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Use of a scaling relationship for synchrotron radiation

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A single table is presented which can be used with a scaling relationship to predict the intensity of synchrotron radiation for any electron energy, wavelength, and observation angle. The accuracy of the scaling relationship and the table has been tested to a level of 1 part in 10^5 using current values for the fundamental constants.

The radiation emitted in electron storage rings and synchrotrons has become increasingly important for applications in many fields of physics, chemistry, biology, and material sciences.[1-3] The broad range of wavelengths (from the infrared down to the x-ray region), the high degree of polarization, and high intensity provide a number of favorable characteristics.

Recently, we have been concerned with using synchrotron radiation as an intensity source for the absolute calibration of spectrometers in the vacuum-ultraviolet region, cf. Ref. 4. As is well known, the absolute intensity of radiation emitted from an electron storage ring can be expressed in terms of the electron energy and current, the local bending radius of the electron orbit, the wavelength of the observed radiation, and the observation angle. The spectral characteristics can be conveniently parametrized in terms of a critical wavelength near the peak of the radiation distribution in the orbital plane.[1-7] In this article we present a method for determining the intensity using a scaling relationship which allows one to determine the intensity of radiation for any electron energy, wavelength, and observation angle by using just one single table which gives the wavelength in terms of the critical wavelength and using a universal function for the intensity.

Analytic expressions for the intensity of synchrotron radiation exist as functions of the electron energy, bending radius, observation angle, and wavelength[8-10]; however, the equations contain terms involving Bessel functions for which accurate numerical tables or computational algorithms are not widely known. Thus, it is difficult to readily determine the intensity of synchrotron radiation for different operating conditions. Furthermore, we know of no reports in which the basic equations have been tested numerically to an accuracy of better than $1{:}10^3$. Such tests are important for calibration purposes if there are no experimental checks of the theory. The theory itself contains a number of approximations which are not necessarily accurate at the 1 part in 10^3 level.

The angular distribution of the intensity of synchrotron radiation emitted from a beam of relativistic electrons of kinetic energy E in a circular orbit can be obtained from Schott's equation[8,11] for discrete wavelengths based on harmonics of the orbital frequency

$$\frac{dN_E(\lambda,\Psi)}{d\Omega}=i\,\frac{\Delta\lambda}{\lambda}\,I_E(\lambda,\Psi)\,, \tag{1}$$

where the intensity function is given as

$$I_E(\lambda,\Psi)=\frac{9e}{8\epsilon_0 hc}\,\frac{\gamma^6}{(\lambda/\lambda_c)^2}\,[\beta^4 J_\nu'^2(\beta\nu\cos\Psi) + \beta^2\tan^2\Psi\, J_\nu^2(\beta\nu\cos\Psi)]\,, \tag{2}$$

with

$$\beta=(1-\gamma^{-2})^{1/2}\,, \tag{3}$$

$$\gamma=\frac{E}{mc^2}\,, \tag{4}$$

$$\nu=\frac{2\pi R}{\lambda}\,, \tag{5}$$

and

$$\lambda_c=\frac{4\pi R}{3\gamma^3}\,. \tag{6}$$

The derivative of J_ν is taken with respect to the complete argument $\beta\nu\cos\Psi$. The angular distribution $dN_E(\lambda,\Psi)/d\Omega$ is given in units of photons per second per steradian, where Ψ is the angle between the line of observation and its projection on the orbital plane which is tangent to the orbital ring, λ is the wavelength, $\Delta\lambda$ is the bandwidth, R is the bending radius of the electron orbit, and i is the current. In the derivation of Eq. (2) it is assumed that quantum effects can be neglected, the electrons emit incoherently, the diameter of the electron beam is infinitesimal, and the observation distance is large compared to the bending radius. In generating the table the values for the fundamental constants e, m, h, c, and ϵ_0 were taken from Cohen and Taylor.[12]

In general, if one wishes to calculate the intensity $dN_E(\lambda,\Psi)/d\Omega$ for a particular set of values of E, λ, and Ψ, one needs to evaluate the Bessel function $J_\nu(\nu z)$ and its derivative $J_\nu'(\nu z)$ for large orders of ν. Since these functions change rapidly with z, our calculations were performed using 16 significant digits.

