PHYSICAL REVIEW A VOLUME 25, NUMBER 2 FEBRUARY 1982

Comment on "Electron diffusion under the influence of an electric field near absorbing boundaries." II

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(Received 20 July 1981)

Chantry's gedanken experiment leads to the conclusion that the relaxation distance of electrons emitted from a source can be eliminated if the electrons have the same velocity distribution as the electrons arriving at an absorbing anode. In this paper Monte Carlo results are reported that confirm this conclusion.

The recent comment by Chantry[1] (the preceding paper) is primarily devoted to the analysis of an assumption that Lowke, Parker, and Hall (LPH) introduce in Sec. V of their paper,[2] where electron back diffusion from a planar source upstream against the field, is shown to be complementary to the forward diffusion in the direction of the applied field, near an absorbing anode. LPH's assumption that the complementarity applies (1) when the source electrons are released with the conventional steady-state (say, equilibrium) energy distribution $f_0(\epsilon)$, appropriate to electrons in an unbounded gas acted upon by the same electric field, and (2) when there is an absorbing electrode at a given distance upstream from the source, is called into question. On the contrary, it is pointed out that for the complementarity to strictly apply, the source must be characterized by the same velocity distribution appropriate to the electrons of a continous stream arriving at an absorbing anode.

A second very interesting suggestion which follows from the gedanken experiment on which Chantry's analysis is based, concerns the relaxation distance of electrons emitted from a source in Monte Carlo simulations. In particular, having recourse to the complementarity theorem, Chantry is led to conclude that the ideal source for Monte Carlo simulations of the type performed by Braglia and Lowke[3] (where electrons are isotropically emitted from a source at a given distance from the anode with the equilibrium energy distribution), should have a distribution of velocities the same as the electrons of a continuous stream impinging on the absorbing anode. In fact, as shown by Braglia and Lowke,[3] using a source that injects electrons with the conventional steady-state solution of the spatially independent Boltzmann's equation, results in a stationary-mean-electron energy at the source position which is substantially lower than that of the injected electrons. It is only after the electrons have drifted downstream a sufficiently long distance that this latter (initial) mean energy is regained and a continuous stream of electrons with the same (position independent) energy distribution of the injected electrons is attained. Clearly, this lowering of the mean energy is a consequence of the prevalent back diffusion of the electrons upstream against the field at the source. In fact, if we put the source very near to a perfectly absorbing cathode, e.g., we inject the electrons from a very small hole in the forward hemisphere with the equilibrium energy distribution, the relaxation distance is strongly minimized. This is shown in Fig. 1 where some results we obtained with Monte Car-

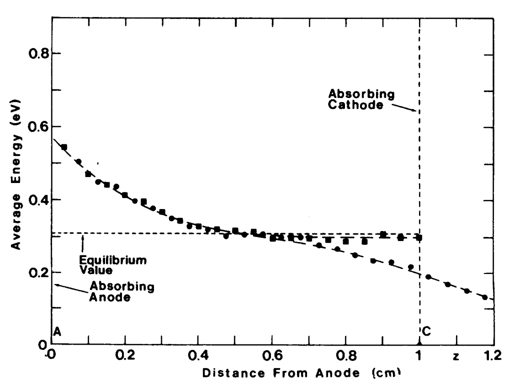

FIG. 1. Average electron energy as a function of distance z from the anode in the absence (●) and the presence (■) of an absorbing cathode. Physical conditions and the electron-atom-interaction law are the same as in Ref. 3.

lo simulations under the same conditions of Ref. 3 are reported. However, reducing the relaxation distance in Monte Carlo simulations by this procedure would be extremely time consuming as only a small percentage of electrons, in the situations of major practical and theoretical interest, would be able to leave the source region and reach the anode, without being reabsorbed by the cathode some time after their release. The ideal situation is certainly constituted by a source which can emit electrons with such a velocity distribution to essentially eliminate the relaxation distance, i.e., a source that is able to reproduce approximately the same behavior of the mean energy observed in Fig. 1 in the presence of the absorbing cathode. Now, in the light of Chantry's discussion, we must expect that this ideal source is a forward-directed source releasing electrons with the same distribution of velocities as the electrons impinging on the anode. Moreover, as a further important conclusion, we must expect that such a source may be placed arbitrarily close to the absorbing anode. In this comment we want to complement the analysis of Ref. 1 by giving some Monte Carlo data we have recently obtained which will permit us to test the validity of Chantry's conclusion.

