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Corrections to scaling in the susceptibility of xenon

Hannes Güttinger and David S. Cannell
Department of Physics, University of California at Santa Barbara, Santa Barbara, California 93106
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We report the first measurements of the susceptibility of a pure fluid near its critical point made with sufficient accuracy to directly reveal the existence of corrections to scaling. The measurements were made on the critical isochore for $T > T_c$, in the range $9.6\times 10^{-5} \leq t \leq 10^{-1}$, and are described to within 0.11% rms deviation by the expression $\chi = \Gamma^{+}t^{-\gamma}(1 + a_1 t^{\Delta} + a_2 t^{2\Delta} + a_3 t^{3\Delta})$, with Δ fixed at 0.496, $\gamma = 1.246 \pm 0.010$, and $a_1 = 1.6 \pm 0.3$. The measurements are consistent with either the high-temperature series result $\gamma = 1.250$, or the renormalization-group result $\gamma = 1.241$. With γ fixed at 1.241, the value assumed by a_1 is 1.3 ± 0.2. The effects of revised scaling are considered using a parametric model, and are found not to affect the results for Γ^{+}, a_1, or γ.

I. INTRODUCTION

The concept of universality[1,2] as refined by renormalization-group calculations[3] is now a central unifying theme in the area of critical phenomena. The behavior of systems involving isotropic short-range forces is understood to be governed only by d, the dimensionality of the system and n, the number of degrees of freedom of the order parameter, provided the system is sufficiently close to the critical point. In this asymptotic region, the singular part of the appropriate free energy assumes a simple scaling form, being a generalized homogeneous function of the relevant scaling fields, as first conjectured by Widom.[4] Consequently, simple power-law dependence on one or another scaling field is observed for various quantities, along well-defined thermodynamic paths, and a number of relationships between the exponents are predicted. In addition, renormalization-group theory provides accurate numerical predictions[5–7] for all the exponents. In general, power-law behavior is observed experimentally, and the predicted scaling relationships between the exponents are observed to hold.[8] However, in almost all cases the observed exponents differ slightly, but significantly, from the predicted values. These differences are now believed to be a consequence of the fact that, of necessity, the experiments are performed in a finite range away from the critical point, rather than asymptotically close to it, and it has been shown[9] that the scaling relations should be obeyed even by the effective exponents observed experimentally.

The idea that the observed behavior should depend slightly on the range in which the experiments are performed was advanced a number of years ago on both experimental and theoretical grounds. Experimentally, Greywall and Ahlers[10] observed that the superfluid density of ^{4}He as a function of $|T - T_\lambda|$, did not exhibit the same behavior at different pressures, and that at higher pressures a simple power law was inadequate to describe the data. Prior to this, systematic deviations from power-law behavior had been observed in measurements of the coexistence curve of xenon,[11] and were also observed for SF_6,[12] at about the same time. Wegner[13] showed that such deviations were predicted by the renormalization-group theory, and obtained an expression for the modified power laws which were predicted to hold over a finite range rather than merely asymptotically. Along the appropriate thermodynamic path, the predicted behavior for any quantity $f(t)$ is of the form

$$f(t) = A|t|^{-\lambda}[1 + a|t|^{\Delta_1} + b|t|^{\Delta_2} + O(|t|^{2\Delta_i})], \quad (1)$$

where t is one of the relevant scaling fields, and the correction terms $a|t|^{\Delta_1}$ and $b|t|^{\Delta_2}$ are the result of including two irrelevant scaling fields. The exponents Δ_1 and Δ_2 are expected to be universal, i.e., the same for all systems having the same values for n and d.

The experimental evidence in support of this picture is of two different types. First, there are several instances in which a single system under different conditions, or a set of systems all of which should, in principle, belong to a given universality class, fail to exhibit the expected universal behavior unless the analysis is expanded to include at least one Wegner correction. The thermal expansion coefficient of ^{4}He near the λ point, measured, at different pressures,[14,15] and to some extent the heat capacities[16] of ^{4}He and of various magnetic systems[17] fall into this category. Second, and more convincingly, there exist experiments which directly reveal significant systematic departures from simple power-law behavior, even after adequate allowance for analytic background contributions. The coexistence curve measurements for several pure fluids[11,12,18–21] and that of the superfluid density of ^{4}He near the λ point[10] are of this nature.

Less compelling, but similar evidence exists for the susceptibility of several pure fluids, where the exponent γ deduced from *PVT* measurements[8] is typically 1.19, while light-scattering measurements,[22,23] nearer to the critical point, give $\gamma=1.22$. This dependence of the effective exponent on the range in which the measurements are made is a consequence of Eq. (1). Additional evidence that γ increases even further in the immediate vicinity of the critical point has been provided by the measurements of Hocken and Moldover[24] who found γ's of 1.23, 1.28, and 1.24 for Xe, SF_6, and CO_2 in a restricted range quite close to T_c. The average value of 1.25 agrees rather well with the theoretical estimates for the Ising model.

