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Atomic-field bremsstrahlung from 50 –140-keV electrons

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Experimental data are presented for the absolute cross section for atomic-field bremsstrahlung produced by 50-, 100-, and 140-keV electron bombardment of thin targets of aluminum, copper, silver, and gold at photon angles of 30°, 45°, 60°, 90°, and 135°. The data are compared with the theory of Pratt *et al.* and generally found to be in very good agreement, although there are some small discrepancies between theory and experiment.

I. INTRODUCTION

Atomic-field bremsstrahlung has been the subject of several reviews. An early, but still useful, review is the paper of Koch and Motz.[1] A comparison of theory and experiment is presented by Tseng and Pratt[2] in their paper on the exact screened calculations. The most recent review is that by Pratt presented at the X80 Conference.[3] Since 1971 there has been extensive theoretical work done largely by Pratt and his co-workers, culminating in a very complete tabulation of the photon energy spectrum from 1 keV to 2 MeV.[4] Recently, Tseng, Pratt, and Lee[5] have provided angular distributions for several elements spanning the entire range of atomic numbers through calculation of the shape function, the ratio of the energy and angular distribution to the photon energy spectrum from 1 to 500 keV. These calculations provide what is expected to be the best prediction of both the photon energy spectrum, or singly differential cross section $d\sigma/dk$, and the energy and angular distribution of the photon, or doubly differential cross section $d^2\sigma/d\Omega\, dk$. Unlike some other approximations such as the work of Elwert and Haug,[6] Brysk, Zerby, and Penny,[7] and Dugne and Proriol,[8] which are discussed in the reviews, the calculations of Tseng, Pratt and Lee are expected to be valid over the entire range of atomic number and radiated photon energy.

The purpose of this work[9] is to report results for the absolute doubly differential cross section for atomic-field bremsstrahlung $d^2\sigma/d\Omega\, dk$ for electron energies of 50, 100, and 140 keV for targets of Al, Cu, Ag, and Au, and for selected photon angles of 30, 45, 60, 90, and 135°. The results will be compared with the calculations of Pratt. While the present results do not cover all areas of interest for comparison between theory and experiment in the 50–140-keV energy range, they do provide the most extensive comparison available and will serve to point out areas where further experimental work would be useful.

II. EXPERIMENTAL ARRANGEMENT

The basic idea of this experiment is to observe the bremsstrahlung photon energy spectrum as a function of photon angle when electrons of incident energy T bombard a thin atomic target. The electron beam is provided by a 150-keV Cockcroft-Walton accelerator. The beam enters a scattering chamber, passes through a thin target located at the center of the chamber and is collected by a Faraday cup. The electron beam is collimated to a spot size of about 2 mm. The targets are thin films of research grade materials prepared by standard vacuum evaporation techniques and are either self-supporting or supported on thin (5–15-μgm/cm^2) carbon films. Target thickness was determined by direct weighing of a measured area on a Cahn Model G2 electrobalance. Targets are of the order of 50-μgm/cm^2 thick and the uncertainty in thickness ranges from about 5% for Al, Cu, and Au to 15% for Ag.

The photon detector is a planar Ge (Li) detector which has a 0.13-mm Be window and is coupled to the scattering chamber through a 0.64-mm Mylar window. The detector could be positioned at 30, 45, 60, 90, and 135° relative to the incident electron beam. A magnetic electron deflector was placed between the target and the mylar window in order to prevent any electrons scattered in the target from reaching the Mylar window.

The efficiency of the detector was measured using calibrated standard point sources located at the position of the target. The efficiency data from the sources was interpolated by fitting a function of the form suggested by Gallagher and Cipolla.[10] Uncertainty in the absolute efficiency ranged from about 1% at 50 keV to as large as 22% at 133 keV. The larger uncertainties were at photon energies below 25 keV and above 100 keV and thus affect the cross section results differently depending on incident electron and radiated photon energy. The uncertainty in efficiency was the largest systematic error for low photon energy for all incident electron energies and for high photon energy for the highest electron energy. At intermediate photon energies, $k/T=0.6$ for example, the uncertainty in efficiency was typically small and of the order of a few percent.

