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Comments

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Comment on "Field dependence of mobility in gases"

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The theory of electron transport in semiconductors has been used by Paranjape to discuss the drift velocities of electrons in gases, especially the onset of nonlinear dependence on field strength. The same problems are here discussed in terms of existing gas theory, and apply to ions as well as electrons. The effect of inelastic collisions is stated more explicitly, in terms of a ratio of thermally averaged cross sections.

The drift velocity (v_d) of electrons in gases eventually becomes nonlinear with increasing electric field strength (E), and the onset of this behavior can be correlated with the sound speed in the gas (c_0).[1] Such a correlation seems odd at first glance, since v_d refers to scattering by individual atoms, whereas c_0 is usually regarded as a collective parameter.[1,2] Paranjape[2] has discussed this effect in terms of an approximate theory based on the analogy with electron transport in semiconductors, the most important feature of which is the representation of the electron distribution function as a displaced Maxwellian with a temperature higher than that of the gas. We offer a few supplementary remarks from the viewpoint of gas theory, of which the main two are as follows:

(1) Although the analogy between electrons in gases and in semiconductors is interesting and remarkable in itself, such substantial progress has taken place in the theory of both electron and ion transport in gases in recent years[3–5] that the analogy is worth examining in reverse. For example, the work of Lin *et al.*[4] demonstrates that a Maxwellian is generally a poor approximation to the electron distribution function, especially when inelastic collisions occur. Accurate computation of electron transport coefficients requires representation of the distribution function by many terms in an expansion about a Maxwellian, and a correspondingly large number of moment equations.

(2) The approximation adopted by Paranjape, while too crude to be of great value in any serious calculation of transport coefficients, could be expected nevertheless to give insight into scaling laws involving the appearance of c_0. Such insight can also be obtained from an existing simple form of gas theory based on Wannier's classic paper on gaseous ion transport,[6] which is known as "free-flight" or "momentum-transfer" theory.[7–10] This theory appears as a first approximation in more elaborate schemes for solving the Boltzmann equation,[3,4] and also avoids another approximation that Paranjape made in his energy-balance equation.

We illustrate briefly with momentum-transfer theory how nonlinearity of v_d with respect to E is controlled by a parameter $v_d/(kT_0/M)^{1/2}$, where T_0 is the temperature and M the molecular mass of the neutral gas. Reference 11 provides perhaps the best background for most of the equations given below for the reader not familiar with the theory. For particles of charge q and mass m in a neutral gas, the momentum-balance equation for constant charged-particle—neutral-momentum-transfer collision frequency ν is[6–8]

$$qE = \mu v_d \nu \,, \tag{1}$$

where $\mu = mM/(m+M)$ is the reduced mass. Like Paranjape, we ignore inelastic collisions (for the moment), but we put no restriction on m. Equation (1) becomes the same as Eq. (5) of Paranjape for $m << M$. The corresponding energy-balance equation can be written as[6,8,11]

$$qEv_d = \tfrac{1}{2}m(\langle v^2\rangle - \langle v'^2\rangle)\nu = \frac{\mu}{m+M}(m\langle v^2\rangle - M\langle V^2\rangle)\nu , \qquad (2)$$

which equates the power gained from the field to the energy-loss rate by collisions, v' being the charged-particle speed after a collision. The second equality in (2), in which V is the neutral-particle speed, follows from an elementary analysis of charged particle-neutral collisions in the center-of-mass frame.[11] Elimination of qE/ν between Eqs. (1) and (2), and substitution of $M\langle V^2\rangle = 3kT_0$, yields an expression for the mean charged-particle energy,

$$\tfrac{1}{2}m\langle v^2\rangle = \tfrac{3}{2}kT_0 + \tfrac{1}{2}mv_d^2 + \tfrac{1}{2}Mv_d^2 . \qquad (3)$$