In light of this situation, experiments of sufficient accuracy to directly reveal the departures from simple power-law behavior assume new importance. Only such results are really suitable for providing values of the leading exponents and amplitudes which are accurate enough to enable stringent testing of the theoretical exponent values and the predicted relationships between the amplitudes[25–27] of the leading singularities. In addition, there is the possibility of obtaining reliable values for the amplitudes of the leading correction terms, the ratios of which are expected to be universal, and are theoretically predicted.[28,9]

In the absence of special symmetries, the above task is further complicated because the relevant scaling fields are analytic but unknown functions of the two intensive thermodynamic variables used to define the state of the system.[13] Although the effect of this revised scaling is to introduce corrections[29–31] which can be quite similar to the Wegner-type corrections, it will be shown in the data analysis section that at least under certain conditions, the leading Wegner correction can still be obtained separately.

The present experiment consists of light-scattering measurements of the isothermal susceptibility and turbidity of xenon as a function of $t\equiv(T-T_c)/T_c$ in the range $9.6\times10^5\leq t\leq 10^{-1}$. The measurements were made on a well-defined path which was very nearly equal to the critical isochore. The susceptibility data systematically and continuously deviate from simple power-law behavior throughout this range, with the effective exponent $\gamma_{\rm eff}$ increasing from 1.14 to 1.246 ± 0.01, as the critical point is approached. Thus, the data are asymptotically consistent with either series calculations[32–34] for the Ising model, $\gamma=1.250\pm0.003$, or the renormalization-group results,[5–7] $\gamma=1.241\pm0.002$, for the Landau-Ginzburg-Wilson Hamiltonian. Over the entire temperature range the data is described to within 0.11% rms deviation by a Wegner-type expansion involving three correction terms.

An analysis of existing data for xenon shows that almost all measurements are consistent with the renormalization-group exponents provided allowance is made for the Wegner correction terms. This analysis yields self-consistent values for the amplitudes of the leading singularities, and it is found that the theoretically predicted relations among these amplitudes are in very good agreement with experiment.

Since knowledge of the susceptibility and turbidity permits calculation of the correlation range near T_c using either the Ornstein-Zernike approximant or more accurate expressions[35] for the angular distribution of the scattered intensity, the present measurements also provide a good measure of the amplitude ξ_0 for the leading singularity of the correlation range. Thus, we can also test the predictions of two scale factor universality[36] to an accuracy of $\sim3\%$. The resulting value for the amplitude ratio $(B^2P_c/k_BT_c\Gamma^+)^{1/3}\xi_0$ is 0.68, in excellent agreement with the theoretically predicted[26] value of 0.67. Here, B is the amplitude of the leading term for the coexistence curve, which is accurately known.[24]

The remainder of this paper consists of four main sections. The experimental method section discusses details of the apparatus, calibration of the detectors at the 0.1% level, and the sample itself. The results section is designed to relate the measured quantities to the susceptibility and turbidity, and it includes a discussion of the various spurious effects for which corrections must be made. The comparison of the data to the theoretical expressions is carried out in the section on data analysis, which includes an analysis of possible systematic effects. This section also addresses the question of the possible effects of revised scaling on the interpretation of the data. We conclude with a discussion section, which includes an analysis of existing data for the heat capacity, coexistence curve, and critical isotherm of xenon, and tests of the various amplitude ratios as well as two-scale-factor universality.

II. EXPERIMENTAL METHOD

The quantities which can actually be measured are the ratio of the photocount rate caused by the scattered intensity at a fixed scattering angle, to that caused by a reference intensity generated from the incident beam, and the ratio of the current induced in a photovoltaic sensor by the beam after transmission through the scattering cell, to the current induced in an identical sensor by a reference beam.

The power scattered per unit beam length per unit solid angle in the fluid, P_s is given by

$$P_s = P_0 A \chi \sin^2\phi\, g(k\xi) , \quad (2)$$

where P_0 is the beam power in the scattering region, ϕ is the angle between the electric field of the incident light and the wave vector of the scattered light, $\chi \equiv (\partial\rho/\partial\mu)_T$ is the susceptibility, and $A = \pi^2 k_B T(\partial n^2/\partial\rho)_T^2/\lambda_0^4$. Here n is the index of refraction of the fluid, and λ_0 is the vacuum wavelength of the incident light. The function $g(k\xi)$ is, for $k\xi \leq 10$, very accurately given[35] by the Ornstein-Zernike approximant $(1+k^2\xi^2)^{-1}$, where k is the scattering wave vector and ξ is the correlation range.

