Nonlinear interaction of a Gaussian electromagnetic beam with an electrostatic upper hybrid wave: Generation of electrostatic lower hybrid wave

M. S. Sodha, P. K. Malhotra, and R. P. Sharma
Centre of Energy Studies, Indian Institute of Technology, Hauz Khas, New Delhi-110 016, India
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This paper presents an investigation of the nonlinear interaction of a Gaussian electromagnetic (EM) beam with an electrostatic upper hybrid wave in a collisionless magnetoplasma. The EM beam is assumed to be propagating in the ordinary mode, having a nonuniform intensity distribution (along its wave front) in a plane transverse to the direction of propagation. Because of the Gaussian intensity distribution of the EM beam, a time-independent component of the ponderomotive force becomes finite and this leads to a coupling between the pump EM beam and the electrostatic upper hybrid wave. This coupling is so strong that the weak electrostatic upper hybrid wave gets excited. The excited electrostatic upper hybrid wave may again interact with the pump EM beam and lead to the generation of an electrostatic lower hybrid wave of significant power.

I. INTRODUCTION

When a high-power Gaussian electromagnetic (EM) beam interacts with a collisionless plasma, a time-independent component of the ponderomotive force becomes finite which leads to the modification in the background electron-ion density.[1] As a result of this, an additional coupling is introduced between the pump Gaussian EM beam and the electrostatic modes of plasma. This coupling is so strong that the electrostatic mode can be excited and under appropriate conditions even focusing of the electrostatic mode can take place.[2] The excited mode can again interact with the pump to give rise to the phenomena of enhanced Raman or Brillouin scattering.[3,4] The second possibility is that the excited electrostatic wave may again interact with the pump wave and generate another electrostatic wave.

The motivation behind the present paper is to study the second possibility, viz., the generation of the electrostatic wave. We have taken the specific case of the interaction of a high-power Gaussian EM beam with the electrostatic upper hybrid wave in a collisionless magnetoplasma. The EM beam is assumed to be propagating perpendicular to the static magnetic field having its electric vector polarized along the static magnetic field (ordinary mode). The excited electrostatic upper hybrid wave interacting with the pump wave is found to generate an electrostatic lower hybrid wave of significant power. The intensity of the generated electrostatic lower hybrid wave is maximum for an optimum intensity of the pump wave.

In Sec. II we have studied the excitation of the electrostatic upper hybrid wave by the Gaussian EM beam. In Sec. III generation of the electrostatic lower hybrid wave has been studied, using the fluid model of plasma. Section IV presents the discussion and important conclusion of the present investigation.

II. EXCITATION OF THE ELECTROSTATIC UPPER HYBRID WAVE

We consider the propagation of a Gaussian EM beam in a collisionless magnetoplasma. The z axis is defined along the static magnetic field ($\vec{B}_0=\hat{z}B_0$). The EM beam is assumed to be propagating along the x axis in ordinary mode, viz., having its electric vector polarized along the static magnetic field. The intensity distribution of the beam at $x=0$, is given by

$$\vec{E}_0=\hat{z}E_0 ,$$
$$E_0E_0^* \mid_{x=0}=E_{00}^2\exp(-r^2/r_0^2) , \qquad (1)$$

where r $[=(y^2+z^2)^{1/2}]$ is the radial coordinate of the cylindrical coordinate system and r_0 is the initial beam width.

Because of the nonuniform intensity distribution of the EM beam in the plane transverse to the direction of propagation, the time-independent component of the ponderomotive force becomes finite. This leads to the modification in the background electron and ion densities. The modified electron-ion density (N_{0e}/N_{0i}) is given by[1]

$$N_{0e} \approx N_{0i} = N_0 \exp\left[-\tfrac{3}{4}\alpha\frac{m_e}{m_i}\vec{E}_0\cdot\vec{E}_0^*\right]. \quad (2)$$

It must be mentioned here that the electrostatic upper hybrid wave is weak and hence it's contribution to the dc component of ponderomotive force is negligible in comparison to the pump Gaussian EM beam.

Following the above mentioned reference, the electric vector $\vec{E}_0$ of the EM beam at finite x, can be written as

$$\vec{E}_0 = \hat{z}\frac{E_{00}}{f_0(x)}\exp\left[\frac{-r^2}{2r_0^2 f_0^2(x)}\right]\exp[-ik_0(x+s_0)],$$

$$s_0 = \frac{r^2}{2}\beta_0(x) + \Phi_0(x), \quad \beta_0 = \frac{1}{f_0(x)}\frac{df_0(x)}{dx},$$

$$k_0 = \frac{\omega_0}{c}\epsilon_0^{1/2}, \quad \epsilon_0 = 1 - \frac{\omega_{pe}^2}{\omega_0^2},$$

$$\omega_{pe}^2 = \frac{4\pi N_0 e^2}{m_e},$$

and f_0 is governed by

$$\frac{d^2 f_0}{dx^2} = \frac{1}{k_0^2 r_0^4 f_0^3} - \left[\tfrac{3}{4}\alpha\frac{m_e}{m_i}E_{00}^2\right] \times \frac{\omega_{pe}^2 \exp\left[-\tfrac{3}{4}\alpha\frac{m_e}{m_i}\frac{E_{00}^2}{f_0^2}\right]}{\omega_0^2 f_0^3 \epsilon_0 r_0^2}. \quad (3)$$

Here

$$\alpha = e^2 m_i / 3k_B T_e(1+T_i/T_e)m_e^2\omega_0^2$$

is the nonlinearity parameter, m_e and m_i are the electron and ion masses, respectively, k_B is the Boltzmann constant, T_e and T_i are the electron and ion temperatures, respectively, N_0 is the electron-ion density in the absence of the EM beam, e is the charge of the electron, c is the speed of light in vacuum, and ω_0 is the frequency of the incident EM beam.

