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Effects of local current gradients on magnetic reconnection

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The excitation of reconnecting modes is commonly considered to rely solely on global properties of the current-density distribution. Here a class of simple resistive magnetohydrodynamic models is studied analytically to show that local properties of the current profile at the reconnecting surface become important in some realistic physical regimes.

For a long time reconnecting modes have attracted great interest in astrophysics[1] and controlled thermonuclear research.[2] These global collective processes change the topology of the equilibrium magnetic configuration in a plasma through the action of a narrow dissipative layer where magnetic energy associated with equilibrium currents is released. The phenomenon presents some analogies with the formation of cat's eyes in shear flows.[3]

Reconnecting modes are found to be excited in many different plasma regimes and geometries (cf. Refs. 2 and 4–10). In all cases one can identify an "outer region" where Ampère's law and the balance of pressure gradients with magnetic forces define the macroscopic structure of the modes. Then in an "inner layer" more physics is involved. There dissipation can be provided by a small amount of resistivity or, for high-temperature regimes, by particle resonance. As a result, in general one has to perform a proper matching of solutions across the dissipative layer. The matching process may involve a *large scale asymmetry,* due to the properties of the equilibrium configuration and the outer boundary conditions (cf. our term A defined below), and a *local asymmetry* due to the presence of current gradients in the dissipative layer [cf. our term a in Eq. (3)].

From the matching procedure given in the past it was argued that the abovementioned asymmetries do not affect the growth of reconnecting modes. On the other hand, in the absence of local asymmetry the inner equations have a simpler treatment and the matching is more easily performed.[2,6–8] In some cases symmetry arguments have been invoked directly in dismissing the asymmetry terms as unimportant.[10] In order to clarify the matching procedure and especially after claims that local current gradients considerably modify the standard dispersion relation for reconnecting modes,[11] we have decided to reconsider the joining of solutions across the dissipative layer for reconnecting modes without using symmetry arguments. In particular, we here reexamine the so-called "constant-ψ" approach[2] in cases where logarithmic terms in the matching region are present due to local current gradients.

To be specific, in the following we refer to the magnetohydrodynamical problem in plane geometry, even if our methods are applicable to more general cases. The magnetic equilibrium configurations of interest are those characterized by

$$\vec{B}=B_y(x)\vec{e}_y+B_z(x)\vec{e}_z$$

and the relevant magnetic perturbation is

$$\vec{B}_1=\tilde{B}(x)\exp(-i\omega t+i\vec{k}\vec{x})$$

so that, at $x=x_0$, $\vec{k}\cdot\vec{B}=0$. It is convenient to choose a frame of reference where $x_0=0$ and $\vec{k}=k\vec{e}_y$. The equilibrium field is taken to have the following expansion around $x=0$:

$$B_y(x)=B_0[\hat{x}+(a/2)\hat{x}^2+\cdots]$$

and

$$B_z(x)=B_{0z}(1+b\hat{x}+\cdots)\ ,$$

where a, b, B_0, and B_{0z} are constants. In addition, we use the dimensionless coordinate $\hat{x}=x/r_B$, where r_B is a typical length for the equilibrium field. The quantity a is then related to the gradient of the current density (dJ_z/dx) at $x=0$. Notice that the case where $B_{0z}=0$ describes the "neutral sheet" configuration, whereas a "sheared magnetic configuration" has a sizable B_{0z}.

Neglecting the effects of inertia and resistivity we have the following "outer" equations for the magnetic perturbation $\psi=\tilde{B}_x/B_0$ and the plasma displacement $\xi=-ik\tilde{\xi}_x$:

$$\psi''-[\hat{k}^2+(B''/B)]\psi=0\ , \tag{1}$$

$$\psi+B\xi=0\ . \tag{2}$$

Here we have used the notation $B=B_y/B_0$ and $\hat{k}=kr_B$. A prime indicates derivative with respect to $\hat{x}$.