The relationship between the scattered-intensity measurements and the susceptibility is complicated by several factors. In the first place, the beam is attenuated as it traverses the cell by the scattering process itself, and the scattered light after leaving the beam in the direction of the detector is also attenuated by being rescattered. However, this turbidity effect can be allowed for very accurately by measuring the attenuation suffered by the beam in traversing the cell. Secondly, some of the light reaching the detector has been scattered more than once while in the cell. This multiple scattering was as great as about 8% of the scattered light, for the point nearest T_c. However, it was possible to estimate it fairly well as described below. The third complication arises from the angular dependence of the scattering as determined by $g(k\xi)$.

Thus, it is necessary to know ξ at each temperature in order to deduce the $k=0$ limit of the scattered intensity from the measurements made at a finite angle. This is a small effect, however, because the measurements were made at a 13° scattering angle which corresponds to $k\xi << 1$.

A. Apparatus

Figure 1 is a schematic diagram of the apparatus which is adapted to simultaneously measure turbidity and scattered intensity. The turbidity was deduced from measurements of the ratio of the current induced in the silicon photovoltaic sensor PV1 by the beam after transmission through the scattering cell to that induced in a similar sensor PV2 by the turbidity reference beam. The turbidity reference beam was derived from the main beam using a multilayer dielectrically coated beam splitter. The output current from the sensor selected by the relay was fed to a current-to-voltage converter in order to obtain linear operation. The output of the current-to-voltage converter passed to an integrating sample and hold circuit, the output of which was converted to digital form and read by the microprocessor.

The scattered-intensity measurements were made by measuring the ratio of the photomultiplier-tube (PMT) count rate induced by the scattered light to

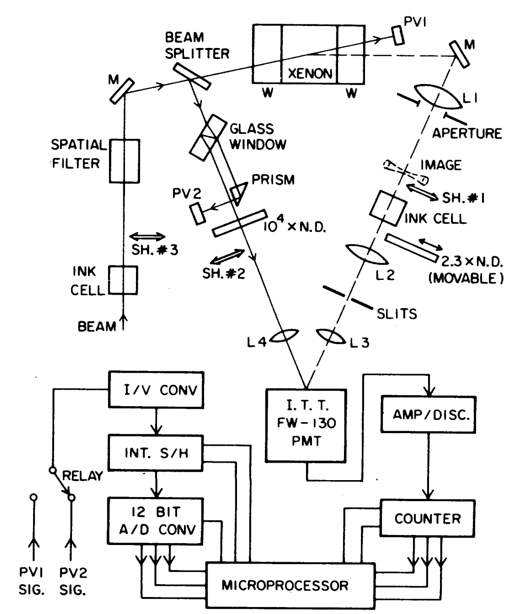

FIG. 1. Schematic diagram of the apparatus.

that induced by the intensity reference. The intensity reference was derived from the turbidity reference by two reflections from a high-quality glass window, followed by an attentuation of 10^4 using a black glass filter.

The collection optics for the scattered light consisted of lenses L1 and L2, the aperture at L1 and the slits, which were actually a rectangular aperture. The scattered light was imaged on the slits, and thus to have been collected, light must have appeared to come from within the region defined by the image of the slits in the cell, and to have been directed into the solid angle defined by the aperture at L1. Lenses L3 and L4 were used to exactly superimpose the scattered light and reference beams on the center of the photocathode.

B. Measurement sequence

An actual measurement sequence consisted of the following steps. With shutter no. 3 open, sensor PV1 was selected by the microprocessor, and shutter no. 2 was closed while shutter no. 1 was opened. This blocked the intensity reference, and permitted the scattered light to reach the PMT. After a timed delay of ~80 msec, during which the integrating sample and hold circuit was discharged and the counter reset to zero, a gate pulse of 1 sec duration was generated by the microprocessor from its 1-MHz crystal controlled clock. During this time, the PMT output was counted, and the output of the current-to-voltage converter was integrated. Immediately thereafter, shutter no. 2 was opened while no. 1 was closed, and sensor PV2 was selected. The results for the scattered intensity and transmitted beam power were then obtained by reading the number already in the counter, and converting and reading the sample and hold output. The measurements were then repeated yielding values for the reference intensity and reference beam power.

When the above sequence had been repeated ten times, shutter no. 3 was closed, thus blocking the entire beam. The measurement sequence was then repeated five times with shutter no. 3 closed yielding an accurate average for the PMT dark count rate and sensor "dark current" for both possible positions of shutters 1 and 2 and for both sensors. The dark count rates obtained in this manner were always within 10 Hz of the true dark counting rate, showing that a negligible amount of light was reaching the PMT from sources other than the reference or scattered light. The dark signals were subtracted from the previously measured values, and the appropriate ratios were calculated and averaged for the ten measurement sequence. This entire procedure was designated as a pass, and a typical measurement at any one temperature consisted of from 50 to 400 passes.

