L\ , \tag{1}$$

where L is called the stopping number per target electron, described by

$$L=L_0(\beta)+zL_1(\beta)+L_2(\beta)\ . \tag{2}$$

In this expression, the first term $L_0(\beta)$, is given by the Bethe formula

$$L_0=\ln\left[\frac{2mc^2\beta^2}{(1-\beta^2)}\right]-\beta^2-\ln I-\frac{C(\beta)}{Z}\ , \tag{3}$$

with mc^2 the rest mass energy of the electron. I is the mean excitation energy of the target atom,[1] and $C(\beta)$ represents a correction for the binding of the electrons in the target atom.[7] The Barkas-effect correction term L_1 is expressed by

$$L_1=\frac{F(b/x^{1/2})}{Z^{1/2}x^{3/2}}\ , \tag{4}$$

with the function F defined and tabulated in Ref. 4, and $x=v^2/(Zv_0^2)=18\,787\beta^2/Z$. The quantity b is considered to be a free parameter in the present study. This parameter, introduced in Refs. 4 and 16, was initially evaluated as $b=1.8\pm0.2$ on the basis of stopping-power measurements with $z=1$ and $z=2$ projectiles traversing thin Al and Ta targets.[17] When the Bloch term was subsequently included in the stopping-power formula for the purpose of analyzing two recent sets of accurate stopping-power measurements (one with Al, Cu, Ag, and Au targets and $z=1$, 2, and 3 projectiles,[18] and the other with a Au target and $z=1-3$ and $5-9$ projectiles[19]), the ensuing extracted value of the free parameter[20] was $b=1.4\pm0.1$. A recent analysis of the stopping power of polystyrene for (2.2 – 5.9)-MeV protons led to a recommended value of the projectile-z^3 parameter[21] of $b=1.90\pm0.05$. Thus the anticipated values of b in the present study were of order unity.

The last term in Eq. (2) was given by Bloch[6] as

$$L_2(y)=\psi(1)-\mathrm{Re}[\psi(1+iy)]\ , \tag{5}$$

where $y=z\alpha/\beta$, with α the fine-structure constant, and ψ is the logarithmic derivative of the gamma function.[22] This term can be represented by the series[22]

$$L_2(y)=-y\sum_{j=1}^{\infty}j^{-1}(j^2+y^2)^{-1}\ , \tag{6}$$

and is therefore always negative. For $y^2<1$, L_2 can be approximated to within 0.4% by the function

$$\bar{L}_2(y)=-y^2[1.202\,06-y^2(1.042-0.8549y^2+0.343y^4)]. \tag{7}$$

It is here assumed that no corrections for atomic binding are needed in L_1 and L_2. The possible need for such corrections has been pointed out in Ref. 4. The contributions to stopping power from "nuclear collisions" are negligible for the projectile energies considered here.[2,23]

B. The mean excitation energies I

The mean excitation energy I is defined[1,2] by

$$\ln I=\textstyle\sum_i f_i(\epsilon)\ln(\epsilon)\ , \tag{8}$$

where the sum includes an integral over continuum states, and $f_i(\epsilon)$ is the dipole oscillator strength of the transition from the ground state to an excited state with energy ϵ of the atom, molecule, liquid, or solid. Completely theoretical calculations, based on theoretical wave functions for atoms, have been available for some time.[10,24,25] However, considerably more complicated calculations are involved when aggregation effects are taken into account.[1,2,16,26,27] A pair of studies illustrates the difference in I values for Al induced by aggregation effects: $I=$ 124.3 eV for isolated Al atoms,[10] whereas fits to accurate experimental data imply[28] a value of $I=167$ eV for Al in its solid state. In more recent studies, calculations based on detailed analyses of experimental and theoretical values of $f_i(\epsilon)$ have been performed.[11,12]

In this paper, the only substances considered are those for which I values have been calculated from optical or theoretical dipole oscillator strengths with an uncertainty of the order of 1%. The values used[9-14] are given in Table I.

C. Shell corrections

A detailed description of the shell corrections is given by Walske.[8] The stopping number $B_i(L_0=B=\sum_i B_i)$ for the ith shell was given as

$$B_i(\theta_i,\eta_i)=S_i(\theta_i)\ln\eta_i+T_i(\theta_i)-C_i(\theta_i,\eta_i)\ , \tag{9}$$

where θ_i depends on Z and is defined in Ref. 8, and

$$\eta_i=mc^2\beta^2/[2\mathscr{R}(Z-d_i)^2]=18\,787\beta^2/(Z-d_i)^2\ , \tag{10}$$

TABLE I. Mean excitation energies (I) and shell correction parameters for K and L shells (θ_K, θ_L, S_K, T_K, S_L, T_L, d_K, and d_L).