The data were collected using standard Nim Bin electronics. Spectra for a given angle were collected in a multichannel analyzer. A background run with the target removed was made. The data were read via magnetic tape to a computer where the background run was scaled and subtracted, and the resultant number of bremsstrahlung photon counts per channel was converted to cross section using the formula

$$\frac{k}{Z^2}\frac{d^2\sigma}{d\Omega\,dk}=\frac{k}{Z^2}\frac{N}{N_0}\frac{1}{\Delta k\,\Delta\Omega\,\tau e(k)}\times 10^{27}\,\text{m barn/ster}, \tag{1}$$

where N is the number of photon counts in energy interval Δk, N_0 is the number of electrons incident on the target of atomic number Z, $\Delta\Omega$ is the solid angle, τ the target thickness, and $e(k)$ the detector efficiency.

A typical plot of the photon energy spectrum and the background for 50-keV electrons on silver is shown in Fig. 1. The background was typically from 1 to 10% of the target-in spectrum. One major source of background is the thick target bremsstrahlung produced in the scattering chamber walls by scattered electrons. The region of the chamber viewed through the target by the photon detector was lined with carbon to reduce this background. A second source of background would be due to electrons striking the window in front of the detector. This was reduced by the magnetic deflector described above. In this type of experiment, there is always some concern that a "target-out" run does not adequately provide a measure of background due to the possibility of additional bremsstrahlung in the chamber walls produced by elastically scattered electrons which only occurs with the target in place. A test for this effect was made by displacing the target off center along the electron beam line so that the elastically scattered electrons would be about the same as they would normally be with the target at the center. The photon detector, however, would not see any bremsstrahlung from the target. No significant difference in the background was observed. Hence we believe that we are adequately accounting for the background and properly subtracting for it.

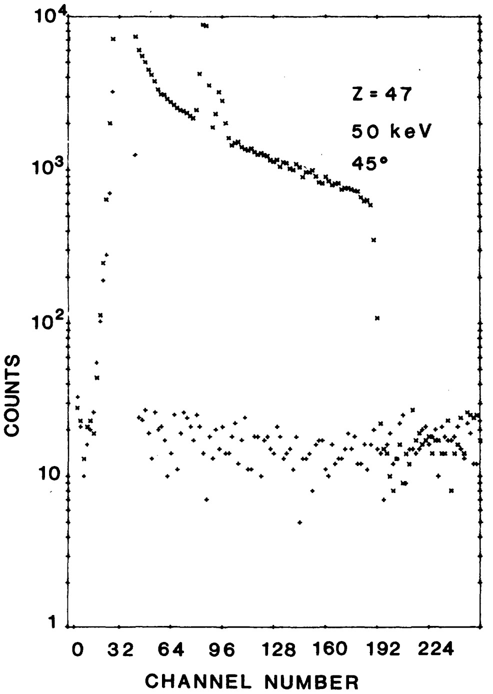

FIG. 1. Plot of typical bremsstrahlung spectrum (×) and the background (+). This case is for 50-keV electrons on silver. The photon angle is 45°.

In summary, the major errors are statistical errors which are typically 5%, target thickness errors which range from 5 to 15% and detector efficiency errors which range from 1 to 22% as discussed above.

III. RESULTS AND DISCUSSION

In order to compare our results with the calculations of Pratt, instead of simply presenting the data for $d^2\sigma/d\Omega\,dk$, we have chosen to calculate the shape function from our data by div-

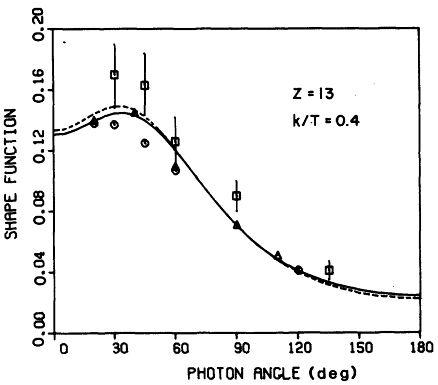

FIG. 2. Shape function S vs photon angle for 50-keV electrons on aluminum for $k/T=0.4$. Squares are this experiment, circles are data of Ref. 11, and triangles are Ref. 12. The solid curve is an interpolation of theoretical data from Tseng *et al.* (Ref. 5). The dashed curve is the first Born approximation.