C. Detector calibration

It is essential when using pulse-counting techniques to calibrate the response of the PMT and counting circuitry, if accurate intensity measurements are to be realized over a wide range of count rates. The limited response rate of the amplifier-discriminator circuit and the inevitable superposition of pulses result in missed pulses at high count rates. Since response can be sensitive to the polarization, and spatial and temporal coherence properties of the detected light, it was felt to be important to calibrate the response using the scattered light itself with the reference beam and all other conditions identical to the actual experimental conditions.

The calibration was accomplished by means of the ink cell and movable 2.3 × neutral density filter shown in Fig. 1. The scattering cell was maintained at a temperature such that the scattered intensity resulted in a 1.3-MHz count rate with water in the ink cell. The ratio of this count rate to the reference rate was measured, as described above, both with and without the filter attenuating the scattered light. This yielded an effective attenuation for the filter at the 1.3-MHz rate. By varying the concentration of ink in the ink cell, it was possible to repeat this measurement at a number of count rates varying from each other by about a factor of 2.5 and extending down to a rate of 10 kHz. The apparent filter attenuation was found to decrease with increasing count rate, declining by ~4.5% by 1.3 MHz. Such a decline is to be expected because of "dead-time" effects which result in missed pulses. It was found that a simple correction of the expected form $N = N_E/(1-\tau_D N_E)$ was sufficient to account for the observed effect. Here N is the number of counts which an ideal system would have recorded, N_E is the number actually recorded, and τ_D is an effective dead time during which the system is incapable of responding to a second pulse if it has just detected a pulse. The dead time was found to depend on the high voltage applied to the tube and on the discriminator setting, increasing with high voltage and increased sensitivity. This is to be expected since the discriminator circuit cannot respond to a second pulse until about 20 nsec after the voltage at its input has returned to below the threshold, which time depends on the amplitude of the pulses generated by the tube. For the experimental conditions used, a τ_D of 61.5 nsec was found and incorporated into the microprocessor's handling of count-rate data. When this was done, the filter attenuation was found to be the same to within $\pm 0.05\%$ from 10 kHz to 1.3 MHz, thus verifying the detector linearity to within 0.1%. The linearity was then reverified quite near T_c and ~ 1 K above T_c, and the different temporal coherence properties of the light were found to play a negligible role, the light having a correlation time much longer than 60 nsec.

In the early stages of this calibration, it was found that the present PMT response could be slightly sensitive ($\sim 1\%$) to the light levels to which it had been exposed during the previous 30 sec. This effect vanished only when the discriminator setting was placed at the minimum in the pulse-height distribution versus discriminator setting curve, as measured with the tube voltage fixed. This effect may be due to a slight change in pulse-height distribution caused by temporary charge buildup on the tube envelope, but it was not investigated.

The linearity of the photovoltaic detectors was determined to be better than 0.1% throughout the intensity range where they were used by the same technique, and by direct comparison with the PMT.

D. Sample

The sample was contained in a BeCu cell of 20-mm internal diameter, between highly polished ($\sim 10-5$) glass windows 12.5-mm thick separated by 19.5 mm. The xenon was specified to contain less than 50 ppm of impurities, mostly krypton; however, the critical temperature was observed to increase at the rate of 58 μK/day. This was almost certainly due to slow contamination by water which had been used to clean the cell. The actual critical temperature was 16.64 °C in fairly good agreement with the accepted value of 16.59 °C. In any event, measurements obtained several months apart were found to be in good agreement provided they were referenced to the appropriate value of T_c. The sample density was adjusted to within better than 0.1% of the critical density, as judged by the temperature dependence of the meniscus height. The estimate of 0.1% includes the possible effects of revised scaling, which allow for a "hook" in the coexistence curve diameter, and the meniscus height measurements were analyzed using the parametric equation of state to allow for gravitational effects. The sample temperature was controlled to within ± 50 μK, and vertical gradients were found to be less than 17 μK/cm by direct measurement. Temperatures were measured to better than 50 μK using an NBS calibrated platinum resistance thermometer and an ac Kelvin bridge.

With the exception of the one data point nearest T_c, all measurements were performed with the sample in complete thermal equilibrium, which involved waiting periods of from 1 to 24 h. The one point which was not taken in complete equilibrium has no effect on any of the fits to the data or the conclusions reached, but was included for completeness. From our experience, it should correspond to a very accurate susceptibility measurement, but the turbidity could be in error by $\sim 1\%$.