Absorber	I (eV)	Uncertainty	Ref.	θ_K	θ_L	S_K	T_K	S_L	T_L	d_K	d_L
H_2	19.26	~1%	11	0.75		1	1.289			0	
He	41.8 42.0	<2%	13 14	0.63		2	1.289			0.3125	
N_2	81.84	~1%	11	0.66	0.35	1.88	2.506	10	28.14	0.327	3.16
O_2	95.02	~1%	11	0.66	0.35	1.88	2.506	10	28.14	0.335	3.54
CH_4	41.66	~1%	12	0.66	0.35	1.73	2.403	10	28.14	0.342	2.82

where $\mathscr{R}=13.6$ eV and d_i is a screening constant.[29] S_i represents essentially the total oscillator strength of all optical transitions from the ith shell to unoccupied levels and the continuum,[7,8] and

$$T_i = S_i \ln[4\mathscr{R}(Z-d_i)^2/I_i] \;, \tag{11}$$

where I_i is the mean excitation energy for the ith shell.

Walske provided[8] asymptotic functions ($\eta_i > 5$), calculated within the hydrogenic approximation, for C_K and C_L. For smaller values of η_i, tables of B_i have been prepared,[8,30,31] and C_i is calculated by solving Eq. (9) for C_i.

The values used for θ, S, T, and d are given in Table I. For H and He, the function

$$C_k = \frac{Z}{2\eta_K}\left[2+\frac{25}{3\eta_K}-\frac{45}{\eta_K^2}\right] \tag{12}$$

is used for the asymptotic expression. Otherwise, the tabulated values of B_K, C_K, B_L, and C_L are used. The L-shell correction is multiplied by $(Z-2)/8$ for $Z \le 10$. It should be mentioned that the binding correction for the L -shell in N_2 for protons reduces L_0 by 1% at 1 MeV and by 5% at 0.3 MeV.

The sum of these corrections $C=C_K+C_L$, can, for large values of η, be compared with asymptotic values C_a calculated by Fano with Eq. (58c) of Ref. 1

$$C_a \sim 2Z\left[\frac{K_1}{2mv^2}+\frac{K_2}{(2mv^2)^2}\right] . \tag{13}$$

For nitrogen, with x defined below Eq. (4),

$$\begin{aligned} K_1/2mv^2 &\sim Z^{0.4}/2x = 0.000\,406/\beta^2 \;, \\ K_2/(2mv^2)^2 &\sim 4Z/x^2 = 3.9\times10^{-6}/\beta^4 \;. \end{aligned} \tag{14}$$

Some values are given in Table II. For large values of β^2, the agreement thus is good.

It should be noted that the binding correction for the K shell, C_K, is negative for alpha particles with energies below 1.5 MeV in C, N_2, and O_2. Calculations with hydrogenic wave functions may be quite inaccurate at these energies.

D. Magnitudes of Barkas effect and Bloch terms

The Barkas-effect term possesses a complicated dependence on projectile velocity and on Z, whereas the Bloch term features a simple monotonically decreasing dependence on β and no variation with Z. In order to provide a basis for comprehension of the magnitudes of these correction terms relative to L_0, values of each are shown

TABLE II. Comparison of Walske (Ref. 8) and Fano (Ref. 1) binding corrections for nitrogen. The values $\eta_K=419\beta^2$ and $\eta_L=1275\beta^2$ were used.

		Ref. 8		Ref. 1
β^2	C_K	C_L	C	C_a
0.01	0.510	0.079	0.589	1.11
0.051	0.107	0.015	0.122	0.132
1.0	0.004 94	0.000 74	0.005 68	0.005 74

as a function of projectile energy in Table III for protons and alpha particles in a N_2 target. The function L_1/L_0 is also given for H_2.

E. Data analysis

Differences between theory and experiment can be seen readily in a plot of the ratio r of experimental to calculated stopping power as a function of particle energy E,

$$r = S_x / S_t \,, \tag{15}$$

where S_x is the value of the stopping power measured in an experiment and S_t is calculated from Eq. (1).

In order to demonstrate further the importance of the various terms in Eq. (2), several ratios will be shown for alpha particles in nitrogen: r_0, calculated with L_0 only (i.e., $L_1 = L_2 = 0$); r_2, calculated with L_0 and L_2 but $L_1 = 0$; and r_3, including all three terms in Eq. (2). Since r_1, calculated with L_0 and L_1, would contain the free parameter b, it will not be shown. In addition, in Fig. 6, in order to show the sensitivity of the results to the choice of b, the calculated function q will be plotted, where q is defined as

$$q = \frac{L_0 + zL_1(b_2) + L_2}{L_0 + zL_1(b_1) + L_2} \,, \tag{16}$$

and b_1 and b_2 signify two different values of the parameter b in Eq. (4).