This EM beam may excite the natural modes of vibration of magnetoplasma, viz., the electrostatic upper hybrid wave, thus leading to the enhancement in the amplitude of the electrostatic upper hybrid wave. Here we study the excitation of an electrostatic upper hybrid wave using the following:

(i) The equation of motion,

$$m_{e,i}\left[\frac{\partial\vec{V}_{e,i}}{\partial t} + (\vec{V}_{e,i}\cdot\vec{\nabla})\vec{V}_{e,i}\right] = q_{e,i}\vec{E} + \frac{q_{e,i}}{c}\vec{V}_{e,i}\times(\vec{B}+\vec{B}_0) - \gamma_{e,i}k_B T_{e,i}\frac{\vec{\nabla}N_{e,i}}{N_{e,i}}. \quad (4)$$

(ii) The equation of continuity,

$$\frac{\partial N_{e,i}}{\partial t} + \vec{\nabla}\cdot(N_{e,i}\vec{V}_{e,i}) = 0 \quad (5)$$

and the wave equation,

$$\nabla^2\vec{E} - \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) = \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} + \frac{4\pi}{c^2}\frac{\partial\vec{J}}{\partial t}, \quad (6)$$

where

$$\vec{J} = q_e N_e \vec{V}_e + q_i N_i \vec{V}_i$$

is the current density.

In the above equations subscripts e and i are for the electron and ion, respectively. Thus, $\vec{V}_{e,i}$ are the total electron and ion velocities, $N_{e,i}$ are the total electron and ion densities in the plasma, $q_{e,i}$ are the electron and ion charges ($q_e = -e$; $q_i = +e$), respectively. $\vec{B}$ is the total time-varying magnetic field in the plasma and $\vec{E}$ is the total electric field in the plasma. $\gamma_{e,i}$ are the ratio of specific heats of the electron and ion gases, respectively.

Near upper hybrid frequency the ion motion is negligible in comparison to the electron motion. To find the electric field $\vec{E}_1$ $(=\hat{x}E_1)$ associated with the electrostatic upper hybrid wave, we expand $\vec{V}_e$, N_e, and $\vec{E}$ as

$$\vec{V}_e = \vec{V}_{0e}(\omega_0,k_0) + \vec{v}_{1e}(\omega_1,k_1),$$
$$N_e = N_{0e} + n_{1e}(\omega_1,k_1), \quad (7)$$
$$\vec{E} = \vec{E}_0(\omega_0,k_0) + \vec{E}_1(\omega_1,k_1),$$

where N_{0e} is the electron density in the presence of the EM beam given by Eq. (2), $\vec{v}_{1e}$ and n_{1e} are the perturbations in the electron velocity and density, due to the electrostatic upper hybrid wave, varying at frequency ω_1. $\vec{V}_{0e}$ is the electron velocity in the presence of the EM beam only.

Equations (4), (5), and (7) give the x and y components of $\vec{v}_{1e}$ as

$$v_{1ex}=(\omega_{ce}^2-\omega_1^2)^{-1}\left[-\frac{i\omega_1 e}{m_e}E_1-\frac{i\omega e v_{\rm th}^2}{m_e(\omega_{ce}^2-\omega_1^2)}\frac{\partial^2 E_1}{\partial x^2}\right] \qquad (8a)$$

and

$$v_{1ey}=(\omega_{ce}^2-\omega_1^2)^{-1}\left[-\frac{e\omega_{ce}}{m_e}E_1-\frac{e\omega_{ce} v_{\rm th}^2}{m_e(\omega_{ce}^2-\omega_1^2)}\frac{\partial^2 E_1}{\partial x^2}\right] . \qquad (8b)$$

Here $\omega_{ce}=eB_0/m_e c$ is the electron cyclotron frequency and

$$v_{\rm th}=(\gamma_e k_B T_e/m_e)^{1/2}$$

is the electron thermal speed.

Equations (6)–(8b) give the following equation satisfied by the electric field E_1 of the excited electrostatic upper hybrid wave:

$$\frac{\omega_1^2\omega_{pe}^2 v_{\rm th}^2}{c^2(\omega_{ce}^2-\omega_1^2)^2}\frac{\partial^2 E_1}{\partial x^2}+\frac{\partial^2 E_1}{\partial y^2}+\frac{\partial^2 E_1}{\partial z^2}+\frac{\omega_1^2}{c^2}E_1+\frac{\omega_1^2\omega_{pe}^2}{c^2(\omega_{ce}^2-\omega_1^2)}\frac{N_{0e}}{N_0}E_1=0 . \qquad (9)$$

In the absence of EM beam, Eq. (9) gives the dispersion relation

$$\omega_h^2-\omega_1^2=\frac{\omega_{pe}^2 v_{\rm th}^2 k_1^2}{(\omega_{ce}^2-\omega_1^2)} . \qquad (10)$$

In the absence of thermal motion, the above equation yields the usual upper hybrid oscillations at frequency

$$\omega_1=(\omega_h)^{1/2} .$$

Here $\omega_h=(\omega_{pe}^2+\omega_{ce}^2)^{1/2}$ is the upper hybrid frequency.