III. RESULTS

A. Turbidity measurements

As mentioned, the turbidity, which is the integral over solid angle of the scattering cross section given by Eq. (2), was determined from measurements of the transmitted beam power relative to a reference beam power. Thus the measured ratio R_τ is given in terms of the turbidity τ, by

$$R_\tau = G_\tau e^{-\tau L} , \tag{3}$$

where L is the distance the beam travels in the cell, and G_τ is an unknown constant which includes the effects of the beam splitter, reflection losses at the cell windows, the relative sensitivities of the two detectors, etc. The value of G_τ can be determined by a measurement made at a value of $T-T_c$ so large (~ 30 K) that $e^{-\tau L}=1$ to within experimental accuracy, and this was occasionally done. However, it was found to be more accurate and much more convenient to choose a fixed reference temperature T_r, and to make all measurements relative to the value observed at T_r. Thus, the quantity actually measured was

$$R_\tau(T)/R_\tau(T_r) = e^{-[\tau(T)-\tau(T)_r)]L} , \tag{4}$$

from which $\tau(T)-\tau(T_r)$ was obtained. Since one of the photovoltaic detectors was maintained at fixed temperature while the other changed with the sample, it was necessary to correct for the temperature dependence of the sensitivity, which was found to be 0.040%/K.

Another effect for which corrections were necessary was the detection of scattered light in addition to the transmitted beam. This was measured as a function of $T-T_c$, by displacing the detector PV1, and measuring the scattered light directly. The scattered light exceeded 0.1% of the transmitted beam in power only in the range $9.6\times10^{-5}\leq t \lesssim 5\times10^{-4}$, rising sharply to 2.95% at $t=9.6\times10^{-5}$.

The only other potential complication involved in these measurements was the slight bending of the beam while in the sample, caused by the gravitationally induced gradient in the index of refraction. This, in addition to its finite diameter, caused the beam to sample a range of turbidities as it traversed the cell, and to be displaced downward on the detector PV1, by ~1.8 mm, for the worst case. The uniformity of the detector response was sufficient to ignore the effect of the displacement, and the net effect of sampling a range of turbidities was calculated using the parametric equation of state, and found to be less than 0.1%; hence, neither of these effects was serious.

Once the temperature dependence of the turbidity was known, the measurements at the reference temperature were corrected slightly for the effect of T_c changing with time. This yielded $\tau(T)-\tau(T_{\mathrm{or}})$, where T_{or} was fixed at $T_c+0.6677$ K.

B. Intensity measurements

The ratio of the count rate produced by the scattered light to that produced by the reference beam R_I contains contributions from both singly and multiply scattered light, in general. The single-scattering contribution is given by

$$R_{\mathrm{IS}}=\frac{G_I T\chi}{1+k^2\xi^2}e^{-\tau l}\,, \tag{5}$$

where τ is the turbidity, l the total path length in the xenon for the beam to reach the scattering region and the scattered light to leave the cell, and $k=2.23\times10^4\ \mathrm{cm}^{-1}$ is the scattering wave vector, corresponding to the scattering angle of 13.4° in xenon. The constant G_I incorporates all constant factors such as collection solid angle, length of beam from which light was collected, etc.

The multiple-scattering contribution is additive and cannot be characterized in any simple manner. It can, however, be measured to an accuracy of $\sim20\%$ by carefully scanning the slit image in the cell across the beam, and measuring the scattered intensity as a function of slit-image position. This must be done well away from T_c, where the scattering cross section is small enough to ensure single scattering only, and then repeated at each temperature for which it is necessary to measure the multiple scattering. Since single scattering emanates from the very well-defined (~0.1-mm) beam, while multiple scattering is present in the entire region near the beam, it is possible to distinguish between the two in this way. The scan made far from $T_c(T-T_c=2.5\ \mathrm{K})$ is an accurate measure of the convolution of the slit image and the beam profile, while the scans made near T_c, show the presence of a spatially nonlocalized source, namely, the multiple scattering. Its intensity can be measured quite accurately outside the immediate location of the beam, but the intensity present at the beam location itself must be inferred by extrapolation. This procedure is somewhat subjective and is the source of the 20% error estimate. It was found in all cases, however, that the spatial dependence of the multiple-scattering contribution was the same to within experimental accuracy. The results obtained for the ratio of multiple to single scattering $I_{\mathrm{M}}/I_{\mathrm{S}}$ as a function of $T-T_c$ are peculiar to the sample size and optical geometry employed here; however, $I_{\mathrm{M}}/I_{\mathrm{S}}$ was found to exhibit a simple power-law divergence $(T-T_c)^{-1.19}$, as might be expected.