III. RESULTS AND DISCUSSION

For alpha particles in N_2, the ratio r_0 is plotted in Fig. 1 along with several sets of data.[23,32–37] For energies above 1.5 MeV, the average difference between theory and experiment seems to be about 2%. It should be noted that a good fit to the current experimental data above 1.5 MeV can be obtained with L_0 alone if the I value is chosen to be a free parameter ($I = 78$ eV). This result is fortuitous and should not be considered a justification for the use of L_0 only. If the Bloch term L_2 had been used in earlier analyses,[1,38] the need for L_1 would have been seen clearly, as is evident from Fig. 2, where the ratio r_2 is shown. In Fig. 1, the function $u = 1 + (zL_1 + L_2)/L_0$ is also plotted for $b = 1.8$ and 2.0. [Note that L_2 is negative; see Eq. (6)]. For energies above 1.5 MeV, $b = 1.8$ would give a good compromise for representing experimental data. For $E < 1.5$ MeV, the agreement between theory and the experiment would be excellent for $b = 2.0$. Finally, in Fig. 3, the ratio r_3 (with $b = 1.8$) is given. Recent measurements by Thwaites[39] are also included in this figure. The agreement between theory and experiment is within experimental uncertainties for energies above 1.5 MeV, except for the Hanke-Laursen data[37] at about 6 MeV, where the difference between experiment and theory is 1.3%, and the uncertainty stated by the authors is only 0.3%. For these data, the overall agreement between theory and experiment would be better for $I = 80$ eV.

From an inspection of the plot, it is quite obvious that no further improvement of the theory is required by the existing experimental data above 1.5 MeV. Furthermore, the fluctuations in the data (with the possible exception of the Hanke values[37]) seem to preclude a meaningful analysis with more than the one parameter used here. In particular, it appears unlikely that modifications of the shell corrections could be derived from existing experimental data. We believe that the error (one standard deviation) of the theory is no more than 2% for energies above 1.5 MeV. For an analysis of the data below 1.5 or 2 MeV, it seems advisable to exclude the measurements with natural alpha particles[32,33,37,39] because of the straggling problem for thicknesses close to the total range.[40] The residual data[34–36] then have a spread of about 5%. For these energies, with $b = 1.8$, further modifications of the theory used here would be needed to obtain agreement between it and experiment, such as the introduction of charge state corrections, or of binding corrections for L_1 and L_2, or changes in the shell corrections.

For alpha particles in H_2 and He, the experimental ratios r_3 for $b = 0.6$ are given in Figs. 4 and 5. The measurements for H_2 are found in Refs. 23, 34–37, 41, and for He in Refs. 23, 35, 37, 39, 42, and 43. For both gases the agreement is acceptable for energies above 3 MeV, except for the data with He by Thwaites,[39] where the difference between theory and experiment is about 5%, and the experimental uncertainty is quoted as $\leq 1\%$. For alpha particles in O_2 (experimental data in Refs. 23, 34, 36, and 37) and CH_4 (Refs. 32–34, 39, 44, and 45) the ratios r_3 are shown in Figs. 6 and 7. As was the case for N_2, the theory for O_2 and CH_4 agrees with experiments for energies above 1.5 MeV. For CH_4, it must be noted, though, that there is considerable spread in the data. Again, the data by Thwaites[39] differ from calculated values by more

TABLE III. Values of the stopping number L_0, the Bloch correction term L_2, and the Barkas-effect term L_1 for protons and alpha particles in nitrogen. L_1 is also given for hydrogen.

E_p (Mev)	L_0	L_2/L_0	zL_1/L_0	zL_1/L_0 (H_2)	$E\alpha$ (MeV)	L_0	L_2/L_0	zL_1/L_0	zL_1/L_0 (H_2)
0.1	0.9778	0.2530	0.2143	0.4116	0.5	1.1561	0.5097	0.3561	0.5618
0.2	1.5539	0.0872	0.1211	0.1333	0.6	1.3078	0.3986	0.3039	0.4175
0.3	1.9245	0.0486	0.0851	0.0706	0.7	1.4412	0.3242	0.2676	0.3261
0.4	2.1935	0.0325	0.0654	0.0453	0.8	1.5601	0.2717	0.2409	0.2638
0.6	2.5836	0.0187	0.0431	0.0243	0.9	1.6662	0.2329	0.2205	0.2191
0.8	2.8648	0.0128	0.0316	0.0157	1.0	1.7626	0.2032	0.1970	0.1856
1.0	3.0892	0.0095	0.0250	0.0112	1.5	2.1392	0.1213	0.1382	0.0989
1.2	3.2781	0.0075	0.0204	0.0085	2.0	2.4150	0.0844	0.1039	0.0636
1.4	3.4374	0.0062	0.0170	0.0067	2.5	2.6299	0.0639	0.0819	0.0452
1.6	3.5786	0.0052	0.0145	0.0055	3.0	2.8077	0.0509	0.0671	0.0343
1.8	3.7049	0.0045	0.0126	0.0046	3.5	2.9611	0.0420	0.0572	0.0271
2.0	3.8182	0.0039	0.0110	0.0039	4.0	3.0962	0.0355	0.0496	0.0221
2.5	4.0586	0.0029	0.0083	0.0028	4.5	3.2186	0.0307	0.0435	0.0185
3.0	4.2549	0.0023	0.0065	0.0021	5.0	3.3274	0.0269	0.0386	0.0158
3.5	4.4210	0.0019	0.0053	0.0017	6.0	3.5178	0.0214	0.0311	0.0120
4.0	4.5648	0.0016	0.0045	0.0014	7.0	3.6820	0.0177	0.0258	0.0095
5.0	4.8027	0.0013	0.0033	0.0010	8.0	3.8256	0.0150	0.0218	0.0078
6.0	4.9947	0.0010	0.0026	0.0007	9.0	3.9524	0.0130	0.0188	0.0065
7.0	5.1560	0.0008	0.0021	0.0006	10.0	4.0660	0.0114	0.0164	0.0055
8.0	5.2948	0.0007	0.0018	0.0005	11.0	4.1685	0.0101	0.0145	0.0048
9.0	5.4166	0.0006	0.0015	0.0004	12.0	4.2623	0.0091	0.0129	0.0042
10.0	5.5249	0.0006	0.0013	0.0003	13.0	4.3484	0.0083	0.0116	0.0037
11.0	5.6224	0.0005	0.0012	0.0003	14.0	4.4284	0.0076	0.0105	0.0033
12.0	5.7118	0.0004	0.0010	0.0003	15.0	4.5026	0.0070	0.0096	0.0030
13.0	5.7929	0.0004	0.0009	0.0002	16.0	4.5722	0.0064	0.0088	0.0027
14.0	5.8679	0.0004	0.0008	0.0002	17.0	4.6372	0.0060	0.0082	0.0025
15.0	5.9274	0.0003	0.0008	0.0002	18.0	4.6982	0.0056	0.0076	0.0023
16.0	6.0022	0.0003	0.0007	0.0002	19.0	4.7557	0.0052	0.0071	0.0021
17.0	6.0630	0.0003	0.0006	0.0002	20.0	4.8100	0.0049	0.0066	0.0020
18.0	6.1201	0.0003	0.0006	0.0001	21.0	4.8614	0.0046	0.0062	0.0018

