Free-electron lasers operating in higher harmonics

W. B. Colson
Quantum Institute, University of California, Santa Barbara, California 93106
(Received 24 February 1981)

The nonlinear wave equation and self-consistent pendulum equation are used to generalize free-electron-laser operation to higher harmonics; this can significantly extend their tunable range to shorter wavelengths.

INTRODUCTION

In a free-electron laser, a beam of relativistic electrons travels through a static periodic magnetic field and oscillates to amplify coherent optical radiation with the same polarization as the magnet.[1] While the laser radiation causes spatial "bunching" on the optical wavelength scale,[2] the large-scale electron trajectories are primarily determined by the magnet. Several theoretical approaches have been used to describe the free-electron laser, and Ref. 3 compiles many of these techniques. The picture of single-particle electron currents driving the nonlinear optical wave equation[4] provides a clear, intuitive description of both electron and wave dynamics; we use this view to analyze the feasibility of operating free-electron lasers at selected frequencies which are odd multiples of the fundamental $3\omega, 5\omega, \ldots$.

Theory and experiment have been primarily applied to free-electron lasers using helical magnets, but many proposed experiments will use linearly polarized magnets, which are magnets with alternating poles. A small periodic longitudinal motion of electrons in the linear magnet causes spontaneous emission and gain in the higher harmonics; this has been proposed as a method of extending their tunable range.[5] Backscattering into higher harmonics has also been described (Ref. 3, Chap. 32, Vol. 7), but this process does not involve gain. Recently,[6] harmonic gain has been calculated for the low-gain case, but an incorrect result is presented; also, we are told of a quantum-mechanical contribution to the topic.[7] We derive a complete nonlinear, self-consistent wave equation for the laser field and show how the coupling between the electrons and light is altered in a nontrivial way. A useful notation allows simple scaling arguments to compare operation in any selected harmonic.

NONLINEAR WAVE EQUATION

General solutions to the electron motion in a purely transverse, periodic magnetic field $\vec{B} = B(0, \sin k_0 z, 0)$ with wavelength $\lambda_0 = 2\pi/k_0$ are difficult; but the physical situation of interest occurs when $\beta_z \approx 1 >> \beta_x, \beta_y$. An electron's path through the magnet is nearly sinusoidal with oscillation amplitude $K/\gamma_0 k_0$, where $K = eB\lambda_0/2\pi mc^2$, $e = |e|$ is the electron charge, m is the electron mass, and $\gamma_0 mc^2$ is the initial electron energy; smaller longitudinal oscillations of amplitude $K^2/8\gamma_0^2 k_0$ cause spontaneous emission and gain into a few higher harmonics.[8]

Calculation of the detailed properties of spontaneous radiation is straightforward using standard classical techniques.[5] In a long magnet ($N = L/\lambda_0 \approx 10^2$), emission is sharply peaked at well-separated harmonics

$$f\omega = 2\pi cf/(1-\beta_0)\lambda_0 \approx 2\gamma_0^2 f k_0 c/(1+\tfrac{1}{2}K^2)$$

in the forward direction, where $\beta_0 c$ is the electron's z velocity; the spectral width is $\sim 1/2N$. Far away from the linearly polarized magnet the element of optical energy received in the fth harmonic per unit solid angle, $d\Omega$, in the forward direction per unit frequency interval, $d(f\omega)$, is

$$\frac{dW_f}{d\Omega\, d(f\omega)} = \left(\frac{eN\gamma_0 f}{1+\frac{1}{2}K^2}\right)^2 \frac{\mathcal{K}_f^2(\xi)}{c}, \quad f = 1,3,5,7,\ldots, \tag{1}$$

where

$$\mathcal{K}_f(\xi) = K(-1)^{(f-1)/2}[J_{(f-1)/2}(f\xi) - J_{(f+1)/2}(f\xi)],$$
$$\xi = K^2/4(1+\tfrac{1}{2}K^2).$$

The radiation is stored in a resonant cavity which we take to be selective to only one of the harmonics. In order to describe stimulated emission, we must calculate the feedback of the light wave on the electron current using Maxwell's nonlinear wave equation. The detailed derivation of the wave equation is presented elsewhere.[9] The optical wave amplitude $E(t)$ and phase $\phi(t)$ slowly evolve into a coherent laser beam.