Another effect which is very common in scattering experiments, particularly at small angles, is that of elastically scattered stray light. If, after two or more scatterings from the glass interfaces of the cell, light appears to come from the illuminated volume in the xenon, and is directed into the collection optics, it will be accepted. Such light would result in a temperature-independent contribution to the scattered intensity, and thus to the susceptibility. In terms of detecting a deviation from simple power-law behavior such a background would be a serious effect, and considerable care was exercised to verify that any such contribution was very small. This was done by using the scanning procedure discussed above to measure the background intensity away from the illuminated region at $T-T_c=29.00$ K, where the scattering from the xenon is at its weakest. The stray light was found to be less than 0.1% of the scattering from the xenon. As a further check similar scans were carried out measuring the scattering from the beam while in the glass of the windows. Since the glass is a very weak scatterer, it was possible to examine the stray light background in the immediate vicinity of the beam, where it is generally greatest, and again it was found to be less than 0.1%.

After division by $(1+I_M/I_S)$, to correct for multiple scattering, the intensity measurements can be interpreted in terms of Eq. (5). The first step in this procedure was to divide each measured value of R_{IS} by the turbidity signal R_τ which had been measured at the same time as R_{IS}, yielding the quantity

$$R'_{IS}=\frac{G'_I T\chi}{1+k^2\xi^2}e^{-\tau\delta L}\,, \tag{6}$$

where $\delta L=-0.027$ cm was the difference between l and L, the path length in the cell for the beam. Since the factor $e^{-\tau\delta L}$ was unity to within 2%, it was not necessary to measure τ with the great accuracy which would otherwise be required, to interpret the measurements.

As in the case of the turbidity measurements, it was found to be very desirable to make each measurement relative to the result obtained at the fixed reference temperature. Thus, each measurement was made in conjunction with an identical measurement at the reference temperature. The ratio of such a pair of measurements is given by

$$\frac{R'_{IS}(T)}{R'_{IS}(T_r)}=\frac{T}{T_r}\frac{\chi(T)}{\chi(T_r)}\left[\frac{1+k^2\xi^2(T_r)}{1+k^2\xi^2(T)}\right]\times e^{-[\tau(T)-\tau(T_r)]\delta L}\,. \tag{7}$$

Since $\xi(T)$ had been measured previously,[37] it was straightforward to deduce the ratio $\chi(T)/\chi(T_r)$ for each temperature. Once the temperature dependence of $\chi(T)$ was known for $T\simeq T_r$, all the measurements were corrected to allow for the slow drift in T_c, yielding $\chi(T)/\chi(T_{or})$. These results, together with the data for $\tau(T)-\tau(T_{or})$, are presented in Table I.

As in the case of the turbidity, the possible effect of the gravitationally induced refractive index gradient on the intensity measurements was considered using a parametric equation of state with accurately known parameters.[38] This was done by numerical ray tracing, and a number of effects were considered. These included changes in the acceptance solid angle and length of beam from which light was collected, as well as changes in the attenuation suffered by various rays in leaving the cell and, of course, variations in the susceptibility across the beam diameter. All of these effects were cumulatively found to be less than 0.15% and were thus neglected.

TABLE I. Susceptibility ratios and turbidity differences measured at various values of the reduced temperature.

t	$\chi(t)/\chi(t_{or})$	$\tau(t)-\tau(t_{or})$
9.607×10^{-5}	50.17	1.1249
1.499×10^{-4}	28.77	0.7604
2.309×10^{-4}	16.84	0.4958
3.133×10^{-4}	11.55	0.3551
4.757×10^{-4}	6.892	0.2135
6.393×10^{-4}	4.794	0.1427
9.115×10^{-4}	3.098	0.0814
1.185×10^{-3}	2.246	0.0490
2.304×10^{-3}	1.0	0
4.569×10^{-3}	0.4366	−0.0227
4.571×10^{-3}	0.4359	−0.0230
4.569×10^{-3}	0.4363	−0.0228
8.759×10^{-3}	0.1994	−0.0324
8.757×10^{-3}	0.1995	−0.0326
1.258×10^{-2}	0.1294	−0.0351
1.258×10^{-2}	0.1296	−0.0354
1.950×10^{-2}	0.076 98	−0.0369
1.950×10^{-2}	0.077 04	−0.0376
3.150×10^{-2}	0.043 80	−0.0388
4.615×10^{-2}	0.028 07	−0.0393
4.615×10^{-2}	0.027 95	−0.0398
6.901×10^{-2}	0.017 54	−0.0404
1.001×10^{-1}	0.011 50	−0.0394