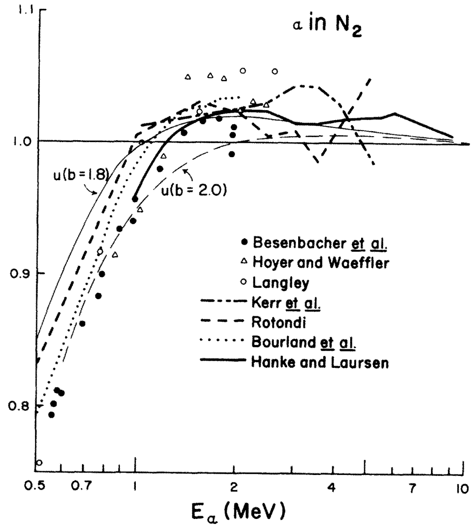

FIG. 1. Plots of the ratio r_0 of experimental to calculated stopping power (without Barkas or Bloch corrections) for alpha particles in N_2. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%), 35 (2 to 6%), and 36 (2%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 32 (4 to 12%), 33 (3 to 8%), 34 (1 to 2%), and 37 (0.3 to 5%). The function $u=1+(zL_1+L_2)/L_0$ is also plotted for $b=1.8$ and $b=2.0$.

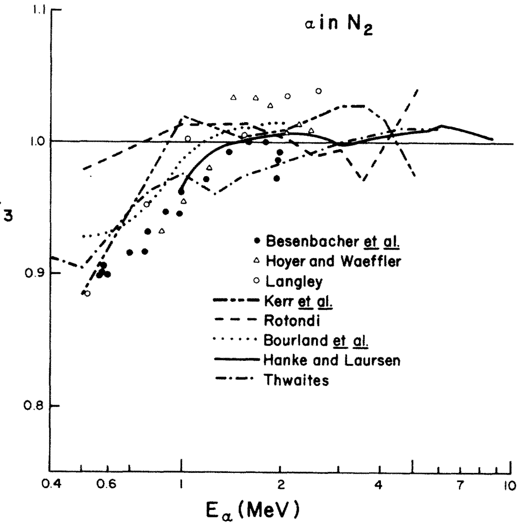

FIG. 3. Plots of the ratio r_3 of experimental and calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b=1.8$) for alpha particles in N_2. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%), 35 (2 to 6%), and 36 (2%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 32 (4 to 12%), 33 (3 to 8%), 34 (1 to 2%), 37 (0.3 to 5%), and 39 (1 to 7%).

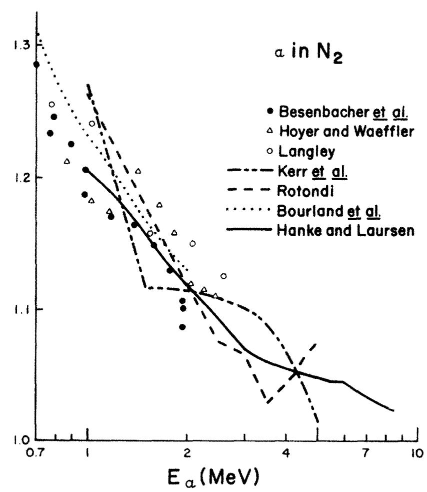

FIG. 2. Plots of the ratio r_2 of experimental to calculated stopping power (with Bethe-Bloch theory) for alpha particles in N_2. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%), 35 (2 to 6%), and 36 (2%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 32 (4 to 12%), 33 (3 to 8%), 34 (1 to 2%), and 37 (0.3 to 5%).