In our simulations, electrons are emitted from a forward-directed source, 1 cm distant from an absorbing anode. To save computer time, the electron-collision frequency v is assumed to be energy independent (in Ref. 2 it was on the contrary assumed that $v \propto \epsilon$, cf. Fig. 1). The background gas atoms are considered to be at rest. As regards the numerical parameters, it is assumed a mass ratio $m/M = 1.37\times10^{-4}$, a collision frequency $v = 3.868\times10^{9}$ sec^{-1}, an electric field $E = 1$ V/cm and a gas pressure $p = 1$ Torr 0 °C. In correspondence of these constants, we obtain an equilibrium-mean-electron energy

$$\bar{\epsilon} = \epsilon_0 = (\tfrac{3}{2})(D/W)E = 0.474 \text{ eV},$$

a drift velocity $W = 4.77\times10^{5}$ cm/sec, and a diffusion coefficient $D = 1.50\times10^{5}$ cm^2/sec. Thus, $D/W = 0.315$ cm, which is the same distance considered in Refs. 2 and 3 and in Fig. 1. Contrary to what happens in Fig. 1, however, the relaxation distance of the mean energy relevant to electrons injected from the source with isotropic Maxwellian energy distribution at the mean energy 0.474 eV (curve 1 of Fig. 3) is now so large that it overlaps the region where the anode perturbation begins to be important (at about 0.4 cm from the electrode). In fact, the behavior of the mean energy $\bar{\epsilon}(z)$ is found to be that which is represented by curve 1 of Fig. 2.[4]

Following Chantry's suggestion,[1] the stationary velocity distribution of the electrons crossing the anode (calculated mean energy 0.79 eV) is then used in a second run as a new initial velocity distribution (curve 2 of Fig. 3). With this initial distribution the new behavior of $\bar{\epsilon}(z)$ represented by curve 2 of Fig. 2 is obtained. Apparently, the relaxation distance is not eliminated, but this is due to the fact that the new initial distribution is not exactly that relevant to electrons of a continuous stream with $\bar{\epsilon} = 0.474$ eV far from the anode. In fact, as mentioned and as curve 1 of Fig. 2 indicates, the source was too near the anode in the first run so that also $\bar{\epsilon}(0)$ is expected (and found) to be too low. Curve 2 gives a higher value of $\bar{\epsilon}(0)$ at the anode (i.e., 0.92 eV) than the initial value 0.79 eV. So, the simulation is repeated with the new electron velocity distribution at the anode represented by curve 3 of Fig. 3 and the result for $\bar{\epsilon}(z)$ is now given by curve 3 of Fig. 2. As one can see, the relaxation distance has been almost completely

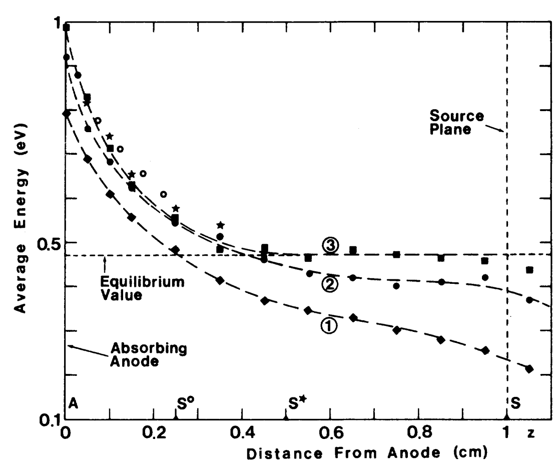

FIG. 2. Average electron energy as a function of distance from the anode, corresponding to the conditions fixed in the text. Curve 1 refers to source electrons emitted with a Maxwellian energy distribution of mean energy $\epsilon_0 = \bar{\epsilon}(1) = 0.474$ eV (curve 1 of Fig. 3). Curve 2 (3) refers to electrons emitted with the same velocity distribution as the electrons impinging on the anode in case 1 (2) [curve 2 (3) of Fig. 3]. In this case $\bar{\epsilon}(1) = 0.79(0.92)$ eV. The stars and the circles refer to electrons emitted with the same velocity distribution as the electrons impinging on the anode in case 3 (curve 4 of Fig. 3). Contrary to the general assumption of a source 1 cm distant from the anode, in these two cases the distances are 0.5 and 0.25 cm, respectively.

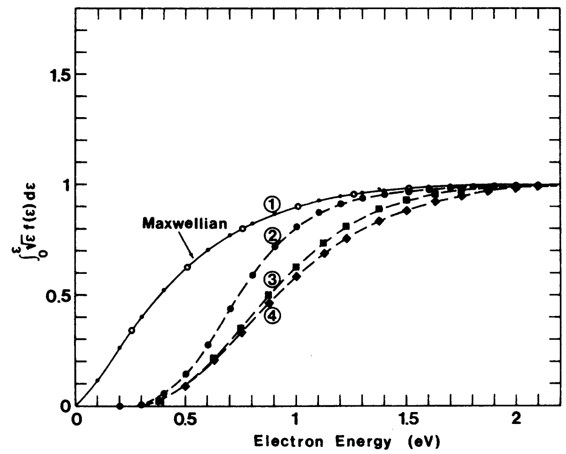

FIG. 3. Initial energy distributions leading to the behaviors of $\bar{\epsilon}(z)$ reported in Fig. 2. The circles represent the distribution obtained (with a distance of 0.25 cm between anode and source) when considering *all* the electrons upstream from the anode. As expected (Ref. 1), the distribution agrees with the (Maxwellian) energy distribution appropriate to the given E/N in the absence of boundaries.

