Acoustic-surface-wave generation along a cylindrical plasma column surrounded by a compressible gas

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(Received 27 August 1980; revised manuscript received 8 June 1981)

Acoustic surface waves can be generated along the plasma column in pressure equilibrium with a gas blanket in the presence of the uniform axial magnetic field. Unlike the case of volume-acoustic-wave generation in the magnetoplasma reported recently, the threshold magnetic field required for the generation of acoustic surface waves increases with increasing gas pressure.

The spontaneous generation and amplification of acoustic waves in weakly ionized plasmas have been discussed by many investigators.[1] In a recent comment Ventrice[2] has reported the generation of acoustic waves in very-low-pressure plasmas in the presence of a uniform magnetic field. It was observed that when a uniform axial magnetic field was applied to the plasma column the acoustic-wave generation occurred at pressures below 0.1 Torr. The threshold value of B_0 that was necessary to induce oscillations increased with decreasing pressure.

In this paper we report that similar theoretical results have been obtained by us[3] in the case of acoustic surface wave generation at the interface between a plasma and a neutral gas while making a detailed study of Alfvén surface waves along a cylindrical plasma column. The acoustic surface waves are generated along a plasma column surrounded by a compressible gas and are subjected to an axial magnetic field only when the strength of the latter is such that $v_A > s$ or $B_0^2 > 4\pi\gamma P_{g0}(\rho_0/\rho_{g0})$. Here v_A is the Alfvén wave velocity and s is the acoustic wave velocity in the neutral gas.

In the magnetohydrodynamic approximation the linearized equations governing the hydrodynamic and electromagnetic properties of an incompressible conducting plasma fluid with an equilibrium mass density ρ_0 in an external magnetic field $\vec{B}_0$ are

$$\vec{\nabla}\cdot\vec{v}=0 , \tag{1}$$

$$\rho_0\frac{\partial\vec{v}}{\partial t}=-\nabla p+\frac{1}{4\pi}(\vec{\nabla}\times\vec{b})\times\vec{B}_0 , \tag{2}$$

$$\frac{\partial\vec{b}}{\partial t}=\vec{\nabla}\times(\vec{v}\times\vec{B}_0) , \tag{3}$$

$$\vec{\nabla}\cdot\vec{b}=0 , \tag{4}$$

where $\vec{v}$, p, and $\vec{b}$ are the perturbed fluid velocity, pressure, and magnetic field, respectively. Equations (1)–(4) can be combined to give

$$\nabla^2\left[\frac{B_0^2}{4\pi}\frac{\partial^2}{\partial z^2}-\frac{\rho_0\partial^2}{\partial t^2}\right]v_z=0 . \tag{5}$$

The linearized hydrodynamic equations for the nonconducting neutral gas are

$$\frac{\partial\rho_g}{\partial t}+\rho_{g0}\vec{\nabla}\cdot\vec{v}_g=0 , \tag{6}$$

$$\rho_{g0}\frac{\partial\vec{v}_g}{\partial t}=-\nabla p_g , \tag{7}$$

$$\frac{\partial p_g}{\partial t}=s^2\frac{\partial\rho_g}{\partial t} . \tag{8}$$

Equations (6)–(8) can be combined to give

$$\frac{\partial^2 p_g}{\partial t^2}-s^2\nabla^2 p_g=0 . \tag{9}$$

As there is no conduction current in the neutral gas and the displacement current is neglected, the perturbed magnetic field in the gas is given by $\vec{\nabla}\times\vec{b}_g=0$, which means $\vec{b}_g$ can be derived from a magnetic potential ψ, i.e.,

$$\vec{b}_g=\nabla\psi , \tag{10}$$

and since $\vec{\nabla}\cdot\vec{b}_g=0$, ψ obeys the equation

$$\nabla^2\psi=0 . \tag{11}$$

Equation (5), for the perturbations of the form $f(r,\varphi,z,t)=\exp i(kz+m\varphi-\omega t)$, is reduced to

$$\left[\frac{k^2B_0^2}{4\pi}-\rho_0\omega^2\right]\nabla^2 v_z=0 . \tag{12}$$

As

$$\frac{k^2B_0^2}{4\pi}\neq\rho_0\omega^2$$

(since we are not interested in bulk Alfvén waves), the solution of Eq. (12) is given as

$$v_z=AI_m(kr)\ ,\quad r<a\ . \tag{13}$$

Using Eq. (13) in Eqs. (1)–(4), the other components of velocity, magnetic field, and pressure in the plasma can be calculated.