Reconnecting modes are characterized by the property $\psi(\hat{x}=0)\neq 0$. Then we see that the "frozen-in law" (2) leads to *singular* perturbations at $x=0$ even for the locally symmetric ($a=0$) case where Eq. (1) is regular. Only by including finite inertia and resistivity effects in a small layer of width δ (to be determined) around the surface $x=0$, is the singularity of the reconnecting problem removed.

In the vicinity of the singular layer we can use the expanded form for B and obtain the following approximate solution for ψ from Eq. (1):

$$\psi_{\pm}^{(\mathrm{out})}=\tfrac{1}{2}(A\pm\Delta)\psi^{(1)}+\psi^{(2)}\ , \tag{3}$$

where

$$\psi^{(1)}\sim\hat{x}+(a/2)\hat{x}^2+\cdots\ ,$$

$$\psi^{(2)}\sim 1+(\hat{k}_1^2/2)\hat{x}^2+\cdots+a\ln|\hat{x}|\psi^{(1)}\ ,$$

and $\hat{k}_1$ is a modified wave number which reduces to $\hat{k}$ for a linear B profile. In Eq. (3) the sign $+$ ($-$) refers to positive (negative) values of $\hat{x}$. In general the solution should be represented by $\psi_{\pm}^{(\mathrm{out})}=A_{\pm}\psi^{(1)}+B_{\pm}\psi^{(2)}$. In (3) we have considered the general case where neither B_+ nor B_- is zero so that for our linear problem we can take $B_+=B_-=1$. The constants $A_{\pm}$ and $B_{\pm}$ and therefore the constants A and Δ in our representation (3) are determined by the outer boundary conditions on Eq. (1). As is best seen for the locally symmetric case ($a=0$) where

$$\psi^{(\mathrm{out})}\sim 1+\tfrac{1}{2}\Delta|\hat{x}|+\tfrac{1}{2}A\hat{x}\ ,$$

the constant A measures the large scale asymmetry of the reconnecting mode.

In our resistive MHD description of the inner layer, dissipation is measured by a small parameter $\epsilon=(D_m/v_A r_B\hat{k})$ which is essentially an inverse magnetic Reynolds number. The small resistivity is associated with the magnetic diffusion coefficient D_m; the quantity v_A is a typical Alfvén speed of the problem. Furthermore, inertia is measured by the dimensionless growth rate $\gamma=(-i\omega/kv_A)$. The relevant ordering of our problem is then $\epsilon<<1$, $\gamma=O(\epsilon^r)$ with $0<r<1$, $(\delta/r_B)=O(\epsilon^s)$ with $s>0$, and $\hat{k}=O(1)$. By rescaling the x coordinate into $\zeta=x/\delta$ and using the power expansion for the B profile, we find the following *regular* fourth-order system for the inner problem:

$$\frac{d^2\psi}{d\zeta^2}=p(\psi+\zeta\hat{\xi})\ ,$$
$$\frac{d^2\hat{\xi}}{d\zeta^2}-q\zeta^2\hat{\xi}=q(\zeta-\hat{a})\psi\ , \tag{4}$$

where $\hat{\xi}=\xi(\delta/r_B)$, $p=(\gamma/\epsilon)(\delta/r_B)^2$, $q=(\gamma\epsilon)^{-1}(\delta/r_B)^4$, and $\hat{a}=(a/p)(\delta/r_B)$.

We define the generalized constant-ψ approach by the ordering $q=O(1)$ and $(p^2/q)<<1$, i.e., $r>\frac{1}{3}$ (in the following we can consider $q=1$ as the definition of δ). In addition, in order to explore the effects of a small to moderate local asymmetry we take $\hat{a}=O(1)$. As for the corresponding hydrodynamic problem[12] we can expand our inner solution as

$$\psi^{(\mathrm{in})}=\psi_{(0)}+p\psi_{(1)}+p^2\psi_{(2)}+\cdots$$

and matching with the outer solution is obtained only by considering at least the truncation $\psi^{(\mathrm{in})}=\psi_{(0)}+p\psi_{(1)}$.