We assume the solution of Eq. (9) is of the form[5]

$$E_1=E_{10}(x,y,z)\exp[i(\omega_1 t-k_1 x)] . \qquad (11)$$

Substituting for E_1 from Eq. (11) in Eq. (9) we obtain

$$-2ik_1 A^0\frac{\partial E_{10}}{\partial x}+\frac{\partial^2 E_{10}}{\partial y^2}+\frac{\partial^2 E_{10}}{\partial z^2}+B^0 E_{10}=0 , \qquad (12)$$

where

$$A^0=\frac{\omega_1^2\omega_{pe}^2 v_{\rm th}^2}{c^2(\omega_{ce}^2-\omega_1^2)^2}$$

and

$$B^0=\frac{\omega_1^2}{c^2}+\frac{\omega_1^2\omega_{pe}^2}{c^2(\omega_{ce}^2-\omega_1^2)}\frac{N_{0e}}{N_0} .$$

Substituting further for E_{10} in Eq. (12) as[5]

$$E_{10}=E_{100}(x,y,z)\exp[-ik_1 s_1(x,y,z)] , \qquad (13)$$

we obtain the following equations after separating real and imaginary parts:

$$2\frac{\partial s_1}{\partial x'}+\left[\frac{\partial s_1}{\partial y}\right]^2+\left[\frac{\partial s_1}{\partial z}\right]^2=\frac{1}{k_1^2 E_{100}}\left[\frac{\partial^2 E_{100}}{\partial y^2}+\frac{\partial^2 E_{100}}{\partial z^2}\right]+\frac{B^0}{k_1^2} \qquad (14)$$

and

$$\frac{\partial E_{100}^2}{\partial x'}+\left[\frac{\partial s_1}{\partial y}\right]\frac{\partial}{\partial y}E_{100}^2+\left[\frac{\partial s_1}{\partial z}\right]\frac{\partial}{\partial z}E_{100}^2+\left[\frac{\partial^2 s_1}{\partial y^2}+\frac{\partial^2 s_1}{\partial z^2}\right]E_{100}^2=0 , \qquad (15)$$

where $x'=x/A^0$.

The solution of Eqs. (14) and (15) in paraxial ray approximation is given by[5]

$$E_{100}=\frac{B_0'}{f_1(x')}\exp\left[-\frac{r^2}{2a_0^2 f_1^2}\right] ,$$

$$s_1=\frac{r^2}{2}\beta_1(x')+\Phi_1(x') , \qquad (16)$$

$$\beta_1(x')=\frac{1}{f_1(x')}\frac{df_1(x')}{dx'} .$$

While solving Eqs. (14) and (15) the initial radial variation of E_{100} has been assumed to be

$$E_{100}\,|_{x=0}=B_0'\exp(-r^2/2a_0^2) ,$$

where a_0 is the half-width of Gaussian distribution.

Using Eqs. (14) and (16) we obtain the following equation for the beam-width parameter f_1 of the excited electrostatic upper hybrid wave:

$$\frac{d^2 f_1}{dx'^2} = \frac{1}{a_0^4 k_1^2 f_1^3} + \frac{f_1 \omega_1^2 \omega_{pe}^2}{k_1^2 c^2(\omega_{ce}^2 - \omega_1^2)} \left[\frac{3}{4}\alpha \frac{m_e}{m_i} \frac{E_{00}^2}{r_0^2 f_0^4} \right] \times \exp\left[-\frac{3}{4}\alpha \frac{m_e}{m_i} \frac{E_{00}^2}{f_0^2} \right] . \tag{17a}$$

From Eq. (17a) it is obvious that f_1 depends on f_0, hence, an analytical solution for f_1 cannot, in general, be obtained. However, in the self-trapping mode ($f_0 = 1$) the following analytical solution of Eq. (17a) can be obtained:

$$f_1^2 = \frac{a'+b'}{2b'} - \frac{a'-b'}{2b'} \cos(2\sqrt{b'}x) , \tag{17b}$$

where

$$a' = 1/k_1^2 a_0^4 (A^0)^2 ,$$

and

$$b' = \frac{\omega_1^2 \omega_{pe}^2}{k_1^2 r_0^2 c^2 (A^0)^2 (\omega_1^2 - \omega_{ce}^2)} \left[\frac{3}{4}\alpha \frac{m_e}{m_i} E_{00}^2 \right] \times \exp\left[-\frac{3}{4}\alpha \frac{m_e}{m_i} E_{00}^2 \right] .$$

III. GENERATION OF THE ELECTROSTATIC LOWER HYBRID WAVE

We consider the electrostatic upper hybrid wave described in the previous section, propagating coaxially along with the EM beam. We use Eqs. (4)–(6) to study the generation of the electrostatic lower hybrid wave, by the interaction between the EM beam and the excited electrostatic upper hybrid wave at frequency $\omega = \omega_0 - \omega_1$. To find the electric field $\vec{\mathscr{E}}(=\hat{x}\mathscr{E})$ associated with the generated electrostatic lower hybrid wave, we expand $\vec{V}_{e,i}$, $N_{e,i}$, and $\vec{E}$ as

$$\vec{V}_{e,i} = \vec{V}_{0e,0i}(\omega_0, k_0) + \vec{v}_{1e,1i}(\omega_1, k_1) + \vec{v}_{e,i}(\omega, k) ,$$
$$N_{e,i} = N_{0e,0i} + n_{1e,1i}(\omega_1, k_1) + n_{e,i}(\omega, k) , \tag{18}$$
$$\vec{E} = \vec{E}_0(\omega_0, k_0) + \vec{E}_1(\omega_1, k_1) + \vec{\mathscr{E}}(\omega, k) ,$$

where $N_{0e,0i}$ are the electron and ion densities in the plasma in the presence of the EM beam only, given by Eq. (2). $\vec{v}_{1e,1i}$ and $n_{1e,1i}$ are the perturbations in the electron and ion velocities and densities due to the excited electrostatic upper hybrid wave, varying at frequency ω_1. $\vec{v}_{e,i}$ and $n_{e,i}$ are the perturbations, in the electron and ion velocities and densities due to the generated electrostatic lower hybrid wave, varying at the difference frequency $\omega = \omega_0 - \omega_1$. $\vec{V}_{0e,0i}$ are the electron and ion velocities in the presence of the EM beam only.