Similarly, Eq. (9) can be reduced to

$$\left[\nabla^2+\frac{\omega^2}{s^2}\right]p_g=0\ , \tag{14}$$

which gives the solution for the perturbed pressure as

$$p_g=BK_m(\tau r)\ ,\quad r>a \tag{15}$$

where

$$\tau=k\left[1-\frac{\omega^2}{k^2s^2}\right]^{1/2}.$$

Solving the Laplace equation (11) we get

$$\psi=CK_m(kr)\ ,\quad r>a\ . \tag{16}$$

Equations (15) and (16) are used to calculate the components of the magnetic field and velocity in the gas. In the above equations $I_m(z)$ and $K_m(z)$ are the modified Bessel functions of order m, and A,B,C are the arbitrary constants to be determined by the following boundary conditions: (i) The tangential component of the electric field seen by the plasma just at the surface must vanish; (ii) total pressure is continuous across the boundary; and (iii) the normal velocity component is continuous. Applying these boundary conditions at $r=a$, we get the dispersion relation for the surface waves as

$$\frac{v_{\mathrm{ph}}^2}{v_A^2}\left[1+\eta\frac{h_m(ka)g_m(\tau a)}{(1-\alpha^2v_{\mathrm{ph}}^2/v_A^2)^{1/2}}\right]=F_m(ka)+1\ , \tag{17}$$

where

$$h_m(ka)=I'_m(ka)/I_m(ka)\ ,$$

$$g_m(\tau a)=-K_m(\tau a)/K'_m(\tau a)\ ,$$

$$F_m(ka)=-I'_m(ka)K_m(ka)/K'_m(ka)I_m(ka)\ ,$$

$$v_{\mathrm{ph}}=\omega/k\ ,\quad \alpha=v_A/s\ ,\quad \eta=\rho_{g0}/\rho_0\ ,$$

and the prime denotes the differentiation with respect to r.

Figure 1 gives the variation of phase velocity with α for various values of η and ka. The $v_{\mathrm{ph}}\approx s$ only for values of $\alpha>1$. As η is increased, i.e., ρ_{g0} increases or the neutral gas pressure increases, the value of α for which $v_{\mathrm{ph}}\approx s$ increases. Thus, the critical magnetic field B_0, for which the acoustic surface waves can be generated, increases with increasing gas pressure. For large ka, considering $\rho_0=10^{-7}$ gm/cm^3 and $s=1.5\times10^5$ cm/sec in the neutral gas, we find that when $\eta=0.1$, i.e., $\rho_{g0}=0.1\rho_0$ which gives the gas pressure $P_{g0}=0.105$ Torr, the critical value of the magnetic field B_0 above which surface sound waves are excited is given as $B_0=168$ gauss. When η is taken to be 0.5, $P_{g0}=0.53$ Torr, the value of B_0 is 269 gauss. Similarly for $\eta=0.8$, $P_{g0}=0.84$ Torr, $B_0=303$ gauss. Hence we note that unlike the case of volume waves, for sound-surface-wave generation, the threshold magnetic field increases with the increasing gas pressure.

The variation of v_{ph} with ka in Fig. 1 shows that depending on the value of η, the acoustic waves exist only above a critical wave number k_c. Thus, the acoustic surface waves associated with the Alfvén surface wave phenomenon lie in the wavelength region $0<\lambda<\lambda_c$, where λ_c is a function of the radial dimensions of the cylinder, magnetic field, and the ratio of the plasma to the neutral gas density.

In the case of volume-acoustic-wave generation,

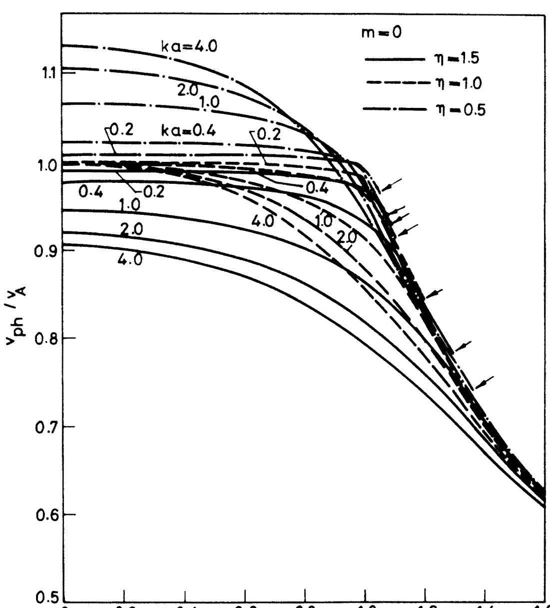

FIG. 1. Variation of v_{ph}/v_A with α for different values of ka and η. Arrows indicate the cutoff points.

the mechanism, even in the case of magnetoplasma,[2] seems to be the transfer of energy from the electron to the neutral gas.[4] In the case of acoustic-surface-wave generation though the plasma and neutral gas interaction takes place at the interface, the acoustic waves are generated by the transfer of energy from the Alfvén surface waves. The Alfvén surface waves can be excited for magnetic fields for which $v_A < s$, but as the magnetic field strength increases such that $v_A > s$ there is a transition from Alfvén-type to acoustic-type surface waves.

The plasma column in pressure equilibrium with a gas blanket and immersed in a magnetic field has been discussed in the literature[5] as a possible configuration for confining hot plasma by a cold gas blanket. This configuration provides a suitable experimental situation for verification of the results reported in this comment.

One of us (C.U.) would like to thank CONACYT for partial financial assistance and the Instituto de Ciencias, Universidad Autonoma de Puebla Pue., Mexico for kind hospitality during part of the preparation of this work.

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[1]For complete list of references see Ref. 2.
[2]C. A. Ventrice, Phys. Rev. A 20, 2625 (1979).
[3]C. Uberoi and K. Somasundaram, Plasma Physics (in press).
[4]U. Ingard, Phys. Rev. 145, 41 (1966).
[5]C. M. Braams, Phys. Rev. Lett. 17, 470 (1966) and the references therein.