Relativistic electrons in both $\vec{B}$ and the radiation fields are governed by the Lorentz force equations. The electron motion contains factors which oscillate periodically each magnet wavelength, but we actually want to describe the slow evolution about these periodic oscillations. This is accomplished by averaging the motion over each magnet wavelength.[6] It is then convenient to define a slowly evolving dimensionless velocity $\nu(t) \equiv L[(k+k_0)\bar{\beta}_z(t) - k]$ using the wave number of the fundamental $k = \omega/c$, and the averaged electron z velocity $c\bar{\beta}_z$. The initial velocity $\nu_0 \equiv \nu(0)$ is called the "resonance parameter"; when $\nu = 0$, exactly one wavelength of light passes over an electron as it passes through one period of the magnet and the coupling between light and electrons is maximized. The dimensionless phase is $\zeta(t) \equiv [(k+k_0)\bar{z}(t) - \omega t]$, where $\bar{z}(t) \equiv \int_0^t c\bar{\beta}_z(t')\,dt'$; note that $\dot{\zeta} \equiv d\zeta/d\tau = \nu$, where $\tau \equiv tc/L$. ζ describes electron dynamics on the optical wavelength scale. The total beam current is the sum of all single-particle currents which we label by initial positions ζ_0 (spread uniformly) and velocities ν_0; we average over sample electrons $\langle\ \rangle$, then weight this result by the macroscopic particle density ρ_0. Furthermore, we note that in long, periodic magnets, the fractional changes in γ are always small $[\lesssim (2N)^{-1}]$.

The coupled wave and electron equations are, respectively,

$$\dot{a} = -r\langle e^{-if\zeta}\rangle,\quad f\ddot{\zeta} = \tfrac{1}{2}|a|\cos(f\zeta+\phi)\ , \tag{2}$$

where $|a| \equiv 4\pi Nef\mathcal{K}_f(\xi)LE/\gamma_0^2mc^2$, and $r \equiv 8\pi^2Nef\mathcal{K}_f^2(\xi)L^2\rho_0/\gamma_0^3mc^2$, and $a = |a|e^{i\phi}$. The second equation is recognized as the self-consistent pendulum equation. The Bessel functions $\mathcal{K}_f(\xi)$ express the reduced coupling between electrons and light resulting from the time electrons spend in periodic longitudinal motion (instead of transferring energy to the optical wave). A helical magnet has $\mathcal{K}_f(\xi) \rightarrow K$ throughout (2) and $\frac{1}{2}|a| \rightarrow |a|$ in the pendulum equation. Since the pendulum equation is periodic in $f\zeta$, we only need to explore one 2π section of phase space; with the transformation $(\zeta,\nu) \rightarrow (f\zeta, f\nu)$ the pendulum phase space can be transformed into the same phase space of the fundamental $(f=1)$. The separatrix $\nu_s^2 = 2|a| \times [1+\sin(\zeta_s+\phi)]$ is a slowly evolving function of $|a|$ and ϕ which guides electrons into bunches about the $\zeta \approx \pi$ phase; this drives the wave equation and is the gain mechanism.[2,3]

It is instructive to solve (2) for weak fields and low gain. We expand the pendulum equation in weak fields ($|a| \ll 1$) and insert ζ into the wave equation. The resulting gain g (the fractional increase in wave energy $|a|^2$) and phase shift $\Delta\phi$ describe the evolution of the optical wave:

$$\frac{g}{r} = \frac{1}{2}\frac{d}{dx}\left(\frac{\cos x - 1}{x^2}\right)_{x=f\nu_0}\ ,$$
$$\frac{\Delta\phi}{r} = \frac{1}{4}\frac{d}{dx}\left(\frac{\sin x - x}{x^2}\right)_{x=f\nu_0}\ . \tag{3}$$

These are fundamental results, and the effects of stronger fields and higher gain are best described as deviations from these expressions. The maximum weak-field gain occurs at $f\nu_0 = 2.6056$ and the maximum gain is $g = 0.067\,52r$; the gain curve is symmetric in $f\nu_0$ and $\Delta\phi$ is antisymmetric. For large r, a large optical phase shift causes the gain curve to distort and become more symmetric about $\nu_0 = 0$. In strong fields ($|a| \gg 1$), electrons become trapped, the gain curve becomes broader in ν_0, and decreases in height; this is the saturation mechanism.

HARMONICS

We now examine Eqs. (2) with particular attention paid to the possibility of operating in higher harmonics ($f = 3, 5, 7, \ldots$). Several points are explored separately:

(1) The optical wavelength in higher harmonics is given by $\lambda_0(1+\frac{1}{2}K^2)/2\gamma_0^2f$; the tunable range can now be adjusted by f as well as K and γ.

(2) The weak-field, low-gain expression (3) gives us a good indication of many of the scaling results. Maximum gain occurs closer to resonance in higher harmonics than in the fundamental; $\nu_0^{max} = 2.6056/f$. This creates a stiff requirement for the electron beam energy and angular spreads since their initial range in ν_0's must avoid the negative-gain region of the gain curve.