The widely known approximate formula for synchrotron radiation derived independently by Schwinger[9] and Ivanenko and Sokolov[10] in which asymptotic representations of the Bessel functions of high order are used is given as

$$I_E(\lambda, \Psi) \simeq \frac{3e}{8\pi^2\epsilon_0 hc} \frac{\gamma^2}{(\lambda/\lambda_c)^2}(1+X^2)^2 \times \left(K^2_{2/3}(\xi) + \frac{X^2}{1+X^2}K^2_{1/3}(\xi)\right), \quad (7)$$

where

$$X = \gamma\Psi, \quad (8)$$

$$\xi = \frac{1}{2(\lambda/\lambda_c)}(1+X^2)^{3/2}, \quad (9)$$

and $K_{1/3}$ and $K_{2/3}$ are modified Bessel functions of the second kind. Inspection of Eq. (7) shows that by using the dimensionless parameters $\tilde{\lambda} = \lambda/\lambda_c$ and $\tilde{\Psi} = \tilde{E}\Psi$ one can write a scaling relationship

$$I_E(\lambda, \Psi) = \tilde{E}^2 \tilde{I}(\tilde{\lambda}, \tilde{\Psi}), \quad (10)$$

where $\tilde{E}$ is the energy in GeV and $\tilde{\Psi}$ is in rad. Thus, if one table of the universal intensity function $\tilde{I}(\tilde{\lambda}, \tilde{\Psi})$ is available for a large range of $\tilde{\lambda}$ and $\tilde{\Psi}$ one can scale the results and produce a table for the intensity function for any electron energy.

Equation (10) is similar in form to parametric equations derived by Green,[7] some of which were reprinted in Ref. 1; however, our equation is more compact and incorporates the angular dependence of the intensity directly. The results of Green presented in Ref. 1 for $\Psi = 0$ only are in graphical form and are accurate to only 10–20%. Reference 7, containing more extensive graphs, is unpublished and not widely available. Our results are accurate to better than 1 part in 10^5.

For practical calculations the critical wavelength λ_c can be expressed as

$$\lambda_c(\text{Å}) = 5.5893 \frac{R}{\tilde{E}^3}, \quad (11)$$

where R is in meters.

The polarization fraction of the intensity is given by

$$P = \frac{I_\parallel - I_\perp}{I_\parallel + I_\perp}, \quad (12)$$

where $I_\parallel$ and $I_\perp$ are the parallel and perpendicular components, respectively, with respect to the orbital plane containing the electron acceleration vector.

Table I lists the universal intensity function $\tilde{I}(\tilde{\lambda}, \tilde{\Psi})$ and the polarization fraction. Figure 1 shows $\tilde{I}(\tilde{\lambda}, \tilde{\Psi})$ vs $\tilde{\lambda}$ for several observation angles $\tilde{\Psi}$.

Precise numerical analysis shows that the scaling relationship, Eq. (10), and $I_E(\lambda, \Psi)$ found from Eq. (7) agree with the more accurate formula Eq. (2) to within less than (a) 1:10^5 for $\tilde{E} \geq 0.5$ and $\tilde{\lambda} \leq 10^4 \tilde{E}^3$, and (b) 1:$10^3$ for $\tilde{E} \geq 0.05$ and $\tilde{\lambda} \leq 10^6 \tilde{E}^3$. Uncertainties in the fundamental constants e and m limit the accuracy of the calculations to five significant figures in general and to three significant figures for values of $\tilde{\lambda}$ which are less than 0.1, see Table I.

A complete table for a particular storage ring can be obtained from the universal Table I by replacing the $\tilde{\lambda}$ and $\tilde{\Psi}$ scales by actual λ and Ψ scales. The wavelengths λ are found by multiplying the $\tilde{\lambda}$ values in Table I by the λ_c for the storage ring considered. The angles Ψ are found by dividing the $\tilde{\Psi}$ values in Table I by $\tilde{E}$.