The turbidity τ is related to χ by[39]

$$\tau=\pi A\chi f(\alpha)\,, \tag{8}$$

where

$$f(\alpha)=\left[\frac{2\alpha^2+2\alpha+1}{\alpha^3}\right]\ln(1+2\alpha)-\frac{2(1+\alpha)}{\alpha^2}\,, \tag{9}$$

and $\alpha=2(k_0\xi)^2$, where k_0 is the wave vector of the light in the fluid. Consequently, the data in Table I may, together with data for ξ, be used to deduce the actual magnitude of χ at each temperature, rather than simply the ratios $\chi(T)/\chi(T_{or})$. Because the function $f(\alpha)$ is essentially constant for $\alpha\lesssim 0.1$, the results are almost independent of the values used for ξ, provided only data in the range $t>4\times10^{-4}$ are analyzed. When this was done the resulting value for $\chi(T_{or})$ was 2.42×10^{-6}g^2/erg cm^3, and the corresponding value for $\tau(T_{or})$ was 0.041 cm^{-1}. In carrying out this analysis, values for ξ were obtained from the fit $\xi=64.66 (T-T_C)^{-0.62}$ Å determined from direct-correlation-range measurements.[37]

Having determined $\chi(T_{or})$ and $\tau(T_{or})$, the data provide values for $\chi(T)$ and $\tau(T)$ for each temperature studied. Thus, over the very limited range $t\leq4\times10^{-4}$, where the function $f(\alpha)$ is somewhat sensitive to the actual value of ξ, it is possible to combine the values for χ and τ, and to obtain values for ξ using Eqs. (8) and (9). When this was done, the values were found to be $\sim5\%$ larger than the fit values, and to be consistent with the expression $\xi=1.84t^{-0.63}(1+0.55t^{1/2})$. The 5% difference is within the relative accuracy with which ξ can be measured directly and inferred from χ and τ, especially in view of the uncertainties introduced by multiple scattering; thus, the agreement is satisfactory. However, the absolute accuracy of the present determination of $\xi_0=1.84$ should be better than that obtained from the direct measurements of ξ, considerably more effort having been devoted to the multiple-scattering problem, which is quite serious in xenon. The susceptibility ratios for $t\leq2.3\times10^{-4}$ presented in Table I have been corrected very slightly ($<0.2\%$) to allow for the change in the factor $(1+\tilde{k}^2\xi^2)$ due to the better determination of ξ near T_c.

IV. DATA ANALYSIS

The primary experimental results of this study are contained in Table I. If correction to scaling terms are important at the present level of accuracy, the susceptibility ratios should deviate systematically from simple power-law behavior. That this is indeed the case may be seen from Fig. 2, which gives the percentage deviation between the measured ratios and the arbitrary function 0.6390 $(T-T_c)^{-1.206}$. Since the value of $\chi(T_{or})$ was determined as mentioned in Sec. III B, it was possible to obtain the actual value of χ for each temperature studied. These values of $(\partial\rho/\partial\mu)_T$ were converted to dimensionless form[40] to yield values for

$$\phi^*\equiv\left[\frac{\partial\rho^*}{\partial\mu^*}\right]_T=\frac{P_c}{\rho_c^2}\left[\frac{\partial\rho}{\partial\mu}\right]_T \tag{10}$$

using $\rho_c=1.11$ g/cm^3 and $P_c=58.40\times10^6$ dyne/cm^2. The resulting values for χ^* were fit to

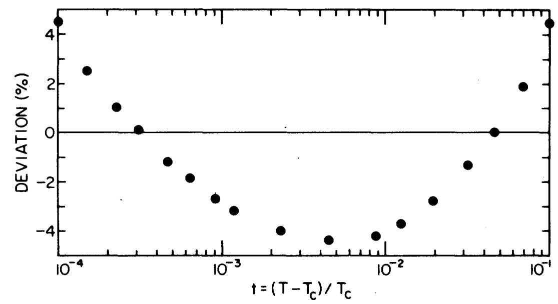

FIG. 2. Percent deviation between the measured susceptibility ratios $\chi(t)/\chi(t_{or})$ and the simple power law 0.6390 $(T-T_c)^{-1.206}$, as a function of reduced temperature.

the expression

$$\chi^*=\Gamma^+t^{-\gamma}(1+a_1t^{\Delta}+a_2t^{2\Delta}+a_3t^{3\Delta}) , \tag{11}$$

with Δ fixed[7] at 0.496. In fitting, the parameters Γ^+, γ, a_1, a_2, a_3, and T_c were independently adjusted.

In order to examine the extent to which systematic effects might be present in the data, the temperature range over which the fit was made was varied by systematically removing points corresponding to values of t less than a given t_{min}. The results obtained for all of the parameters were essentially unchanged as t_{min} varied from 9.6 $\times10^{-5}$ to 9.1×10^{-4}. The results for the various parameters are given in the first part of Table II. A similar test was made by removing all points corresponding to values of t greater than a given value t_{max}, and the results are given in the second part of the table. Here it is seen that as the data range is narrowed the parameters a_2 and a_3 become indeterminate as would be expected. Their presence in the fit is, however, sufficient to seriously affect the result obtained for a_1. When they are removed, the results for Γ^+, γ, and a_1 are in good agreement with the other fits.

