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Goos-Hänchen shifts from absorbing media

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Theoretical curves for the Goos-Hänchen shift of a well-collimated beam of polarized radiation at an interface where the reflecting medium possesses nonzero extinction are presented. Under certain circumstances the beam shift can be negative and several wavelengths in magnitude.

The phenomena of reflection of a plane electromagnetic wave from an interface between two adjacent linear media, where the incident medium has an index of refraction greater than the reflecting medium and such that total internal reflection occurs, is known to be accompanied by a small lateral shift in the plane of incidence. This shift was qualitatively predicted by Newton, and later by Picht[1] before it was observed experimentally by Goos and Hänchen.[2] Several theories have since been developed to explain this phenomena. These include the stationary-phase approach by Artmann[3] and von Fragstein,[4] the imposition of energy conservation by Renard,[5] time-delayed scattering processes by Chiu and Quinn,[6] and the angular spectrum approach developed by Brekhovskikh,[7] and later on by McGuirk and Carniglia.[8] An extensive amount of literature exists concerning the Goos-Hänchen (GH) shift; an extensive review of the subject up to 1970 has been given by Lotsch.[9]

There appears to be general agreement concerning the extent of the GH shift for both polarizations for incident angles just beyond the critical angle for total internal reflection. Experimental evidence in the visible and microwave portions of the spectrum indicate fairly good agreement with the stationary-phase and bounded-beam theories.[10] However, with the exception of some brief discussion, the emphasis of theoretical and experimental studies of the GH shift have been oriented toward the case where the reflecting medium is nonabsorbing. It is the intent of this paper to illustrate the behavior of the GH shift when the reflecting medium is absorptive. It is demonstrated that there is a shift when the incident medium is a vacuum and that the shift occurs for all angles of incidence. We shall also, by way of a specific example, show that a peculiar negative shift may occur and possibly be experimentally observed.

Consider the situation shown in Fig 1. A well-collimated wave is incident from a nonabsorbing medium onto an interface with the reflecting medium at an angle θ_0 from the normal. The incident medium is assigned an index of refraction n_1 and the reflecting medium has complex index $\hat{n}_2 = n_2 + ik_2$, where k_2 is the extinction coefficient. The incident wave has two possible polarizations: *p*, where the electric-field vector lies within the plane of incidence, and *s*, where the electric-field vector is orthogonal to the plane of incidence. The GH shift is defined as the distance between the ideally reflected beam and the real reflected beam, as indicated in Fig. 1. When there is a nonzero shift, one may think of the radiation as being reflected ideally from a hypothetical surface, and depending on the sign of the shift *D*, this surface may be either above or below the actual interface.

We shall briefly summarize the assumptions behind our computations and the formulas used to determine the extent of the GH shift. We shall adopt the coordinate conventions and nomenclature developed in Ref. 8. Further, the theoretical development of the angular spectrum approach and the application to the GH shift is summarized in

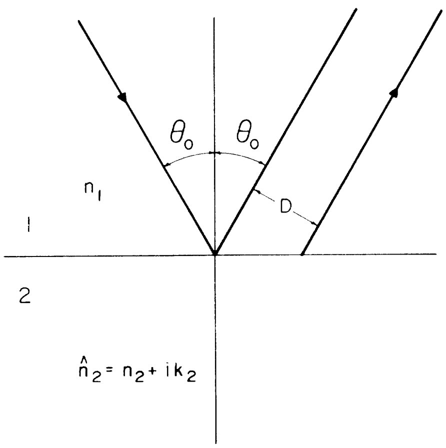

FIG. 1. Geometry indicating the GH shift, defined as D. Here θ_0 is the angle of incidence, n_1 the real index of refraction of the incident medium, and $\hat{n}_2 = n_2 + ik_2$ the complex index for the reflecting medium where k_2 is the extinction coefficient.