As an example of the use of Table I we consider calculating the intensity $dN_E/d\Omega$ for the Stanford Synchrotron Radiation Laboratory[1] (SSRL) operating at 2.5 GeV. To determine the intensity of 9 Å radiation emitted at $\Psi = 0.2$ mrad, we let

$E = 2.5$ GeV ($\tilde{E} = 2.5$),

$R = 12.7$ m (radius of curvature),

$i = 10$ mA (current of electrons),

$\Delta\lambda/\lambda = 0.01$ (1% bandwidth averaging linearly).

The intensity from Eqs. (1) and (10) is

$$\frac{dN_E}{d\Omega} = i\frac{\Delta\lambda}{\lambda} I_E(\lambda, \Psi),$$

where

$$I_E(\lambda, \Psi) = \tilde{E}^2 \tilde{I}(\tilde{\lambda}, \tilde{\Psi}).$$

We note that $\tilde{\lambda} = \lambda/\lambda_c \approx 2.0$ since $\lambda_c = 4.54$ Å from Eq. (11), and

$$\tilde{\Psi} = \tilde{E}\Psi = 5 \times 10^{-4} \text{ rad}.$$

From Table I $\tilde{I}(\tilde{\lambda}, \tilde{\Psi}) = 1.2082 \times 10^{22}$ photons/sec A sr from which $dN_E/d\Omega \simeq 7.55 \times 10^{18}$ photons/sec sr.

Practical calculations are concerned with the flux of radiation passing through an aperture. An expression for the flux through a rectangular aperture centered in the orbital plane and subtending an acceptance angle $\Delta\phi$ in the orbital plane and an opening angle $2\Psi_0$ perpendicular to the orbital plane is given by

$$N_E(\lambda, \Psi_0) = i\frac{\Delta\lambda}{\lambda}\Delta\phi F_E(\lambda, \Psi_0), \quad (13)$$

where

TABLE I. Universal intensity function $\tilde{I}(\tilde{\lambda},\tilde{\Psi})$ in photons/sec A sr and polarization fraction P (in parentheses) using Eqs. (2)–(6) and (12). The angle $\tilde{\Psi}=\tilde{E}\Psi$ is in rad, where $\tilde{E}$ is the electron energy in GeV. The parameter $\tilde{\lambda}=\lambda/\lambda_c$, where λ_c is the critical wavelength. A number given as 1.2345 20 is to be interpreted as 1.2345×10^{20}. A complete table for a particular storage ring can be obtained by replacing $\tilde{\lambda}$ by λ and $\tilde{\Psi}$ by Ψ. The wavelength scale λ is obtained by multiplying $\tilde{\lambda}$ by λ_c. The angle scale Ψ is obtained by dividing $\tilde{\Psi}$ by $\tilde{E}$.