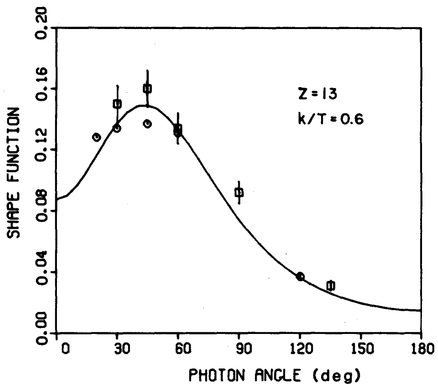

FIG. 3. Same as Fig. 2 except $k/T=0.6$.

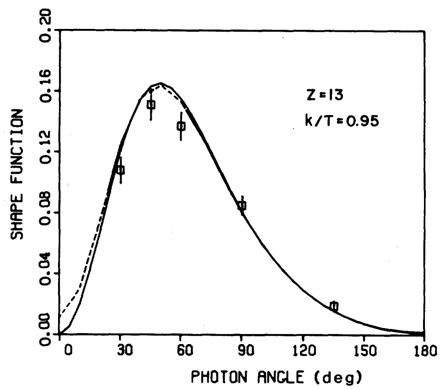

FIG. 5. Same as Fig. 2 except $k/T=0.95$.

iding the data by the photon energy spectrum given by Pratt *et al.*[4] as follows:

$$S(T,Z,\theta,k/T)=\frac{k}{Z^2}\frac{d^2\sigma}{d\Omega\, dk}\Big/\frac{k}{Z^2}\frac{d\sigma}{dk}\,, \tag{2}$$

where k/T is the ratio of photon energy to incident electron energy. The experimental values of S can then be compared directly with the values calculated from Tseng, Pratt, and Lee.[5]

In Figs. 2–5 we have plotted the shape functions S versus photon angle θ for 50-keV Al data for $k/T=0.4$ to 0.95. In Figs. 6–8 we present the data for Au at 50 keV for k/T from 0.4 to 0.8. The squares are our data points. The circles are the data of Rester, Edmonson, and Peasley[11] which was also taken with a high resolution Ge(Li) detector. The triangles are several points from the work of Motz and Placious[12] which were obtained with a Na(I) detector. The error bars shown represent the total error. The errors in the data of Rester and Motz are comparable to those of this experiment. The solid curve in each figure is obtained by interpolating the parameters for the shape functions given by Tseng, Pratt, and Lee[5] for $Z=13$ and $Z=79$. Also shown is a dashed curve which is the shape function calculated from the first Born approximation.[13]

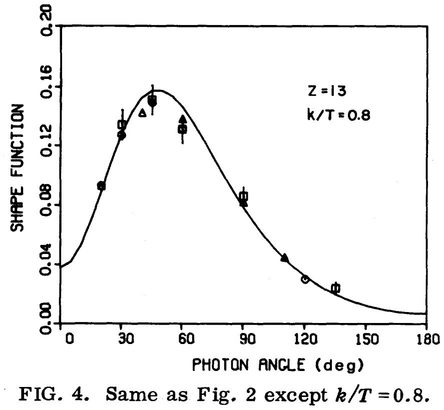

FIG. 4. Same as Fig. 2 except $k/T=0.8$.

Generally the agreement appears to be very good, although there appear to be some systematic differences. The present experiment agrees well with the data of Rester *et al.* and Motz and Placious for all values except for $k/T=0.4$ for Al where our results are about 1 standard deviation higher. This could be due in part to the fact that we have not corrected our data for any effect of electron-electron bremsstrahlung which would enhance the data at lower k/T. A customary correction for electron-electron bremsstrahlung is obtained by replacing Z^2 by $Z(Z+1)$ in the first Born approximation. Since there exists no data on the electron-electron effect which could be used to test this prescription, we have decided not to make any correction. Because the kinetic energy cut off of electron-electron bremsstrahlung is energy and angular dependent[1] we could expect to see a larger cross section when k/T is less than about 0.6. We expect the effect to be smaller for