Ref. 8. If we denote the reflected beam as $E_r(x',z')$, and the ideally reflected beam by $E_I(x',z')$, such that the latter suffers no shift or alteration in shape, merely a change in propagation direction after reflection, then the reflected beam to a first-order expansion in the reflectivity is

$$E_r(x',z') = r_0 \left[1 - \frac{i}{\rho(0)} \frac{d\rho(k_x)}{dk_x} \bigg|_{k_x=o} D_{x'} \right] \times E_I(x'+D,z') , \quad (1)$$

where

$$r(k_x) = \rho(k_x) e^{-i\delta(k_x)} \quad (2)$$

is the reflectivity in terms of the amplitude $\rho(k_x)$ and phase $\delta(k_x)$. Here r_0 is evaluated at $k_x = 0$, and $D_{x'}$ is the derivative operator with respect to x'. Under the circumstances of total internal reflection, the amplitude of the reflectivity is unity and formula (1) reduces to

$$E_r(x',z') = e^{i\delta(0)} E_I(x'+D,z') . \quad (3)$$

Here we can relate the wave-vector component k_x to the angle of incidence by

$$\theta = \theta_0 - \arcsin(k_x/k) , \quad (4)$$

where $k = 2\pi/\lambda$.

Consequently, the GH shift D reduces to the expression [Eq. (25) in Ref. 8]

$$D = -\frac{\lambda}{2\pi} \frac{d\delta(\theta)}{d\theta} , \quad (5)$$

which is exactly that expression used by Artmann.[3] Now when the reflecting medium possess nonzero extinction (such as a metal), then the assumption that $\rho(\theta) = 1$ no longer holds for any angle except $\theta = 90°$. However, if we assume that the incident beam is well collimated and that the angular derivative of the reflectivity amplitude is small, then the second term in Eq. (1) may be safely assumed to be small. It should be noted that when higher-order terms in the Taylor-series development of $r(k_x)$ are included, and a beam profile is assumed, that unusual phenomena will accompany the GH shift.[8]

We have numerically evaluated Eq. (5) for a variety of materials. The phase shifts for the s and p polarizations are

$$\delta_s = \text{Im} \left[\ln \left[\frac{n_1 \cos\theta_0 - (\hat{n}_2^2 - n_1^2 \sin^2\theta_0)^{1/2}}{n_1 \cos\theta_0 + (\hat{n}_2^2 - n_1^2 \sin^2\theta_0)^{1/2}} \right] \right] \quad (6a)$$

and

$$\delta_p = \text{Im} \left[\ln \left[\frac{n_2^2 \cos\theta_0 - n_1 (\hat{n}_2^2 - n_1^2 \sin^2\theta_0)^{1/2}}{n_2^2 \cos\theta_0 + n_1 (\hat{n}_2^2 - n_1^2 \sin^2\theta_0)^{1/2}} \right] \right] . \quad (6b)$$

Using Eqs. (5), (6a), and (6b), we may compute the GH shift for any desired situation where the incident medium is nonabsorbing and the reflecting medium may be either dielectric, metal, or a vacuum. The phase-shift curves, though using a different sign convention than that adopted here, along with the amplitude reflectivity curves, are profusely illustrated for a variety of cases by Whitaker.[11]

In Fig. 2 we show the GH shift as a function of polarization and angle of incidence for several choices of incident and reflecting media. For comparison, the situation when the incident medium is glass and the reflecting medium is a vacuum is presented to illustrate the GH shift beyond the critical angle. Note that very close to the critical angle the GH shift is extremely large, a prediction by the stationary-phase theory that is alleviated when a bounded-beam theory is imposed.[10] Also, according to the theory developed here and further elaborated upon in Refs. 8 and 9, as the angle of incidence approaches grazing incidence there appears to be a nonzero shift. That this may or may not be physically true has been the subject of con-