$\tilde{\lambda}$ \ $\tilde{\Psi}$	0.0 0	1.00 -4	2.00 -4	3.00 -4	4.00 -4	5.00 -4	6.00 -4	7.00 -4	8.00 -4	9.00 -4	1.00 -3
2.0 -2	4.04 2 (1.00000)	2.35 1 (0.92970)	3.27 -3 (0.76767)	4.20 -10 (0.59469)	1.22 -20 (0.45191)						
3.0 -2	4.68 9 (1.00000)	7.15 8 (0.93011)	2.01 6 (0.76873)	5.43 1 (0.59603)	5.47 -6 (0.45318)	6.57 -16 (0.34626)					
4.0 -2	1.467 13 (1.00000)	3.63 12 (0.93051)	4.58 10 (0.76977)	1.795 7 (0.59734)	1.062 2 (0.45443)	4.01 -6 (0.34730)	3.74 -16 (0.26949)				
5.0 -2	1.748 15 (1.00000)	5.78 14 (0.93090)	1.797 13 (0.77079)	3.49 10 (0.59864)	2.382 6 (0.45566)	2.84 0 (0.34834)	2.77 -8 (0.27031)	9.85 -19 (0.21362)			
6.0 -2	4.099 16 (1.00000)	1.643 16 (0.93128)	9.31 14 (0.77180)	5.27 12 (0.59993)	1.832 9 (0.45689)	2.185 4 (0.34937)	4.73 -3 (0.27113)	9.48 -12 (0.21426)			
8.0 -2	1.996 18 (1.00000)	1.018 18 (0.93202)	1.222 17 (0.77375)	2.633 15 (0.60244)	7.00 12 (0.45930)	1.481 9 (0.35140)	1.545 4 (0.27275)	4.79 -3 (0.21553)	2.614 -11 (0.17343)		
1.0 -1	1.959 19 (1.00000)	1.154 19 (0.93273)	2.171 18 (0.77565)	1.041 17 (0.60489)	9.39 14 (0.46167)	1.115 12 (0.35341)	1.189 8 (0.27435)	7.59 2 (0.21679)	1.909 -4 (0.17441)	1.231 -12 (0.14270)	
1.5 -1	3.719 20 (1.00000)	2.655 20 (0.93439)	9.077 19 (0.78012)	1.265 19 (0.61075)	5.80 17 (0.46738)	6.874 15 (0.35829)	1.625 13 (0.27829)	5.85 9 (0.21988)	2.430 5 (0.17683)	8.74 -1 (0.14461)	2.040 -7 (0.12007)
2.0 -1	1.4989 21 (1.00000)	1.1770 21 (0.93592)	5.417 20 (0.78428)	1.284 20 (0.61627)	1.327 19 (0.47285)	4.962 17 (0.36302)	5.524 15 (0.28212)	1.494 13 (0.22292)	7.96 9 (0.17922)	6.766 5 (0.14649)	7.37 0 (0.12156)
3.0 -1	5.435 21 (1.00000)	4.689 21 (0.93863)	2.899 21 (0.79181)	1.166 21 (0.62646)	2.710 20 (0.48311)	3.193 19 (0.37201)	1.671 18 (0.28952)	3.393 16 (0.22883)	2.323 14 (0.18390)	4.658 11 (0.15020)	2.367 8 (0.12452)
4.0 -1	9.611 21 (1.00000)	8.685 21 (0.94099)	6.209 21 (0.79848)	3.248 21 (0.63570)	1.1293 21 (0.49261)	2.359 20 (0.38049)	2.676 19 (0.29657)	1.4873 18 (0.23452)	3.648 16 (0.18845)	3.552 14 (0.15383)	1.2327 12 (0.12743)
5.0 -1	1.2963 22 (1.00000)	1.2036 22 (0.94308)	9.376 21 (0.80447)	5.732 21 (0.64415)	2.5352 21 (0.50146)	7.460 20 (0.38850)	1.3449 20 (0.30332)	1.367 19 (0.24002)	7.210 17 (0.19287)	1.811 16 (0.15737)	1.990 14 (0.13028)
6.0 -1	1.5389 22 (1.00000)	1.4543 22 (0.94495)	1.1985 22 (0.80990)	8.121 21 (0.65195)	4.212 21 (0.50975)	1.5567 21 (0.39610)	3.820 20 (0.30980)	5.807 19 (0.24535)	5.099 18 (0.19718)	2.410 17 (0.16085)	5.711 15 (0.13309)
8.0 -1	1.8165 22 (1.00000)	1.7538 22 (0.94817)	1.5479 22 (0.81944)	1.1901 22 (0.66591)	7.521 21 (0.52489)	3.689 21 (0.41024)	1.3299 21 (0.32202)	3.340 20 (0.25551)	5.546 19 (0.20548)	5.774 18 (0.16759)	3.573 17 (0.13857)
1.0 0	1.9276 22 (1.00000)	1.8840 22 (0.95088)	1.7300 22 (0.82759)	1.4322 22 (0.67814)	1.0173 22 (0.53847)	5.906 21 (0.42318)	2.6787 21 (0.33340)	9.087 20 (0.26510)	2.2105 20 (0.21341)	3.694 19 (0.17408)	4.067 18 (0.14388)
1.5 0	1.9196 22 (1.00000)	1.9046 22 (0.95614)	1.8376 22 (0.84385)	1.6720 22 (0.70331)	1.3823 22 (0.56730)	1.0016 22 (0.45144)	6.155 21 (0.35885)	3.1127 21 (0.28700)	1.2580 21 (0.23179)	3.9498 20 (0.18934)	9.363 19 (0.15650)