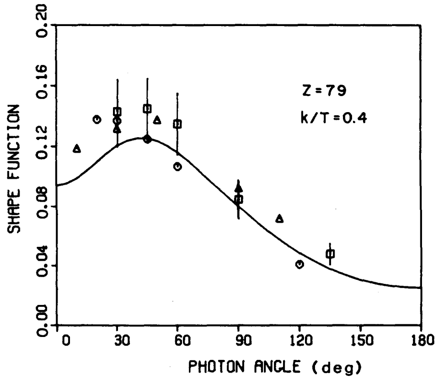

FIG. 6. Shape function S vs photon angle for 50-keV electrons on gold for $k/T=0.4$. Squares are this experiment, circles are data of Ref. 11, and triangles are for Ref. 12. The curve is an interpolation of theoretical data from Tseng *et al.* (Ref. 5).

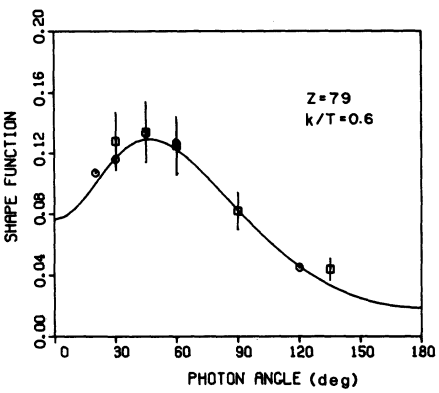

FIG. 7. Same as Fig. 6 except $k/T = 0.6$.

larger Z since it may go as $Z(Z+1)$ which would suggest about an 8% effect for Al and a 1% effect for Au. It is certainly negligible for all cases for k/T greater than 0.6. Hence, this effect may account for the differences observed in Al for $k/T = 0.4$, but it is not as likely to explain the somewhat smaller difference seen in Au at $k/T = 0.4$.

Generally the data are in very good agreement with the theoretical curve calculated from Tseng, Pratt, and Lee,[5] although the data do appear to be somewhat higher at lower k/T (perhaps in part due to the uncorrected electron-electron effect) and lower at high k/T. As pointed out in Ref. 5, the shape function predicted by the first Born approximation agrees well with the exact calculation. However, for Au and $k/T = 0.8$ there is some difference as can be seen from comparing the two curves in Fig. 8. Clearly, the data agree better with the exact calculation where it differs from the first Born approximation.

The comparison between theory and experiment can be made more apparent and more concise by presenting the data in a different way vs k/T. In Fig. 9 we present data for Al for 50, 100, and 140 keV versus k/T. Here we choose to plot the energy spectrum $(k/Z^2)(d\sigma/dk)$ versus k/T. The curve is from Pratt *et al.*[4] The data points are computed from Eq. (2) by dividing our experimental data for $(k/Z^2)(d^2\sigma/d\Omega\, dk)$ by the shape function values computed for the appropriate angle from our interpolation of the values given by Tseng, Pratt, and Lee.[5] This presentation has the advantage of enabling us to plot all the data for a given energy and target regardless of angle on a single curve. In effect we are using the theoretical shape function to transform the doubly differential cross section data into a measure of the singly differential photon energy spectrum. Agreement between theory and experiment is measured by how closely the data points cluster about the curve, independent of angle. It seems clear that within the experimental errors, the data and theory are in very good agreement, except perhaps for a systematic trend most noticeable at 50 keV for the data to be higher at low k/T and lower at high k/T. The data points plotted are for those angles for which the statistical error is

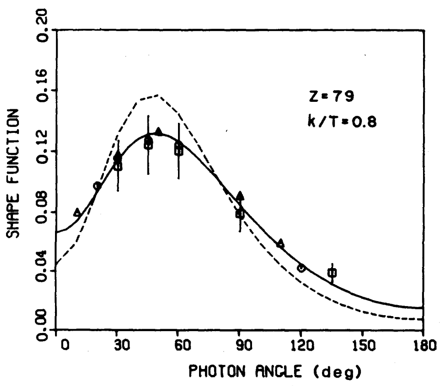

FIG. 8. Same as Fig. 6 except $k/T = 0.8$. The dashed curve is the first Born approximation.