Equations (4)–(6) and (18) give the following equation satisfied by the electric field $\mathscr{E}$ of the generated electrostatic lower hybrid wave:

$$\frac{\partial^2 \mathscr{E}}{\partial y^2} + \frac{\partial^2 \mathscr{E}}{\partial z^2} + \left[\frac{\omega_{pe}^2 \omega^2 v_{\text{th}}^2}{c^2 \omega_{ce}^4} + \frac{\omega_{pi}^2 V_{\text{th}}^2}{c^2 \omega^2} \right] \frac{\partial^2 \mathscr{E}}{\partial x^2} + \left[\frac{\omega^2}{c^2} + \frac{\omega_{pe}^2 \omega^2}{c^2 \omega_{ce}^2} \frac{N_{0e}}{N_0} - \frac{\omega_{pi}^2}{c^2} \frac{N_{0i}}{N_0} \right] \mathscr{E}$$
$$= -\frac{\omega_{pe}^2 \omega^2 m_e}{2ec^2 \omega_{ce}^2} \frac{N_{0e}}{N_0} \left[V_{0e} \frac{\partial}{\partial z} v_{1ex}^* \right] - \frac{\omega_{pi}^2 i\omega m_e}{2ec^2 \omega_{ce}^2} \frac{N_{0e}}{N_0} \left[V_{0e} \frac{\partial}{\partial z} v_{1ey}^* \right] - \frac{i\omega \omega_{pe}^2 v_{\text{th}}^2}{2ec^2 \omega_{ce}^2} \frac{1}{N_0} \frac{\partial}{\partial x} \left[\frac{\partial V_{0e}}{\partial z} n_{1e}^* \right]$$
$$- \frac{\omega^2 \omega_{pe}^2 v_{\text{th}}^2 m_e}{2ec^2 \omega_{ce}^3} \frac{N_{0e}}{N_0} \frac{\partial^2}{\partial x^2} \left[V_{0e} \frac{\partial}{\partial z} v_{1ex}^* \right] - \frac{i\omega \omega_{pe}^2 v_{\text{th}}^2 m_e}{2ec^2 \omega_{ce}^3} \frac{N_{0e}}{N_0} \frac{\partial^2}{\partial x^2} \left[V_{0e} \frac{\partial}{\partial z} v_{1ey}^* \right]$$
$$- \frac{\omega_{pi}^2 m_i}{2ec^2} \frac{N_{0i}}{N_0} \left[V_{0i} \frac{\partial}{\partial z} v_{1ix}^* \right] + \frac{i\omega_{ci} \omega_{pi}^2 m_i}{2ec^2 \omega} \frac{N_{0i}}{N_0} \left[V_{0i} \frac{\partial}{\partial z} v_{1iy}^* \right] - \frac{i\omega_{pi}^2 V_{\text{th}}^2 m_i}{2ec^2 \omega} \frac{1}{N_0} \frac{\partial}{\partial x} \left[\frac{\partial V_{0i}}{\partial z} n_{1i}^* \right]$$
$$+ \frac{\omega_{pi}^2 V_{\text{th}}^2 m_i}{2ec^2 \omega^2} \frac{N_{0i}}{N_0} \frac{\partial^2}{\partial x^2} \left[V_{0i} \frac{\partial}{\partial z} v_{1ix}^* \right] - \frac{i\omega_{ci} \omega_{pi}^2 V_{\text{th}}^2 m_i}{2ec^2 \omega^3} \frac{N_{0i}}{N_0} \frac{\partial^2}{\partial x^2} \left[V_{0i} \frac{\partial}{\partial z} v_{1iy}^* \right] , \tag{19}$$

where $\omega_{pi}=(4\pi N_0 e^2 / m_i)^{1/2}$ is the ion plasma frequency, $\omega_{ci}=eB_0 / m_i c$ is the ion cyclotron frequency, $V_{\rm th}=(\gamma_i k_B T_i / m_i)^{1/2}$ is the ion thermal speed, and v_{1ix}, v_{1iy} are the x and y components, respectively, of $\vec{v}_{1i}$. In writing Eq. (19) we have used the lower hybrid condition $\omega_{ci} << \omega << \omega_{ce}$.

In the absence of the EM beam and the electrostatic upper hybrid wave Eq. (20) gives the dispersion relation

$$k^2\left[\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{v_{\rm th}^2}{\omega_{ce}^2}+\frac{\omega_{pi}^2}{\omega^2}\frac{V_{\rm th}^2}{\omega^2}\right]=1+\frac{\omega_{pe}^2}{\omega_{ce}^2}-\frac{\omega_{pi}^2}{\omega^2}\ . \tag{20}$$

In the absence of thermal motion the above equation yields the usual lower hybrid oscillations at frequency

$$\omega_{\rm lh}=\frac{\omega_{pi}}{(1+\omega_{pe}^2/\omega_{ce}^2)^{1/2}}\ . \tag{21}$$

The solution of Eq. (19) may be written in the form[3]

$$\mathscr{E}=\mathscr{E}_1(x,y,z)\exp(-ikx)+\mathscr{E}_2(x,y,z)\exp[-i(\Delta k)x]\ , \tag{22}$$

where $\mathscr{E}_1$ and $\mathscr{E}_2$ are slowly varying complex functions of space.