TABLE I. (*Continued.*)

$\tilde{\lambda}$ \ $\tilde{\psi}$	0.0 0	5.00 -4	1.00 -3	1.50 -3	2.00 -3	2.50 -3	3.00 -3	3.50 -3	4.00 -3	4.50 -3	5.00 -3
2.0 0	1.7973 22 (1.00000)	1.2082 22 (0.47531)	4.1351 20 (0.16830)	4.1141 16 (0.07578)	7.923 8 (0.04142)	1.870 -4 (0.02573)					
3.0 0	1.5441 22 (1.00000)	1.3142 22 (0.51412)	1.6291 21 (0.18988)	3.960 18 (0.08538)	3.1239 13 (0.04612)	1.2843 5 (0.02830)	4.325 -8 (0.01897)				
4.0 0	1.3470 22 (1.00000)	1.2744 22 (0.54490)	2.9805 21 (0.20926)	3.5739 19 (0.09450)	5.7003 15 (0.05070)	3.0916 9 (0.03084)	1.4301 0 (0.02049)	1.4014-13 (0.01452)			
5.0 0	1.1977 22 (1.00000)	1.1995 22 (0.57027)	4.0821 21 (0.22689)	1.2729 20 (0.10320)	1.2326 17 (0.05517)	1.2510 12 (0.03334)	4.416 4 (0.02201)	1.7836 -6 (0.01550)	2.6981-20 (0.01145)		
6.0 0	1.0820 22 (1.00000)	1.1207 22 (0.59173)	4.8782 21 (0.24307)	2.8729 20 (0.11152)	9.254 17 (0.05952)	6.618 13 (0.03580)	4.2037 7 (0.02351)	9.4027 -2 (0.01647)	2.9241-13 (0.01211)		
8.0 0	9.1445 21 (1.00000)	9.8131 21 (0.62649)	5.7665 21 (0.27195)	7.4984 20 (0.12716)	1.0842 19 (0.06794)	8.900 15 (0.04063)	2.0941 11 (0.02647)	7.0760 4 (0.01840)	1.7130 -4 (0.01343)	1.4785-15 (0.01019)	
1.0 1	7.9855 21 (1.00000)	8.7077 21 (0.65378)	6.0873 21 (0.29718)	1.2698 21 (0.14166)	4.5161 19 (0.07600)	1.6025 17 (0.04533)	3.2929 13 (0.02939)	2.2585 8 (0.02031)	2.9682 1 (0.01474)	4.2809 -8 (0.01113)	3.8748-19 (0.00866)
1.5 1	6.1943 21 (1.00000)	6.8343 21 (0.70285)	5.9396 21 (0.34904)	2.3117 21 (0.17396)	2.7228 20 (0.09480)	6.7963 18 (0.05659)	2.5104 16 (0.03647)	9.5317 12 (0.02499)	2.5748 8 (0.01797)	3.4173 2 (0.01344)	1.5370 -5 (0.01037)
2.0 1	5.1512 21 (1.00000)	5.6806 21 (0.73638)	5.4462 21 (0.39001)	2.8785 21 (0.20190)	6.1548 20 (0.11196)	4.0704 19 (0.06722)	6.3716 17 (0.04329)	1.7995 15 (0.02955)	6.9677 11 (0.02114)	2.8051 7 (0.01572)	8.892 1 (0.01206)
3.0 1	3.9576 21 (1.00000)	4.3313 21 (0.78056)	4.5177 21 (0.45220)	3.2134 21 (0.24866)	1.2420 21 (0.14251)	2.1720 20 (0.08688)	1.4394 19 (0.05623)	3.0224 17 (0.03834)	1.6771 15 (0.02732)	2.0476 12 (0.02020)	4.5751 8 (0.01539)
4.0 1	3.2768 21 (1.00000)	3.5580 21 (0.80917)	3.8354 21 (0.49832)	3.1430 21 (0.28691)	1.6273 21 (0.16917)	4.6193 20 (0.10478)	6.2931 19 (0.06835)	3.6011 18 (0.04674)	7.5620 16 (0.03330)	5.0837 14 (0.02456)	9.5348 11 (0.01866)
5.0 1	2.8286 21 (1.00000)	3.0507 21 (0.82961)	3.3378 21 (0.53454)	2.9650 21 (0.31924)	1.8250 21 (0.19287)	6.9187 20 (0.12126)	1.4514 20 (0.07977)	1.5151 19 (0.05478)	7.0688 17 (0.03908)	1.3223 16 (0.02882)	8.8883 13 (0.02187)
6.0 1	2.5075 21 (1.00000)	2.6892 21 (0.84514)	2.9629 21 (0.56407)	2.7696 21 (0.34719)	1.9096 21 (0.21422)	8.7706 20 (0.13654)	2.4521 20 (0.09058)	3.8204 19 (0.06250)	3.0344 18 (0.04469)	1.1228 17 (0.03299)	1.7674 15 (0.02502)
8.0 1	2.0725 21 (1.00000)	2.2034 21 (0.86746)	2.4372 21 (0.60993)	2.4161 21 (0.39359)	1.9133 21 (0.25147)	1.1147 21 (0.16416)	4.4577 20 (0.11063)	1.1450 20 (0.07709)	1.7677 19 (0.05545)	1.5341 18 (0.04105)	6.9937 16 (0.03117)
1.0 2	1.7872 21 (1.00000)	1.8879 21 (0.88297)	2.0858 21 (0.64440)	2.1334 21 (0.43106)	1.8303 21 (0.28321)	1.2276 21 (0.18864)	6.0781 20 (0.12893)	2.1058 20 (0.09070)	4.8423 19 (0.06564)	7.0065 18 (0.04878)	6.0450 17 (0.03712)
1.5 2	1.3650 21 (1.00000)	1.4207 21 (0.90733)	1.5639 21 (0.70339)	1.6543 21 (0.50081)	1.5688 21 (0.34641)	1.2638 21 (0.23998)	8.2953 20 (0.16883)	4.2757 20 (0.12127)	1.6704 20 (0.08908)	4.7749 19 (0.06687)	9.6389 18 (0.05123)