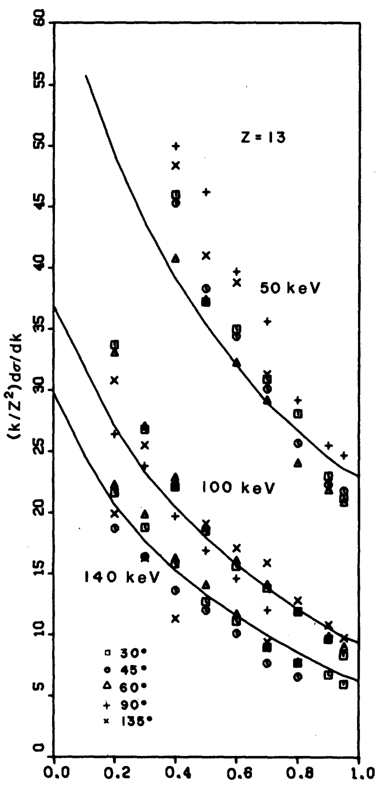

FIG. 9. Plot of $(k/Z^2)\, d\sigma/dk$ for 50-, 100-, and 140-keV electrons on aluminum. Data points are shown for different photon angles. The statistical error on each data point is less than 10%. The curve is from Pratt *et al.* (Ref. 4).

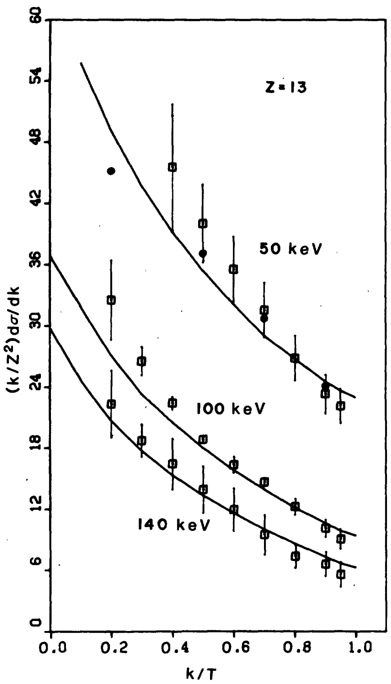

FIG. 10. Plot of $(k/Z^2)d\sigma/dk$ for 50-, 100-, and 140-keV electrons on aluminum. The data points (squares) are averages of the data at different photon angles. The circles are data from Motz and Placious (Ref. 12).

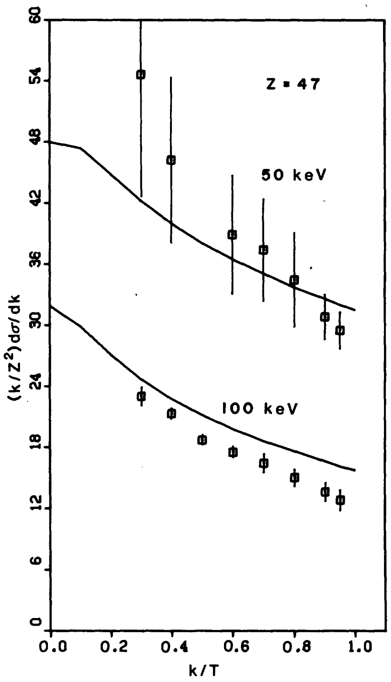

FIG. 12. Same as Fig. 10 except for 50- and 100-keV electrons on silver.

less than 10%. The target thickness error is a scale error and not a point-to-point error in this plot. The efficiency error is a point-to-point error, but is the same for each angle.

To summarize the comparison for all the data of this experiment, we present plots in Figs. 10–13 of the data for $(k/Z^2)(d\sigma/dk)$ computed as des-

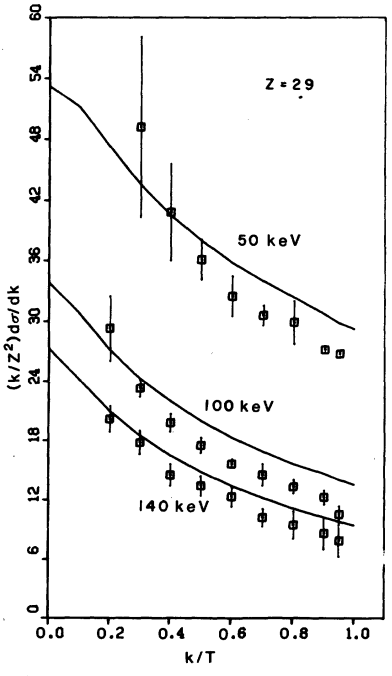

FIG. 11. Same as Fig. 10 except for copper.