The three terms on the right have simple physical meanings: The first is the thermal energy, the second is the drift energy obtained from the field, and the third is that part of the energy that has been obtained from the field but randomized by collisions. Hence, the mean energy in the center-of-mass frame is

$$\epsilon = \tfrac{1}{2}\mu\langle(\vec{v}-\vec{V})^2\rangle = \tfrac{1}{2}\mu(\langle v^2\rangle + \langle V^2\rangle) = \tfrac{3}{2}kT_0 + \tfrac{1}{2}Mv_d^2 , \qquad (4)$$

where we have assumed $\langle\vec{V}\rangle = 0$. Momentum-transfer theory consists of adopting these equations for a general energy-dependent collision frequency $\nu(\epsilon)$. Skullerud[12] has tested Eqs. (1)–(4) for several cases where $\nu = \nu(\epsilon)$, and found that they are accurate to within about 10% under most circumstances. We therefore feel confident in drawing conclusions of a semiquantitative nature (specifically, scaling laws) based upon these equations adapted to the general case.

Clearly, (1) and (4) furnish an implicit relation for v_d that cannot in general be solved unless the explicit energy-dependence of $\nu(\epsilon)$ is known. The criterion for nonlinearity, however, is readily extracted. When $Mv_d^2 < 3kT_0$, that is, when

$$\eta \equiv v_d/(3kT_0/M)^{1/2} < 1 , \qquad (5)$$

then ν depends only on T_0 and $v_d \propto E$. In the opposite extreme of $\eta > 1$, v_d will generally be nonlinear in E. For example, for the case of constant cross section (rigid spheres), $\nu \propto \epsilon^{1/2} \propto v_d$, and hence $v_d \propto E^{1/2}$. Thus scaling occurs according to η, and it is no surprise that nonlinearity sets in when $\eta \simeq 1$. This argument is independent of the mass of the charged particles and is valid for either ions or electrons. This similarity between ions and electrons was earlier noted empirically by Huang and Freeman.[13]

The essential feature of the above argument that makes the scaling so transparent is the simplicity of the random field-energy term, $(1/2)Mv_d^2$. This simplicity is remarkable, and was missed in Paranjape's energy balance arguments, but in gases it follows from simple physical considerations.[11]

The choice[1,2] of the nonlinear scaling parameter to be $\eta = v_d/c_0$ is virtually the same as Eq. (5), since $c_0 = (\gamma kT_0/M)^{1/2}$. This choice, of course, does not imply any collective nature for η, since single-collision scattering is clearly the determining factor in Eq. (5). Any mystery about collective behavior in fact resides only in c_0, since it is not so clear (although it is familiar) how sound propagation arises in a gas that is controlled by binary collisions among randomly moving molecules.[14]

Similarities between electron transport in semiconductors and in gases have been noted before,[15] and even theories at a more sophisticated level[16,17] bear a striking resemblance. This is as it should be, since the fundamental assumption in both cases is that the electron distribution can be described by the Boltzmann equation.

Finally, inelastic collisions cause the charged particles to lose energy faster, suggesting a larger value for η before nonlinearity occurs. Paranjape noted that this effect in semiconductors arises through the introduction of optical-mode scattering in addition to acoustic-mode scattering. For gases, the effect of inelastic collisions with both electrons[4] and ions[18] is to replace Eq. (4) by

$$\epsilon = (\tfrac{3}{2}kT_0 + \tfrac{1}{2}Mv_d^2)[1 + (M/m)\xi(\epsilon)]^{-1} , \qquad (6)$$

where $\xi(\epsilon)$ is a dimensionless ratio characterizing fractional energy loss due to inelastic collisions, which can be expressed as the ratio of a thermally averaged cross section for inelastic energy loss to one for momentum transfer, and which vanishes for $\eta << 1$. The form of Eq. (1) remains the same, however, although inelastic collisions can change the dependence of ν on ϵ. The condition corresponding to $\eta \gtrsim 1$ for elastic collisions is $\epsilon \gtrsim 2[(3/2)kT_0]$, which with (6) translates into

$$\eta \gtrsim [1 + 2(M/m)\xi]^{1/2} . \qquad (7)$$

Thus nonlinearity occurs for a larger value of η (and v_d) when inelastic collisions are present, in agreement with Paranjape. The effect is probably small for ions, but may be large for electrons because of the mass factor M/m.[4,18]

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