TABLE I. (*Continued.*)

$\tilde{\lambda}$ \ $\tilde{\psi}$	0.0 0	2.00 -3	4.00 -3	6.00 -3	8.00 -3	1.00 -2	1.20 -2	1.40 -2	1.60 -2	1.80 -2	2.00 -2
2.0 2	1.1272 21 (1.00000)	1.3502 21 (0.39466)	2.8575 20 (0.11012)	1.3767 18 (0.04026)	2.7058 13 (0.01849)	3.7519 5 (0.00994)	6.2091 -7 (0.00596)				
3.0 2	8.6050 20 (1.00000)	1.0536 21 (0.46548)	4.3675 20 (0.14684)	1.4083 19 (0.05619)	1.1252 16 (0.02617)	6.9875 10 (0.01410)	1.0384 3 (0.00844)	1.1263 -8 (0.00545)			
4.0 2	7.1043 20 (1.00000)	8.6896 20 (0.51630)	4.9838 20 (0.17825)	4.1443 19 (0.07094)	2.1092 17 (0.03355)	2.7709 13 (0.01817)	3.9013 7 (0.01087)	2.4228 -1 (0.00701)	2.7192-12 (0.00479)		
5.0 2	6.1228 20 (1.00000)	7.4387 20 (0.55529)	5.1474 20 (0.20572)	7.5392 19 (0.08470)	1.1645 18 (0.04065)	9.5454 14 (0.02214)	2.0618 10 (0.01328)	5.7810 3 (0.00857)	1.0319 -5 (0.00584)	5.7370-17 (0.00417)	
6.0 2	5.4223 20 (1.00000)	6.5347 20 (0.58652)	5.1003 20 (0.23014)	1.0876 20 (0.09759)	3.5194 18 (0.04750)	9.7746 15 (0.02603)	1.3035 12 (0.01565)	4.6337 6 (0.01010)	2.4279 -1 (0.00689)	1.0341-10 (0.00491)	
8.0 2	4.4762 20 (1.00000)	5.3109 20 (0.63407)	4.7775 20 (0.27208)	1.6234 20 (0.12125)	1.3224 19 (0.06052)	1.6876 17 (0.03358)	2.1895 14 (0.02030)	1.8619 10 (0.01314)	6.6650 4 (0.00897)	6.4347 -3 (0.00639)	1.0720-11 (0.00471)
1.0 3	3.8576 20 (1.00000)	4.5159 20 (0.66910)	4.3926 20 (0.30721)	1.9675 20 (0.14256)	2.7850 19 (0.07274)	8.8689 17 (0.04083)	4.5049 15 (0.02484)	2.5735 12 (0.01612)	1.1616 8 (0.01102)	2.9040 2 (0.00785)	2.8147 -5 (0.00579)
1.5 3	2.9440 20 (1.00000)	3.3614 20 (0.72771)	3.5774 20 (0.37567)	2.2971 20 (0.18822)	6.7705 19 (0.10046)	7.2867 18 (0.05787)	2.2823 17 (0.03572)	1.6520 15 (0.02337)	2.1880 12 (0.01604)	4.1899 8 (0.01146)	9.1545 3 (0.00846)