In the above defined constant-ψ scheme the matching is possible and easily performed. The lowest-order solution of (4) satisfies $d^2\psi_{(0)}/d\zeta^2=0$. In order to match with $\psi^{(\mathrm{out})}$ [cf. Eq. (3)] we take $\psi_{(0)}=1+\psi_1\zeta$, with $\psi_1=(A/2)(\delta/r_B)$. This function does not include the discontinuity Δ of the outer solution [cf. Eq. (3)]. Therefore we proceed to evaluate $\psi_{(1)}$ in the above expansion of $\psi^{(\mathrm{in})}$. In order to do this we calculate the lowest-order solution $\hat{\xi}_{(0)}$ by considering the second equation of system (4). By applying the method of Laplace transforms (for the symmetric case $\hat{a}=0$, cf. Ref. 13) we find the particular solution

$$\hat{\xi}_{(0)}=-\psi_1-(1-\psi_1\hat{a})I_1(\zeta)+\hat{a}I_2(\zeta)\ , \tag{5}$$

where

$$I_1(\zeta)=(\sqrt{q}/2)\zeta\int_0^1 dt(1-t^2)^{-1/4}\times\exp[-(\sqrt{q}/2)t\zeta^2]$$

and

$$I_2(\zeta)=(\sqrt{q}/2)\int_0^1 dt(1-t^2)^{-3/4}\times\exp[-(\sqrt{q}/2)t\zeta^2]\ .$$

This solution is just what we need for determining the lowest-order expression of the dispersion relation. In fact the correction $\psi_{(1)}$ obeys the equation

$$d^2\psi_{(1)}/d\zeta^2=\psi_{(0)}+\zeta\hat{\xi}_{(0)} \; .$$

Since the matching to zeroth order in p has been already performed by the solution $\psi_{(0)}=1+\psi_1\zeta$, we can now proceed to first order in p in the matching process by considering

$$d\psi^{(\mathrm{in})}/d\zeta\sim\psi_1+p\,d\psi_{(1)}/d\zeta$$

rather than the function

$$\psi^{(\mathrm{in})}\sim 1+\psi_1\zeta+p\psi_{(1)} \; .$$

Using expression (5) we obtain

$$d\psi^{(\mathrm{in})}/d\zeta\sim p[\psi_{11}+I_3(\zeta)]+\psi_1 \; , \tag{6}$$

where

$$I_3(\zeta)=\int_0^\zeta d\zeta'(\psi_{(0)}+\zeta'\hat{\xi}_{(0)}) \; .$$

At large values of ζ we find $I_3(\zeta)\rightarrow I_3^{\pm}(\zeta)$ for $\zeta\rightarrow\pm\infty$:

$$I_3^{\pm}(\zeta)=\psi_1\hat{a}\zeta\pm(1-\psi_1\hat{a})\alpha_1+\hat{a}(\alpha_2+\ln|\zeta|) \; , \tag{7}$$

where

$$\alpha_1=\int_0^\infty d\zeta'[1-\zeta' I_1(\zeta')]\approx 1 \; ,$$

and

$$\alpha_2=\int_0^\infty d\zeta'[\zeta' I_2(\zeta')-(1+\zeta')^{-1}] \; .$$

By considering the expression

$$d\psi_{\pm}^{(\mathrm{out})}/d\hat{x}\sim a+a\ln|\hat{x}|+(A\pm\Delta)/2 + (aA/2)\hat{x}+\cdots$$

from Eq. (3), we see that *the logarithmic terms are automatically matched.* Proper matching with Eq. (6) then requires

$$p(r_B/\delta)\psi_{11}=a[1+\ln(\delta/r_B)-\alpha_2] \tag{8}$$

and

$$\Delta=2p(r_B/\delta)\alpha_1(1-\psi_1\hat{a}) \; . \tag{9}$$

Equation (8) is just a definition of the integration constant ψ_{11} in expression (6). The second relation (Eq. 9) is the desired dispersion relation. Using the relation $q=1$ as a definition of δ, we can rewrite the dispersion relation as

$$2\alpha_1\gamma^{5/4}\epsilon^{-3/4}=\Delta+\alpha_1 aA\gamma^{1/4}\epsilon^{1/4} \; . \tag{10}$$