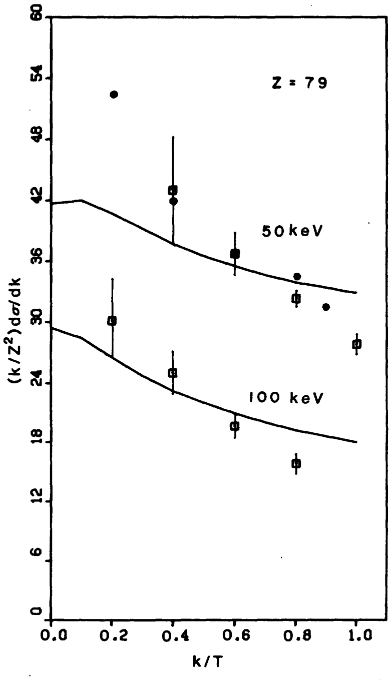

FIG. 13. Same as Fig. 10 except for 50- and 100-keV electrons on gold.

cribed above for Al and *averaged* over photon angle to obtain the best measure of the photon energy spectrum at each energy for each target. The squares are the results of this experiment. The circles are from Motz and Placious.[12] The errors plotted are the combined statistical and detector efficiency errors. The errors in the data of Motz and Placious are comparable. First, it should be again noted that the data and the theory have not been normalized to each other, so that, in general, the absolute agreement between theory and experiment is very good. This agreement also implies good agreement for the shape function calculated from Tseng, Pratt, and Lee over the angular range included, 30° to 135°. The agreement between the present experiment and the results of Motz and Placious at 50 keV is also very good.

There are, however, some apparent systematic differences. The data for copper are systematically lower than the theory at each energy over most of the k/T range. This would be expected if there were simply a scale error in the target thickness of the copper targets used. All the data generally appear to be higher than theory at low k/T and lower at high k/T. This trend is most evident for 50-keV Al (Fig. 10) and for Au (Fig. 13). The trend seems to decrease somewhat at the higher energy. We can offer no conclusive experimental explanation for this trend although several possibilities have been considered. First, the data for k/T less than 0.6 may be enhanced by a contribution from electron-electron bremsstrahlung. This might partly explain the trend seen in the Al data, but it would not explain the Au data where the relative contribution of electron-electron to electron-atom bremsstrahlung is expected to be much smaller. Second, the efficiency of the detector could be underestimated for the lower photon energies (below k of 25 keV). An underestimate of efficiency would raise the apparent cross section for low k/T for 50-keV and could explain the higher values at $k/T = 0.3-0.4$ seen for each Z at 50 keV as well as the higher value at $k/T = 0.2-0.3$ for 100 keV. We have measured the efficiency and believe our results to be correct as presented and within the errors. A check of this explanation could be provided by repeating the measurements for low photon energy with a Si(Li) detector which has a higher efficiency for k less than 25 keV. We were not able to do this in this experiment. Third, there could be some undetected background which enhances the spectrum at lower k/T. This is certainly not unreasonable since any thick target bremsstrahlung background produced by electrons stopping in the window or chamber walls could produce such an enhancement. Again, we believe we have adequately corrected for background and do not believe this to be a significant effect. Furthermore, such an effect would be expected to be larger at higher electron energy which does not appear to be the case. In any case, we do not feel that any of these possibilities would change our results outside the errors given.

IV. CONCLUSIONS

In conclusion, we find generally very good agreement between experiment and the theory of Pratt *et al.*[4,5] over the range of parameters studied in this experiment. There does remain, however, the small systematic trend for the experimental data to be higher than theory at lower k/T and lower at high k/T. We should stress that the agreement is good in absolute value both for the photon energy spectrum and for the shape functions from 30° to 135°. Clearly, several areas remain for further experimental study in this energy region. First, it would be interesting to have data for angles between 0° and 30° where Pratt's calculation for the shape function are more significantly different from that of the first Born approximation. Second, it would be useful to have data at lower photon energy with a higher resolution and more efficient Si(Li) detector to see whether the systematic trend observed here persists.

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