2.0 3	2.4303 20 (1.00000)	2.7281 20 (0.76491)	3.0056 20 (0.42672)	2.2948 20 (0.22603)	9.7285 19 (0.12497)	1.9219 19 (0.07356)	1.4937 18 (0.04601)	3.8472 16 (0.03034)	2.7596 14 (0.02093)	4.6244 11 (0.01499)	1.5168 8 (0.01108)
3.0 3	1.8547 20 (1.00000)	2.0370 20 (0.81088)	2.2940 20 (0.49992)	2.0617 20 (0.28641)	1.2502 20 (0.16699)	4.5217 19 (0.10177)	8.7081 18 (0.06510)	7.9747 17 (0.04356)	3.0966 16 (0.03033)	4.5397 14 (0.02186)	2.2351 12 (0.01622)
4.0 3	1.5310 20 (1.00000)	1.6587 20 (0.83900)	1.8736 20 (0.55129)	1.8135 20 (0.33354)	1.3094 20 (0.20227)	6.3920 19 (0.12667)	1.9353 19 (0.08254)	3.3395 18 (0.05594)	3.0157 17 (0.03928)	1.3073 16 (0.02848)	2.4933 14 (0.02122)
5.0 3	1.3194 20 (1.00000)	1.4159 20 (0.85836)	1.5958 20 (0.59009)	1.6079 20 (0.37199)	1.2853 20 (0.23268)	7.4989 19 (0.14900)	2.9757 19 (0.09864)	7.5051 18 (0.06759)	1.1242 18 (0.04784)	9.3386 16 (0.03487)	4.0131 15 (0.02608)
6.0 3	1.1684 20 (1.00000)	1.2450 20 (0.87267)	1.3978 20 (0.62079)	1.4426 20 (0.40430)	1.2317 20 (0.25941)	8.0826 19 (0.16927)	3.8382 19 (0.11360)	1.2463 19 (0.07861)	2.6153 18 (0.05604)	3.3508 17 (0.04105)	2.4746 16 (0.03082)
8.0 3	9.6447 19 (1.00000)	1.0176 20 (0.89271)	1.1328 20 (0.66693)	1.1991 20 (0.45623)	1.1075 20 (0.30466)	8.3975 19 (0.20496)	4.9838 19 (0.14072)	2.2170 19 (0.09904)	7.0867 18 (0.07149)	1.5602 18 (0.05286)	2.2666 17 (0.03996)
1.0 4	8.3115 19 (1.00000)	8.7103 19 (0.90626)	9.6208 19 (0.70046)	1.0303 20 (0.49673)	9.9434 19 (0.34199)	8.2043 19 (0.23568)	5.5581 19 (0.16485)	2.9838 19 (0.11768)	1.2270 19 (0.08586)	3.7365 18 (0.06400)	8.1446 17 (0.04868)
1.5 4	6.3428 19 (1.00000)	6.5788 19 (0.92698)	7.1576 19 (0.75573)	7.7310 19 (0.56907)	7.8593 19 (0.41338)	7.2210 19 (0.29771)	5.8153 19 (0.21568)	4.0017 19 (0.15827)	2.2988 19 (0.11800)	1.0778 19 (0.08945)	4.0322 18 (0.06894)