This equation reduces to the standard[2] dispersion relation $2\alpha_1\gamma^{5/4}\epsilon^{-3/4}=\Delta$ in the symmetric case. Even if the new term in Eq. (10) is formally a correction $O(\epsilon^{2/5})$, it may affect appreciably the dispersion relation depending on the properties of the equilibrium configuration and the location of the resistive layer. Hints at large asymmetries in relevant modes are given in Fig. 2 of Ref. 7. Numerical calculations of the MHD stability of DITE with respect to $m=2$, $n=1$ reconnecting modes indicate[14] that the ratio $|A/\Delta|$ for such modes is typically around 6, so that in some cases $|aA/\Delta|\sim 35$. This means that for current profiles comparable to DITE, the new term is appreciable when $\epsilon\sim 10^{-4}$, which is a rather large value for typical tokamak regimes, but may be interesting for different situations.

The present paper provides one clear-cut example where effects of local current gradients on magnetic reconnection can be handled analytically and estimated to be *large* or *small* by inspection of a simple dispersion relation. The method of analysis used could be applied or extended to cover different configurations and regimes.

In conclusion our analysis shows in detail that asymmetries due to the current profile may change the lowest-order form of the dispersion relation of "standard" reconnecting modes. This does not exclude the possibility of modes of a different kind that might originate in the presence of such asymmetries. Neither does our argument apply to cases where $p^2/q=O(1)$, such as the resistive internal-kink modes of Ref. 8, where the generalized constant-ψ approach is not permitted. Finally, other physical phenomena (such as those studied in Ref. 15) are associated with "asymmetries" in the relevant equations and might be related to effects analogous to those of local current gradients. In this regard, extensions of the present work and comparison with already existing results deserve a more complete investigation.

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[1]J. W. Dungey, Philos. Mag. 44, 725 (1953).
[2]H. P. Furth, J. Killeen, and M. N. Rosenbluth, Phys. Fluids 6, 459 (1963).
[3]C. C. Lin, *The Theory of Hydrodynamic Stability* (Cambridge University Press, Cambridge, England, 1955).
[4]B. Coppi, Phys. Fluids 8, 2273 (1965).
[5]G. Laval, R. Pellat, and M. Vuillemin, in *Plasma Physics and Controlled Nuclear Fusion Research* (International Atomic Energy Agency, Vienna, 1965), Vol. 2, p. 259.
[6]B. Coppi, J. M. Greene, and J. L. Johnson, Nucl. Fusion 6, 101 (1966).
[7]H. P. Furth, P. H. Rutherford, and H. Selberg, Phys. Fluids 16, 1054 (1973).
[8]G. Ara, B. Basu, B. Coppi, G. Laval, M. N. Rosenbluth, and B. V. Waddell, Ann. Phys. (N.Y.) 112, 443 (1978); B. Coppl, R. Galvão, R. Pellat, M. N. Rosenbluth, and P. H. Rutherford, Fiz. Plazmy 2, 961 (1976) [Sov. J. Plasma Phys. 2, 533 (1976)].
[9]D. Biskamp, Nucl. Fusion 18, 1059 (1978).
[10]B. Coppi, J. W.-K. Mark, L. Sugiyama, and G. Bertin, Ann. Phys. (N.Y.) 119, 370 (1979).
[11]M. Dubois and A. Samain, Plasma Phys. 21, 101 (1979).
[12]C. C. Lin, Q. Appl. Math. 3, 117 (1945).
[13]B. Basu and B. Coppi, Nucl. Fusion 17, 1245 (1977).
[14]F. Alladio (private communication).
[15]D. Dobrott, S. C. Prager, and J. B. Taylor, Phys. Fluids 20, 1850 (1977).