Substituting for $\mathscr{E}$ from Eq. (22) in Eq. (19) and equating the coefficients of $\exp(-ikx)$ and $\exp[-i(\Delta k)x]$ on both sides of the resulting equation, we obtain

$$-k^2A'\mathscr{E}_1-2ikA'\frac{\partial\mathscr{E}_1}{\partial x}+\frac{\partial^2\mathscr{E}_1}{\partial y^2}+\frac{\partial^2\mathscr{E}_1}{\partial z^2}+B'\mathscr{E}_1=0 \tag{23}$$

and

$$-(\Delta k)^2A'\mathscr{E}_2-2i(\Delta k)A'\frac{\partial\mathscr{E}_2}{\partial x}+\frac{\partial^2\mathscr{E}_2}{\partial y^2}+\frac{\partial^2\mathscr{E}_2}{\partial z^2}+B'\mathscr{E}_2=z\frac{E_{00}B_0'}{f_0f_1}\frac{N}{N_0}\frac{\omega e}{2c^2\omega_0}D\exp\left[-r^2\left[\frac{1}{2r_0^2f_0^2}+\frac{1}{2a_0^2f_1^2}\right]\right]\times\exp[-i(k_0s_0-k_1s_1)]\ , \tag{24a}$$

where

$$D=\frac{\omega_{pe}^2}{m_e(\omega_{ce}^2-\omega_1^2)}\left[\frac{1}{a_0^2f_1^2}\left[1-\frac{k_1^2v_{\rm th}^2}{(\omega_{ce}^2-\omega_1^2)}\right]\left[\frac{\omega\omega_1}{\omega_{ce}^2}+1-\frac{\omega\omega_1(\Delta k)^2v_{\rm th}^2}{\omega_{ce}^4}-\frac{(\Delta k)^2v_{\rm th}^2}{\omega_{ce}^2}\right]+\frac{k_1(\Delta k)v_{\rm th}^2}{\omega_{ce}^2}\frac{1}{r_0^2f_0^2}\right]$$
$$+\frac{\omega_{pi}^2}{m_i(\omega_{ci}^2-\omega_1^2)}\left[\frac{1}{a_0^2f_1^2}\left[1-\frac{k_1^2V_{\rm th}^2}{(\omega_{ci}^2-\omega_1^2)}\right]\left[\frac{\omega_1}{\omega}+\frac{\omega_{ci}^2}{\omega^2}+\frac{\omega_1(\Delta k)^2V_{\rm th}^2}{\omega^3}+\frac{\omega_{ci}^2(\Delta k)^2V_{\rm th}^2}{\omega^4}\right]+\frac{k_1(\Delta k)V_{\rm th}^2}{\omega^2}\frac{1}{r_0^2f_0^2}\right]\ , \tag{24b}$$

where

$$A'=\frac{\omega_{pe}^2\omega^2v_{\rm th}^2}{c^2\omega_{ce}^4}+\frac{\omega_{pi}^2V_{\rm th}^2}{c^2\omega^2}\ ,$$

$$B'=\frac{\omega^2}{c^2}+\frac{\omega_{pe}^2\omega^2}{c^2\omega_{ce}^2}\frac{N}{N_0}-\frac{\omega_{pi}^2}{c^2}\frac{N}{N_0}\ ,$$

and $\Delta k=k_0-k_1$ and $N=N_{0e}\approx N_{0i}$ (quasineutrality condition). To solve Eqs. (23) and (24a) we use the eikonal approximation.[5] Thus,

$$\mathscr{E}_1=\mathscr{E}_{10}(x,y,z)\exp[-iks(x,y,z)] \tag{25}$$

and

$$\mathscr{E}_2=\mathscr{E}_{20}(x,y,z)\exp\{-i[k_0s_0(x,y,z)-k_1s_1(x,y,z)]\}\ . \tag{26}$$

Substituting Eq. (25) in Eq. (23) we obtain the following equations, after separating real and imaginary parts:

$$2\frac{\partial s}{\partial x''}+\left[\frac{\partial s}{\partial y}\right]^2+\left[\frac{\partial s}{\partial z}\right]^2 =\frac{1}{k^2\mathscr{E}_{10}}\left[\frac{\partial^2\mathscr{E}_{10}}{\partial y^2}+\frac{\partial^2\mathscr{E}_{10}}{\partial z^2}\right]+\frac{B'}{k^2}-A' . \quad (27)$$

and

$$\frac{\partial\mathscr{E}_{10}^2}{\partial x''}+\left[\frac{\partial s}{\partial y}\right]\frac{\partial\mathscr{E}_{10}^2}{\partial y}+\left[\frac{\partial s}{\partial z}\right]\frac{\partial\mathscr{E}_{10}^2}{\partial z} +\left[\frac{\partial^2 s}{\partial y^2}+\frac{\partial^2 s}{\partial z^2}\right]\mathscr{E}_{10}^2=0 . \quad (28)$$

Here $x''=x/A'$. Solution of Eqs. (27) and (28) in paraxial ray approximation is given by[5]

$$\mathscr{E}_{10}=\frac{B_0''}{f(x'')}\exp\left[\frac{-r^2}{2b_0^2f^2}\right] ,$$

$$s=\frac{r^2}{2}\beta(x'')+\Phi(x'') , \quad (29)$$

$$\beta(x'')=\frac{1}{f(x'')}\frac{df(x'')}{dx''} .$$

Using Eqs. (27) and (29) we obtain the following equation for the dimensionless beam-width parameter f of the generated lower hybrid wave:

$$\frac{d^2f}{dx''^2}=\frac{1}{b_0^4k^2f^3}-\frac{\omega^2 f}{c^2k^2}\left[\frac{\omega_{pi}^2}{\omega^2}-\frac{\omega_{pe}^2}{\omega_{ce}^2}\right] \times\left[\tfrac{3}{4}\alpha\frac{m_e}{m_i}\frac{E_{00}^2}{r_0^2f_0^4}\right]\exp\left[-\tfrac{3}{4}\alpha\frac{m_e}{m_i}\frac{E_{00}^2}{f_0^2}\right] . \quad (30a)$$