eliminated. Approximately, this is also the final result since a further run with the new initial energy distribution represented by curve 4 of Fig. 3 [mean energy $\bar{\epsilon}(0)=0.98$ eV$\simeq 2\epsilon_0$][4] only produces a small improvement (cf. the results of Fig. 2 relevant to the anode-source distances of 0.5 and 0.25 cm). Clearly, similar conclusions can also be drawn when considering the relaxation of transport

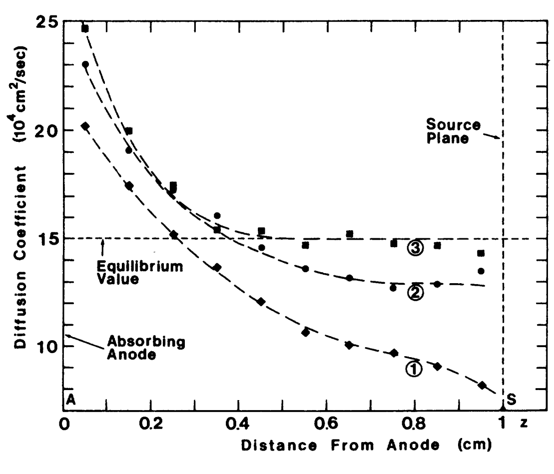

FIG. 4. Observed diffusion coefficients $D=D(z)$ corresponding to the behavior of $\bar{\epsilon}(z)$ reported in Fig. 2.

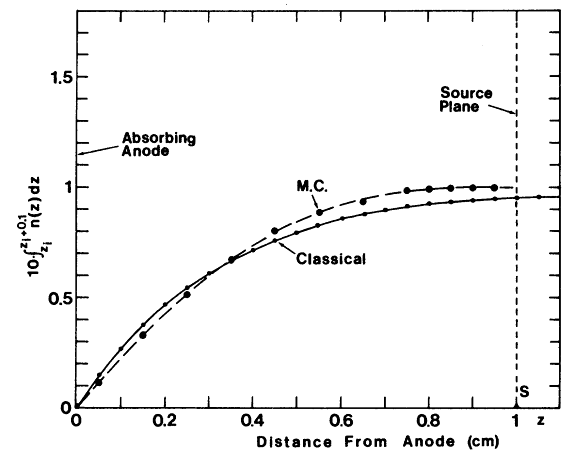

FIG. 5. Normalized electron density profiles $n(z)$ near the anode obtained via the Monte Carlo simulation and via the solution of the continuity equation. No appreciable difference is observed between density profiles obtained with Monte Carlo simulation in correspondence of different initial energy distributions at the source (at least on the scale of this figure).

coefficients which depend on the electron energy distribution. This is confirmed by the results for $D(z)$ reported in Fig. 4.

Before concluding our analysis, it is also interesting to compare the observed behavior of the (normalized) electron density near the anode with that provided by the conventional continuity equation, where both W and D are constant quantities independent of position. This is done in Fig. 5. As one can see, there is substantial difference between the data obtained via the Monte Carlo simulation and via the continuity equation. In Ref. 2 it was found that for $v \propto \epsilon$ the profile of the electron density was between those provided by the continuity equation in correspondence of the two diffusion coefficients D_T and D_L perpendicular and parallel to the field. For constant v, $D_T=D_L=D$, but the density profile provided by Monte Carlo simulation is not coincident with that obtained from the continuity equation as one might be led to expect.[5] This is certainly due to the fact that $D(z)$ increases near the anode [$W=(eE/m)v$ is, on the contrary, a constant] so that the continuity equation treatment fails in this region.

The author is indebted to P. J. Chantry for reading and commenting on the manuscript and communicating his results prior to their publication, and to R. E. Robson for communicating Ref. 4 prior to its publication.

[1]P. J. Chantry, Phys. Rev. A 25, 1209 (1982).
[2]J. J. Lowke, J. H. Parker, Jr., and C. A. Hall, Phys. Rev. A 15, 1237 (1977).
[3]G. L. Braglia and J. J. Lowke, J. Phys. D 12, 1831 (1979).
[4]R. E. Robson, Aust. J. Phys. 34, 223 (1981). In this paper, Robson discusses the constant collision frequency case and finds analytically that $\bar{\epsilon}(0)=(5/3)\epsilon_0$ where ϵ_0 is the conventional (steady-state) mean-electron energy in the absence of absorbing electrodes. This conclusion is in good although not perfect agreement with the results reported in Fig. 2.
[5]The electron density obtained via Monte Carlo simulation is normalized to unity from about 0.7 to 0.9 cm from the anode, as there is a good indication that $n(z)$ is already constant in this region. This assumption may be only approximately true. Even in this case, however, Fig. 5 serves well to point out the differences that exist between the behavior of $n(z)$ as obtained by the Monte Carlo method or the continuity equation near the absorbing anode.