(3) Since the natural energy spread of the electron beam must fit into the narrower gain curve, we must have $\delta\nu_0 \lesssim \pi/f$. In terms of a real fractional energy spread this becomes $\delta\gamma/\gamma \lesssim 1/4Nf$ where N is the number of magnet periods.

(4) For an initial angular spread, there is the similar restriction in f since a change in electron angle also changes ν_0 through $\bar{\beta}_z$. The requirement is $\Delta\theta^2 \lesssim (1+\frac{1}{2}K^2)/2N\gamma^2 f$.

These restrictions on the energy spread $\delta\gamma/\gamma$ and the angular spread $\Delta\theta$ are the most serious problems.

(5) The gain in a free-electron laser is decreased in higher harmonics due to the factor $f\mathcal{K}_f^2$ in r. See Fig. 1. Gain decreases rapidly in f, but the decrease can be diminished using higher values of K. Practical values[10] can reach $K \approx 10$, but $K \approx 2-4$ seems to be adequate to reach higher harmonics.

(6) After a pass through the laser, the final electron energy spread is given by $\delta\gamma/\gamma \approx |a|/8\pi Nf$ in weak fields, and $\delta\gamma/\gamma \approx 1/4Nf$ in strong fields. These results may be important for recirculating electron beams in a storage ring[6] or Van de Graaff.[11]

(7) The laser saturates when $|a| \gtrsim 2\pi$; this gives the final optical-field strength. The optical power at saturation actually increases in the higher harmonics in proportion to $(f\mathcal{K}_f)^{-2}$.

(8) At shorter wavelengths, the optical-mode area in the resonator tends to decrease. The mode area at

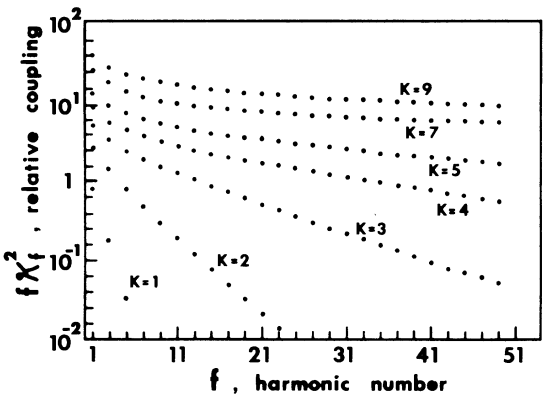

FIG. 1. Gain r is proportional to the new coupling factor $f\mathcal{K}_f^2$ in linearly polarized magnets. In higher harmonics $f=3,5,7,\ldots$ the coupling decreases rapidly unless K is large.

the optical beam waist is $\pi w_0^2 = z_0\lambda/f$, where $2z_0$ is the confocal mirror spacing. As the harmonic number increases, the resonator should be adjusted so that the overlap with the electron beam cross section is maximized. With $f=10$, for example, one could decrease the beam waist w_0 by 2.15 and increase z_0 by 2.15. If the increase in f is shared between w_0^2 and z_0^{-1}, the optical cavity for higher harmonics can be made reasonable.

CONCLUSION

The use of free-electron lasers in higher harmonics is promising; a fixed facility then has a much broader tunable range by another factor of $f \sim 10$ or 20. The major limitation seems to be in the electron beam quality (as usual); the necessary energy and angular spreads decrease with f. A shorter magnet length L may relieve this restriction somewhat. The gain also decreases in higher harmonics, but if $K \approx 2-4$ this penalty does not seem too severe. Van de Graaff free-electron lasers[11] tend to have high gain (larger r because of lower γ_0) and excellent beam quality, but produce long wavelengths ~ 200 μm; higher harmonics may help to reach shorter wavelengths (~ 10 μm) without changing γ_0. Storage rings also have excellent beam quality, but not such large gain (smaller r because of higher γ_0). Even so, with sufficiently high K, higher harmonics could, in principle, extend these free-electron lasers to new shorter wavelengths in the uv and towards x rays. For instance, when γ is increased to achieve an 11-fold decrease in optical wavelength, the normal-gain process ($r \sim \gamma_0^{-3}$ and $\lambda \sim \gamma_0^{-2}$) drops by a factor of 36 (decreasing λ_0 is worse). But, if the $f=11$th harmonic is used with $K=5$, then only a factor of 2.5 in gain is lost; comparisons of higher harmonics are even more dramatic, but the excess beam quality necessary is less likely.

ACKNOWLEDGMENTS

The author wishes to acknowledge helpful discussions with Dr. J. M. J. Madey and Dr. P. Michelson. This research was supported by NASA Grant No. NAG 2-48, NATO Collaborative Grant No. 1876, and Air Force Grant No. AFOSR-81-0061.

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