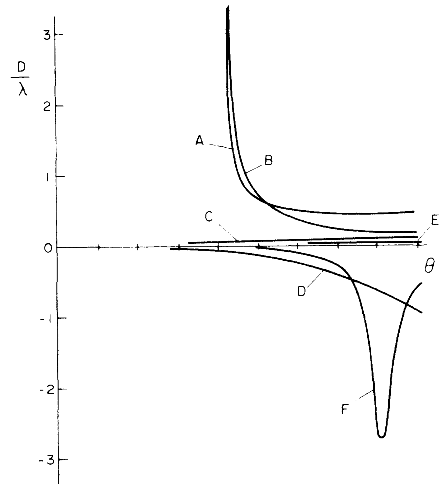

FIG. 2. Curves denoting the normalized GH shift as a function of the angle of incidence. Graduations are in 10° increments. The material and beam parameters are as follows: A, s polarization with $n_1=1.5$, $\hat{n}_2=1.0+i0.0$; B, p polarization with $n_1=1.5$, $\hat{n}_2=1.0+1.0+i0.0$; C, s polarization with $n_1=1.0$, $\hat{n}_2=0.17+i2.94$ (silver); D, same as C but p polarization; E, s polarization with $n_1=1.0$, $\hat{n}_2=5.15+i1.83$ (germanium); F, same as E but with p polarization. For silver, these indices correspond to the incident wavelength being 0.5 μm, and for germanium 0.562 μm.

troversy.[12]

When the incident medium is a vacuum and the reflecting medium is germanium (where $n_2=5.15+i\,1.83$), we see a striking shift in the negative direction near $\theta=82°$ for the p polarization. This seems to be counterintuitive, and has been mentioned previously by Wolter[13] and Lotsch.[9] To the author's knowledge, there has been no direct experimental evidence of the negative shift. Perhaps using microwave radiation and suitably absorbing materials as reflecting media, this shift may be observed. Unfortunately, the region around the greatest negative GH shift corresponds to the Brewster dip in the reflectivity so that there may be difficulty in seeing a reflected beam.

For the case where the reflecting medium is silver, we see that the shift may be positive or negative, for s and p polarizations, respectively. However, there no longer is the large dip that occurs when germanium is used. (The index used for silver is $n_2=0.17+i\,2.94$.[14]) When Goos and Hänchen performed their original experiments, they ingeniously used a prism that would allow the incident beam to undergo a large number of internal reflections so that for each reflection a small shift would cumulatively add to previous shifts to give a larger observable shift. For the sake of comparison to a reference beam that did not undergo any shift, the ideally reflected beam, they put a silver coating on the sides of the prism so that the total internal reflection phenomena would be frustrated. (Reference 15 shows a good picture of their experimental apparatus.) The beam coming out of the prism could therefore be compared with an adjacent beam that was supposed to be reflected in an ideal manner. Fortunately, for silver as the reflecting medium and glass as the incident medium, our analysis indicates that the shift near the critical angle is very small for each polarization relative to the shift from a glass and air combination. Consequently, the results of Goos and Hänchen agreed quite well with the theoretical predictions, since silver coatings act as ideal reflectors near the critical angle for glass.

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1. J. Picht, Ann. Phys. (Leipzig) 3, 433 (1929); Z. Phys. 30, 905 (1929); Optik 12, 41 (1955).
2. F. Goos and H. Hänchen, Ann. Phys. (Leipzig) 1, 333 (1947).
3. K. Artmann, Ann. Phys. (Leipzig) 2, 87 (1948).
4. C. von Fragstein, Ann. Phys. (Leipzig) 4, 271 (1949).
5. A. Renard, J. Opt. Soc. Am. 54, 1190 (1964).
6. K. Chiu and J. Quinn, Am. J. Phys. 40, 1847 (1972).
7. L. Brekhovskikh, *Waves in Layered Media* (Academic, New York, 1960), pp. 100–122.
8. M. McGuirk and C. Carniglia, J. Opt. Soc. Am. 67, 103 (1977).
9. H. Lotsch, Optik 32, 116 (1970); 32, 189 (1970); 32, 299 (1971); 32, 553 (1971).
10. J. Cowen and B. Aničin J. Opt. Soc. Am. 67, 1307 (1977).
11. M. Whitaker, J. Phys. A 12, 297 (1979).
12. C. Carniglia, J. Opt. Soc. Am. 66, 1425 (1976); H. Lotsch, *ibid.* 66, 1426 (1976).
13. H. Wolter, Z. Naturforsch 5a, 143 (1950).
14. *AIP Handbook*, 3rd ed. (McGraw-Hill, New York, 1972), p. 6–149.
15. N. Harrick, *Internal Reflection Spectroscopy* (Interscience, New York, 1967), pp. 33–35.