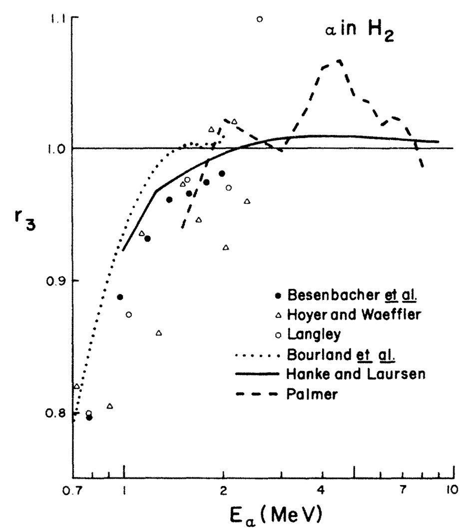

FIG. 4. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b=0.6$) for alpha particles in H_2. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%), 35 (2 to 6%), and 36 (2%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 34 (1 to 2%), 37 (0.3 to 5%), and 41 (2 to 5%).

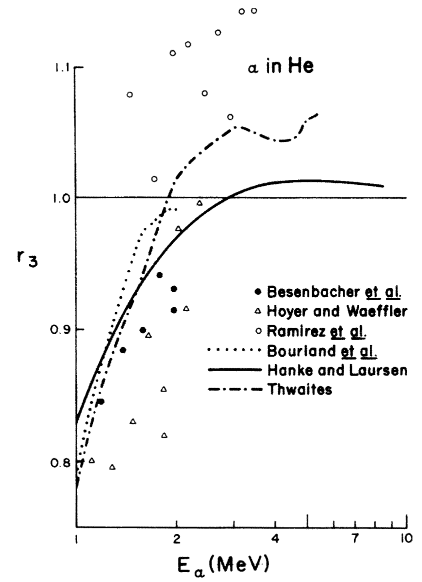

FIG. 5. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b=0.6$) for alpha particles in He. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%), 35 (2 to 6%), and 42 (information not given), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 34 (1 to 2%), 37 (0.3 to 5%), and 39 (1 to 7%). Two data points from Ref. 42 were omitted in order to condense the graphical information: $r_3=1.266$ at $E_\alpha=3.173$ MeV, and $r_3=1.194$ at $E_\alpha=3.407$ MeV.

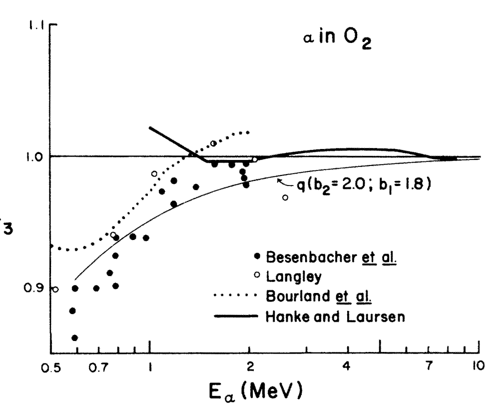

FIG. 6. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b=1.8$) for alpha particles in O_2. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%) and 36 (2%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 34 (1 to 2%) and 37 (0.3 to 5%). The function q [see Eq. (16)] is also plotted for $b_1=1.8$ and $b_2=2.0$.

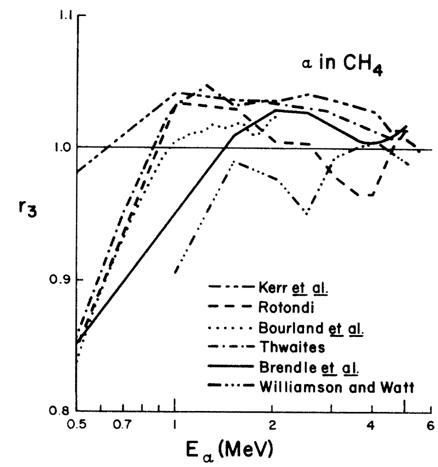

FIG. 7. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b=1.8$) for alpha particles in CH_4. Curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 32 (4 to 12%), 33 (3 to 8%), 34 (1 to 2%), 39 (1 to 7%), 44 (2%), and 45 (1.5 to 1.7%).

than the quoted uncertainty. The fact that the theory agrees with experiment to lower energies for N_2, O_2, and CH_4 than for H_2 and He suggests a reexamination of both theory and experiment. In particular, the Barkas effect should be carefully evaluated for the very light elements, as has been suggested previously.[27,46] Charge state effects, as well as binding corrections for L_1 and L_2, should also be considered.