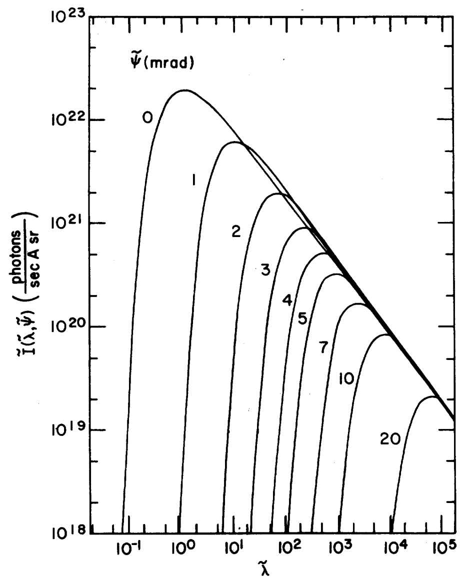

FIG. 1. Spectral distribution for several angles $\tilde{\Psi}$ of the universal intensity function $\tilde{I}(\tilde{\lambda}, \tilde{\Psi})$ as a function of $\tilde{\lambda} = \lambda/\lambda_c$, where λ_c is the critical wavelength. For an electron energy E, the intensity $dN_E(\lambda, \Psi)/d\Omega$ in photons/sec sr per bandwidth $(\Delta\lambda/\lambda)$ is found by multiplying $\tilde{I}(\tilde{\lambda}, \tilde{\Psi})$ by $i(\Delta\lambda/\lambda)\tilde{E}^2$, where $\tilde{E}$ is the electron energy in GeV. The angle $\tilde{\Psi}$ is found by multiplying Ψ by $\tilde{E}$.

$$F_E(\lambda, \Psi_0) = \int_{-\Psi_0}^{\Psi_0} I_E(\lambda, \Psi) \cos\Psi \, d\Psi . \qquad (14)$$

A scaling relationship also holds to a good approximation for the flux function $F_E(\lambda, \Psi_0)$ for those cases where the radiation pattern is confined within a small angle to the orbital plane. This scaling law is linear in $\tilde{E}$, that is

$$F_E(\lambda, \Psi_0) = \tilde{E}\tilde{F}(\tilde{\lambda}, \tilde{\Psi}_0) , \qquad (15)$$

and agrees with Eq. (14) to a few parts in 10^5 for $\tilde{\lambda} \leq 5 \times 10^5 \, \tilde{E}^3$.

A table for the universal flux function $\tilde{F}(\tilde{\lambda}, \tilde{\Psi}_0)$ and a more extensive one for $\tilde{I}(\tilde{\lambda}, \tilde{\Psi})$ accurate to five significant figures will be published elsewhere.[13]

It should be mentioned that in practice a main source of error will be due to the limited accuracy to which the electron motion in a storage ring can be parametrized. The high accuracy of our tables, however, will assist in checking computational algorithms in addition to providing an overview of the spectral characteristics.

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