From Eqs. (30a), it is obvious that f depends on f_0, hence, an analytical solution for f cannot, in general, be obtained. However, in the self-trapping mode ($f_0=1$) the following analytical solution of Eq. (30a) can be obtained:

$$f^2=-\frac{(a''-b'')}{2b''}+\frac{(a''+b'')}{2b''}\cosh(2\sqrt{b''}x) , \quad (30b)$$

where

$$a''=1/k^2b_0^4A'^2$$

and

$$b''=\frac{\omega^2}{c^2k^2A'^2r_0^2}\left[\frac{\omega_{pe}^2}{\omega_{ce}^2}-\frac{\omega_{pi}^2}{\omega^2}\right]\left[\tfrac{3}{4}\alpha\frac{m_e}{m_i}E_{00}^2\right] \times\exp\left[-\tfrac{3}{4}\alpha\frac{m_e}{m_i}E_{00}^2\right] .$$

B_0'' and b_0 in Eq. (29) are the unknowns to be evaluated later by using the boundary condition.

Substituting for $\mathscr{E}_2$ from Eq. (26) in Eq. (24a) we obtain

$$\mathscr{E}_{20}\simeq F_2\exp\left[-r^2\left[\frac{1}{2r_0^2f_0^2}+\frac{1}{2a_0^2f_1^2}\right]\right] , \quad (31)$$

where

$$F_2=\frac{z}{D_r}\frac{E_{00}B_0'}{f_0f_1}\frac{N}{N_0}\frac{\omega e}{2c^2\omega_0}D \quad (32)$$

and D is given by Eq. (24b). Here

$$D_r=[-(\Delta k)^2A'+B'] . \quad (33)$$

Equations (22), (25), (26), (29), and (31) give the electric field of the generated electrostatic lower hybrid wave as

$$\mathscr{E}=F_1\exp\left[\frac{-r^2}{2b_0^2f^2}\right]\exp\{i[\omega t-k(x+s)]\} +F_2\exp\left[-r^2\left[\frac{1}{2r_0^2f_0^2}+\frac{1}{2a_0^2f_1^2}\right]\right] \times\exp\{i[\omega t-(\Delta kx+k_0s_0-k_1s_1)]\} , \quad (34)$$

where

$$F_1=\frac{B_0''}{f(x'')} . \quad (35)$$

To find the unknowns B_0'' and b_0 we use the boundary condition $\mathscr{E}=0$ at $x=0$ which yields

$$\frac{1}{b_0^2}=\frac{1}{r_0^2}+\frac{1}{a_0^2} \quad (36)$$

and

$$B_0''=-\frac{z}{D_r|_{x=0,y=0}}(E_{00}B_0')\frac{\omega e}{2c^2\omega_0}\frac{N}{N_0}\Big|_{x=0,y=0}D' , \quad (37)$$

where

$$D'=D|_{x=0,y=0} . \quad (38)$$

The intensity of the generated electrostatic lower hybrid wave is given by

$$\mathscr{E}\mathscr{E}^* = F_1^2 \exp\left[-\frac{r^2}{b_0^2 f^2}\right] + F_2^2 \exp\left[-r^2\left[\frac{1}{r_0^2 f_0^2} + \frac{1}{a_0^2 f_1^2}\right]\right]$$
$$+2F_1F_2 \exp\left[-r^2\left[\frac{1}{2b_0^2 f^2} + \frac{1}{2r_0^2 f_0^2} + \frac{1}{2a_0^2 f_1^2}\right]\right]\cos[k(x+s) - \Delta k x - (k_0 s_0 - k_1 s_1)] \ . \quad (39)$$

The total power associated with the generated electrostatic lower hybrid wave is given by[6,7]

$$P = \frac{1}{16\pi}\left|\frac{d\omega}{dk}\right|\frac{\partial(\omega\epsilon)}{\partial\omega}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathscr{E}\mathscr{E}^* dy\, dz \ . \quad (40)$$

Using the following expression for the dielectric constant ϵ:

$$\epsilon = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2} - k^2\left[\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{v_{\text{th}}^2}{\omega_{ce}^2} + \frac{\omega_{pi}^2}{\omega^2}\frac{V_{\text{th}}^2}{\omega^2}\right] ,$$

and Eqs. (20), (39), and (40) we obtain this expression for the power

$$P \simeq P_0 \frac{1}{2cr_0^2}\frac{\omega}{k}\left[1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2}\right](B_0')^2\left[\frac{\omega e}{2c^2\omega_0}\right]^2$$
$$\times\left[\frac{D'^2}{(D_r|_{x=0,y=0})^2 f^2}\left[\left.\frac{N}{N_0}\right|_{x=0,y=0}\right]^2 (b_0^2 f^2)^2 + \frac{D^2}{D_r^2 f_0^2 f_1^2}\left[\frac{N}{N_0}\right]^2 \frac{1}{\left[\dfrac{1}{r_0^2 f_0^2} + \dfrac{1}{a_0^2 f_1^2}\right]^2}\right.$$
$$\left. - \frac{2DD'}{(D_r|_{x=0,y=0})D_r}\frac{1}{f_0 f_1 f}\left[\left.\frac{N}{N_0}\right|_{x=0,y=0}\right]\left[\frac{N}{N_0}\right]\frac{\cos[k(x+s) - \Delta k x - (k_0 s_0 - k_1 s_1)]}{\left[\dfrac{1}{2b_0^2 f^2} + \dfrac{1}{2r_0^2 f_0^2} + \dfrac{1}{2a_0^2 f_1^2}\right]^2}\right] , \quad (41)$$

where D and D' are given by Eqs. (24b) and (38), respectively. Here $P_0 = (c/8\pi)\vec{E}_0\cdot\vec{E}_0 \pi r_0^2$ is the total power associated with the incident EM beam.