For protons in all the gases, Figs. 8–12, the agreement between theory and experiment is within the apparent uncertainties of the measurements for energies above 0.3 MeV. Measurements are given for H_2 in Refs. 23, 36, and 47–49, for He in Refs. 23, 47, 48, and 50, for N_2 in Refs. 23, 36, 47, 48, and 51–54, for O_2 in Refs. 47 and 51, and for CH_4 in Refs. 47, 48, 50, 53, and 54. For N_2 (Fig. 10), the experimental values seem to be smaller than S_t on the average in the energy range from 0.5 to 1 MeV. This does not appear to be the case for protons in air, as shown in Fig. 13, where data from Refs. 46–49, 51, and 55 are shown.

The comments concerning the need for L_1 and L_2, which were made for the case of alpha particles in nitrogen in connection with Fig. 2, apply as well for all the other projectile-target combinations. The values of b giving a good fit to the experimental data are shown in Table IV, and in Table V, calculated values of the stopping power for protons and alpha particles are given.

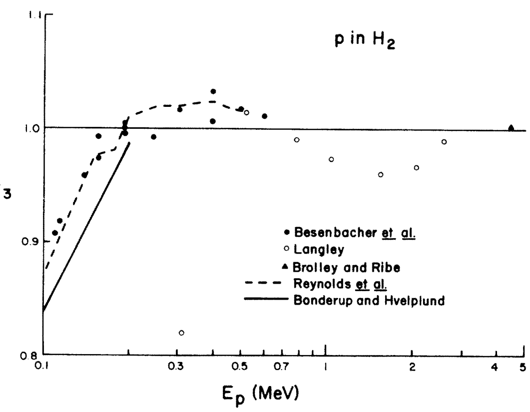

FIG. 8. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b=0.6$) for protons in H_2. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%), 36 (2%), and 48 (2%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 47 (1.7 to 3.4%) and 49 (4%).

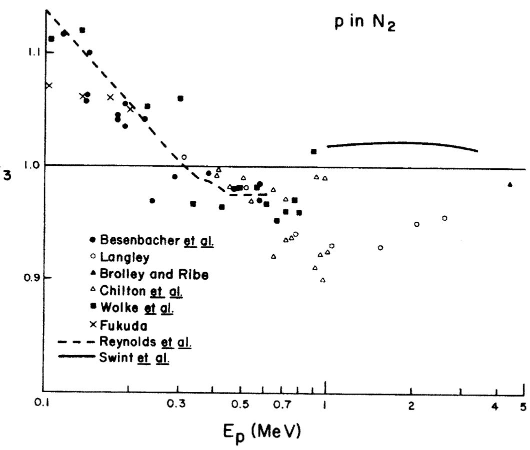

FIG. 10. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b=1.8$) for protons in N_2. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%), 36 (2%), 48 (2%), 51 (2.5 to 4.5%), 52 (information not given), and 54 (1.3%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 47 (1.7 to 3.4%) and 53 (2 to 5%).

IV. CONCLUSIONS

The theory outlined above agrees satisfactorily with experimental data for protons with energies above 0.3 MeV in all gases, and for alpha particles with energies above 1.5 MeV for the gases N_2, O_2, and CH_4. For alpha particles in H_2 and He, the agreement is acceptable above 3 MeV. A single

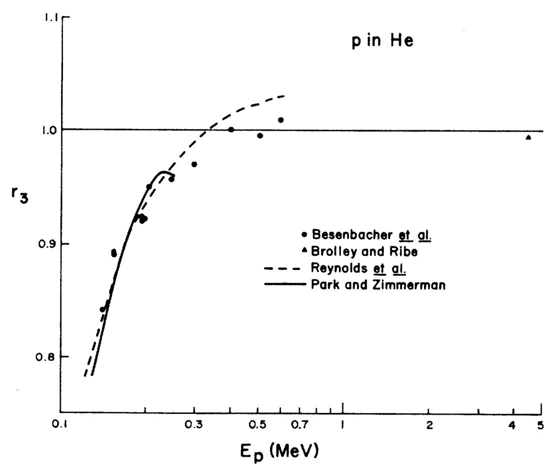

FIG. 9. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b=0.6$) for protons in He. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%) and 48 (2%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 47 (1.7 to 3.4%) and 50 (1.5 to 4%).

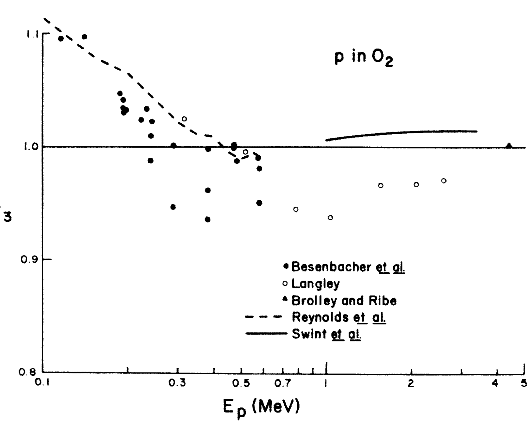

FIG. 11. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b=1.8$) for protons in O_2. Data points represent measurements, with cited accuracy in parentheses, from Refs. 23 (1.5%), 36 (2%), and 48 (2%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 47 (1.7 to 3.4%) and 53 (2 to 5%).

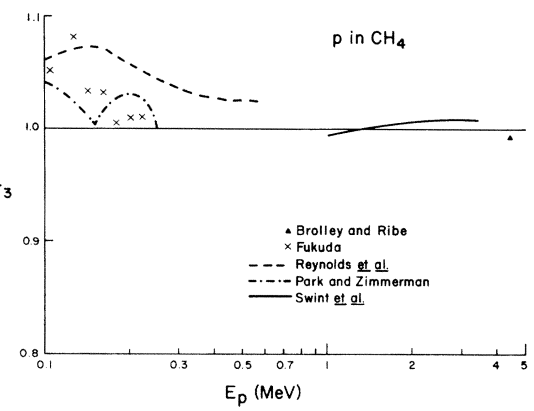

FIG. 12. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b = 1.8$) for protons in CH_4. Data points represent measurements, with cited accuracy in parentheses, from Refs. 48 (2%) and 54 (1.3%), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 47 (1.7 to 3.4%), 50 (1.5 to 4%), and 53 (2 to 5%).

parameter b for the Barkas effect is determined experimentally. The apparently best values appear in Table IV. For protons with energies between 0.4 and 1 MeV an error of as much as 5% in the theory cannot be excluded on the basis of the experiments. It should be noted that the theory agrees with experiment to about $\pm 2\%$ for alpha particles in this velocity range.

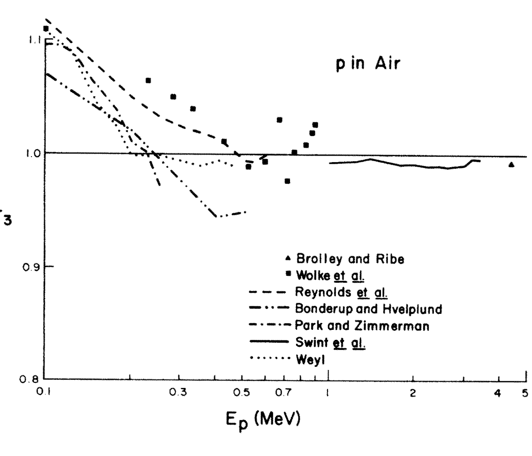

FIG. 13. Plots of the ratio r_3 of experimental to calculated stopping power (with Bethe-Bloch theory plus the Barkas-effect correction with $b = 1.8$) for protons in air. Data points represent measurements, with cited accuracy in parentheses, from Refs. 48 (2%) and 52 (information not given), whereas curves signify values derived by authors from fits to experimental data, with cited accuracy in parentheses, from Refs. 47 (1.7 to 3.4 %), 49 (4%), 50 (1.5 to 4%), 53 (2 to 5%), and 55 (1.5 to 3%).

TABLE IV. Approximate values of the projectile-z^3 parameter, b, which apparently provide best fits to the measurements.

Target	b
H_2	0.6
He	0.6
N_2	1.8
O_2	1.8
CH_4	1.8

Uncertainties in the experimental measurements frequently seem to be larger than estimated by the authors. This is especially disturbing for the measurements with protons above 0.6 MeV. Moreover, the presentation by authors of smooth functions rather than direct experimental data may introduce other systematic deviations.

Changes of the order of 2% in the I values would not change the conclusions reached here. The deviations between theory and experiment for small energies (below 0.2 MeV for protons, 1 MeV for alpha particles) do not show the same trend: for protons in H_2 and He, and for alpha particles in all gases, the calculated values S_t are too large, whereas for protons in N_2, O_2, and CH_4, S_t is too small. Accurate measurements for energies up to about 3 MeV for protons, and 4 MeV for alpha particles, are needed to demonstrate that the theory presented here is incomplete or incorrect. Improvements in shell correction calculations would be highly desirable and could readily be achieved with existing techniques, e.g., by using Hartree-Slater wave functions.[10]

Finally, a comparison can be made with the theory presented by Lindhard and Winther[56] and evaluated by Bonderup.[57] The stopping power for protons and alpha particles in nitrogen evaluated from this theory [based on calculations of the stopping function $L(y)$ by Bichsel and Laulainen[58]] differs by less than 1% from the values used in the present study, except near 0.7 MeV/amu where the difference amounts to 2%. Therefore, this theory[56–58] could also be used for the calculation of stopping-power values. Stopping-power values given by Andersen and Ziegler[15] for protons and α particles in H_2, He, N_2, and O_2 in general differ by

TABLE V. (a) Values of the stopping power S for protons in several gases as a function of particle energy E. (b) Values of the stopping power S for alpha particles in several gases as a function of particle energy E.