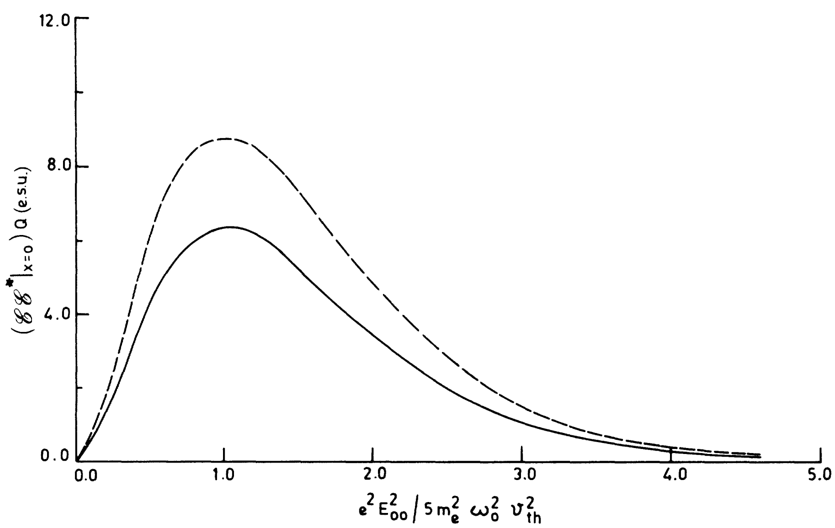

FIG. 1. Variation of the intensity of the generated electrostatic lower hybrid wave at $x=0$ and $30zc/\omega_0 r_0^2 = 0.01$ with the intensity of the pump wave. The solid curve is for $v_{\text{th}}^2/c^2 = 0.0085$ and the dashed curve is for $v_{\text{th}}^2/c^2 = 0.01$, $Q = 10^9$.

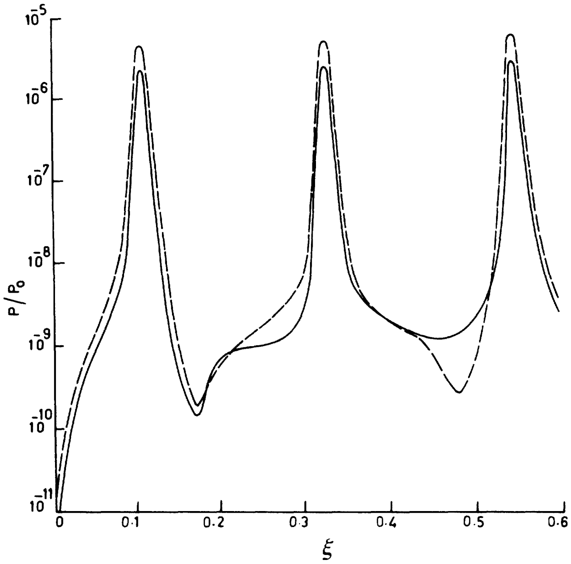

FIG. 2. Variation of the normalized power (P/P_0) of the generated electrostatic lower hybrid wave with the normalized distance $\xi=30xc/\omega_0 r_0^2$ for $v_{th}^2/c^2=0.01$. The solid curve is for $\omega_{ci}/\omega=0.02$ and the dashed curve is for $\omega_{ci}/\omega=0.016$. $P_0 \simeq 5\times 10^{10}$ W.

IV. DISCUSSION

To have an appreciation of the power associated with the generated electrostatic lower hybrid wave, we have chosen the following set of parameters:

$$\omega_0=1.0\times 10^{11}\ \text{rad sec}^{-1}\ ,$$

$$\omega_1=9.999\times 10^{10}\ \text{rad sec}^{-1}\ ,$$

$$\frac{\omega_{pe}^2}{\omega_0^2}=0.5,\quad \frac{\omega_{ci}}{\omega}=0.016 \text{ and } 0.02\ ,$$

$$r_0=100(c/\omega_0)\ \text{cm},\quad a_0=10r_0\ ,$$

$$\frac{v_{th}^2}{c^2}=0.0085 \text{ and } 0.01\ ,$$

$$T_e/T_i=4\ ,$$

$$\frac{B_0'}{E_{00}}=7\times 10^{-2}\ .$$

The results have been depicted in the form of graphs (see Figs. 1–3).