(a)

E (MeV)	S (MeV cm^2/g) H_2	He	N_2	O_2	CH_4
0.4	1372.56	547.50	408.03	381.88	661.04
0.5	1158.53	467.69	356.36	336.34	567.96
0.6	1007.40	410.75	317.53	301.20	500.02
0.7	894.24	367.51	287.08	273.21	448.42
0.8	805.96	333.44	262.84	250.56	407.55
0.9	734.94	305.83	242.88	231.92	374.26
1.0	676.43	282.89	226.12	216.16	346.47
1.2	585.41	246.81	199.52	190.99	302.82
1.4	517.60	219.61	179.08	171.70	269.90
1.6	464.94	198.29	162.93	156.40	244.04
1.8	422.76	181.07	149.81	143.85	223.12
2.0	388.13	166.84	138.86	133.42	205.80
2.2	359.16	154.87	129.56	124.56	191.22
2.4	334.52	144.64	121.57	116.95	178.74
2.6	313.29	135.80	114.60	110.32	167.94
2.8	294.80	128.06	108.46	104.48	158.48
3.0	278.53	121.23	103.02	99.29	150.13
3.2	264.09	115.15	98.15	94.65	142.68
3.4	251.19	109.71	93.78	90.47	136.00
3.6	239.59	104.80	89.82	86.68	129.97
3.8	229.09	100.35	86.21	83.22	124.50
4.0	219.54	96.29	82.91	80.07	119.50
5.0	182.31	80.39	69.85	67.57	99.91
6.0	156.53	69.31	60.60	58.72	86.21
7.0	137.54	61.09	53.69	52.08	76.04
8.0	122.93	54.75	48.31	46.90	68.18
9.0	111.32	49.69	43.99	42.75	61.91
10.0	101.86	45.56	40.44	39.32	56.77
11.0	93.99	42.11	37.47	36.45	52.49
12.0	87.33	39.19	34.95	34.01	48.85
13.0	81.62	36.67	32.77	31.90	45.73
14.0	76.67	34.49	30.86	30.06	43.01
15.0	72.33	32.57	29.19	28.45	40.63
16.0	68.50	30.88	27.71	27.01	38.51
17.0	65.08	29.37	26.38	25.73	36.63
18.0	62.02	28.01	25.19	24.57	34.94
19.0	59.25	26.78	24.11	23.53	33.41
20.0	56.75	25.67	23.13	22.57	32.02

less than 2% from corresponding data given in Table V. The stopping-power values given in the Northcliffe and Schilling tables[59] exceed the corresponding values shown in Table V by as much as 50% at low energies. The computer program utilized for the calculations described in the foregoing paper is available from the first author (H.B.).

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TABLE V. (*Continued*).

E (MeV)	(b) S (MeV cm^2/g) H_2	He	N_2	O_2	CH_4
1.0			2004.67	1827.20	3423.30
1.2			1856.89	1706.62	3084.55
1.4			1721.73	1598.51	2807.38
1.6			1602.47	1498.72	2579.35
1.8			1498.47	1408.89	2388.54
2.0	4587.28	1899.21	1407.14	1328.74	2226.25
2.5	3860.15	1605.75	1225.07	1163.99	1911.15
3.0	3350.44	1400.40	1088.57	1038.08	1683.22
3.5	2970.44	1247.59	983.51	939.51	1508.77
4.0	2674.77	1128.31	899.08	860.24	1369.89
4.5	2437.36	1032.15	830.00	794.65	1257.03
5.0	2242.04	952.70	771.57	739.54	1163.17
6.0	1938.60	828.57	678.84	651.87	1015.45
7.0	1712.87	735.56	608.30	584.51	903.89
8.0	1537.74	662.97	552.56	531.30	816.31
9.0	1397.54	604.57	507.14	487.99	745.59
10.0	1282.55	556.44	469.35	451.98	687.19
11.0	1186.38	516.04	437.33	421.48	638.01
12.0	1104.64	481.58	409.84	395.23	596.00
13.0	1034.26	451.82	385.93	372.39	559.62
14.0	972.95	425.82	364.97	352.31	527.78
15.0	919.04	402.90	346.37	334.50	499.65
16.0	871.22	382.53	329.79	318.60	474.61
17.0	828.51	364.29	314.87	304.28	452.19
18.0	790.09	347.86	301.36	291.36	431.96
19.0	755.35	332.97	289.06	279.60	413.62
20.0	723.76	319.41	277.81	268.84	396.90
21.0	694.91	307.00	267.48	258.97	381.58
22.0	668.45	295.60	257.96	249.86	367.51
23.0	644.08	285.10	249.17	241.44	354.53
24.0	621.56	275.37	241.02	233.61	342.52
25.0	600.69	266.35	233.44	226.33	331.36
26.0	581.28	257.95	226.36	219.52	320.96
27.0	563.18	250.11	219.74	213.15	311.26
28.0	546.26	242.77	213.53	207.18	302.17
29.0	530.41	235.89	207.69	201.56	293.65
30.0	515.53	229.42	202.20	196.27	285.63
31.0	501.52	223.33	197.02	191.29	278.08

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