It is obvious from the expression for the electric field of the generated electrostatic lower hybrid wave [Eq. (34)] that it has two components, the first one is supported by the thermal effects in the plasma, and the second component arises as a result of the finite source terms by the beating of the EM beam and the excited electrostatic upper hybrid wave. The first component which is supported by the thermal effects in the plasma can be Landau damped depending upon the parameters of the plasma. Under such conditions, the intensity of the generated electrostatic lower hybrid wave at $x=0$ can be written as

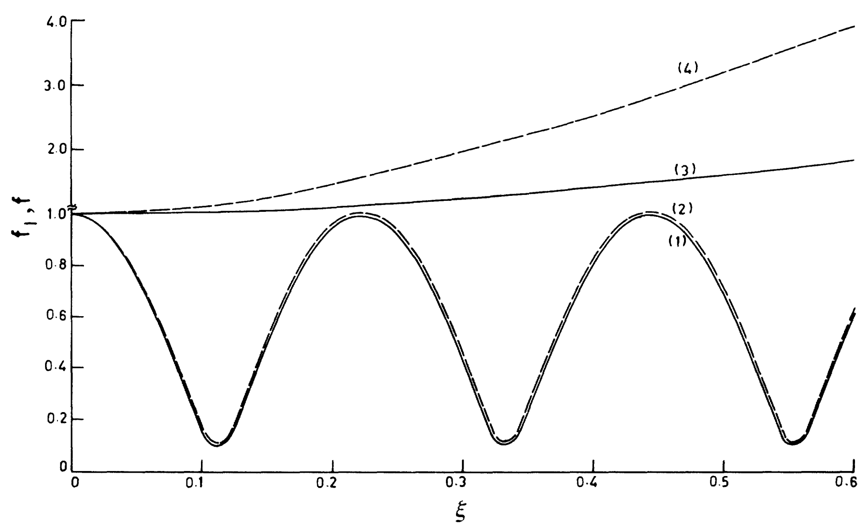

FIG. 3. Variation of the beam-width parameters of the excited electrostatic upper hybrid wave and the generated electrostatic lower hybrid wave (f_1 and f) with the normalized distance $\xi=30xc/\omega_0 r_0^2$. Curves (1) and (2) are for f_1 for $\omega_{ci}/\omega=0.016$ and 0.02, respectively. Curves (3) and (4) are for f for $\omega_{ci}/\omega=0.016$ and 0.02, respectively.

$$\mathscr{E}\mathscr{E}^*\,|_{x=0}=\frac{z^2}{(D_r\,|_{x=0})^2}(E_{00}B_0')^2\left[\frac{N}{N_0}\bigg|_{x=0}\right]^2 \times\left[\frac{\omega e}{2c^2\omega_0}\right]^2(D\,|_{x=0})^2 \times\exp\left[-r^2\left[\frac{1}{r_0^2}+\frac{1}{a_0^2}\right]\right],$$

where $D\,|_{x=0}=D'$ is given by Eq. (38).

From the above equation it is obvious that the intensity of the generated electrostatic lower hybrid wave depends upon (i) the modified background density and (ii) the intensity of the pump. By increasing the pump intensity the modified density decreases and hence there is an optimum value of the pump intensity at which the intensity of the generated electrostatic lower hybrid wave is maximum. This is evident from Fig. 1 where we have plotted the intensity of the generated electrostatic lower hybrid wave versus the normalized intensity of the pump EM beam. It is interesting to note from Fig. 1 that as we change the value of the background electron (or ion) temperature the magnitude of the intensity of the generated electrostatic lower hybrid wave also changes. This may be attributed to the fact that due to the change in the background electron (or ion) temperature the modified density N_{0e} (or N_{0i}) is affected and hence the intensity of the generated electrostatic lower hybrid wave which depends on N_{0e} (or N_{0i}) changes.

Figure 2 shows the variation of the normalized power P/P_0 associated with the generated electrostatic lower hybrid wave with the normalized distance of propagation $\xi=30xc/\omega_0 r_0^2$, for different values of the static magnetic field when the pump EM beam is propagating in the self-trapping mode ($f_0=1$). The power associated with the generated electrostatic lower hybrid wave exhibits maxima and minima with the distance of propagation because of the focusing and defocusing effects of the excited electrostatic upper hybrid wave, and the generated electrostatic lower hybrid wave, as depicted in Fig. 3. For the typical set of parameters chosen here the maximum power is more at lower static magnetic field. The variation in the peak power of the generated electrostatic lower hybrid wave at different values of the static magnetic field is due to the fact that by changing the static magnetic field the focusing of the electrostatic upper hybrid wave, defocusing of the generated electrostatic lower hybrid wave and the dispersive properties of the electrostatic upper hybrid wave, and the generated electrostatic lower hybrid wave are changed.

We conclude from the present investigation that the generation of the electrostatic lower hybrid wave depends on the parameters of the plasma, EM beam, and the value of the static magnetic field. For the set of parameters chosen here the maximum power associated with the generated electrostatic lower hybrid wave is found to be $\simeq 2\times 10^5$ W (corresponding to initial pump power $P_0\simeq 5\times 10^{10}$ W). However, for an optimum value of the above-mentioned parameters the maximum power can be even higher and may lead to the heating of the ions after damping.

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[1]M. S. Sodha, A. K. Ghatak, and V. K. Tripathi, *Progress in Optics* (North-Holland, Amsterdam, 1976), Vol. 13, p. 171.

[2]M. S. Sodha, R. P. Sharma, and S. C. Kaushik, J. Appl. Phys. 47, 3518 (1976).

[3]M. S. Sodha, G. Umesh, and R. P. Sharma, Plasma Phys. 21, 687 (1979).

[4]M. S. Sodha, G. Umesh, and R. P. Sharma, J. Appl. Phys. 50, 4678 (1979).

[5]S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Usp. Fiz. Nauk 93, 19 (1967) [Sov. Phys.—Usp. 10, 609 (1968)].

[6]T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962), Chap. 3.

[7]V. L. Ginzberg, *Propagation of Electromagnetic Waves in Plasmas* (North-Holland, Amsterdam, 1961), Chap. IV, Sec. 22.