In fitting over the entire range, three terms are, in fact, required as evidenced by substantial improvements in the accuracy of the fit as each term was added. The error bars quoted are simply one standard deviation allowing for the correlations between the parameters. They are small, in general, because the data are very precise. Since even small systematic errors could change the fit values by far more than these errors, the standard deviations must not be considered very meaningful.

In order to examine the extent to which the data are consistent with the exponent value $\gamma=1.241$ predicted by renormalization-group calculations,

TABLE II. Parameters obtained by fitting dimensionless susceptibility data to the form $\chi=\Gamma^{+}t^{-\gamma}(1+a_1t^{\Delta}+a_2t^{2\Delta}+a_3t^{3\Delta})$.

t_{min}	t_{max}	γ	Γ^{+}	a_1	a_2	a_3
9.6×10^{-5}	10^{-1}	1.246 ± 0.002	0.0551 ± 0.0012	1.62 ± 0.14	$-2.7\ \pm0.5$	$3.6\pm\ 0.8$
2.3×10^{-4}	10^{-1}	1.245 ± 0.005	0.0556 ± 0.0026	1.57 ± 0.26	$-2.5\ \pm0.8$	$3.4\pm\ 1.2$
4.7×10^{-4}	10^{-1}	1.244 ± 0.010	0.0562 ± 0.0055	1.50 ± 0.48	$-2.4\ \pm1.4$	$3.1\pm\ 1.9$
9.1×10^{-4}	10^{-1}	1.255 ± 0.023	0.0519 ± 0.0126	1.94 ± 1.00	$-3.4\ \pm2.6$	$4.3\pm\ 3.3$
9.6×10^{-5}	4.6×10^{-2}	1.249 ± 0.002	0.0538 ± 0.0012	1.88 ± 0.17	-4.33 ± 0.9	$7.7\pm\ 1.9$
9.6×10^{-5}	2.0×10^{-2}	1.252 ± 0.004	0.0527 ± 0.0025	2.28 ± 0.46	$-7.3\ \pm3.2$	$17.4\pm\ 9.7$
9.6×10^{-5}	8.8×10^{-3}	1.238 ± 0.008	0.0597 ± 0.0049	0.38 ± 0.98	$+11.7\ \pm9.2$	-66.8 ± 39.0
9.6×10^{-5}	8.8×10^{-3}	1.240 ± 0.002	0.0584 ± 0.0009	1.07 ± 0.06	(0)	(0)
9.6×10^{-5}	10^{-1}	(1.241)	0.0577 ± 0.0001	1.29 ± 0.03	-1.55 ± 0.2	$1.9\pm\ 0.5$
2.3×10^{-4}	10^{-1}	(1.241)	0.0576 ± 0.0001	1.34 ± 0.04	-1.84 ± 0.3	$2.4\pm\ 0.5$
4.7×10^{-4}	10^{-1}	(1.241)	0.0574 ± 0.0002	1.37 ± 0.06	-2.00 ± 0.4	$2.6\pm\ 0.6$
9.1×10^{-4}	10^{-1}	(1.241)	0.0576 ± 0.0003	1.35 ± 0.10	-1.93 ± 0.5	$2.5\pm\ 0.9$
9.6×10^{-5}	4.6×10^{-2}	(1.241)	0.0578 ± 0.0001	1.26 ± 0.05	-1.35 ± 0.4	$1.6\pm\ 1.2$
9.6×10^{-5}	2.0×10^{-2}	(1.241)	0.0578 ± 0.0001	1.14 ± 0.08	$+0.32\pm1.1$	$-4.9\pm\ 4.5$
9.6×10^{-5}	8.8×10^{-3}	(1.241)	0.0583 ± 0.0002	0.71 ± 0.10	$+8.8\ \pm1.0$	$-55\ \ \pm11.0$

the analysis was repeated with γ fixed at 1.241. The results are shown in the third and fourth parts of the table. Again, the results are invariant under changing t_{min}, and behave as expected when t_{max} is varied. Since the fit mentioned above with $a_2=a_3=0$ resulted in $\gamma=1.240$, when γ was freely adjusted, this was not repeated with γ fixed at 1.241. The accuracy with which the data are described by Eq. (11) is indicated in Fig. 3, which gives the percentage deviations using the parameters obtained with $\gamma=1.241$, for the entire range in t. All the other fits gave deviations which were quite similar.

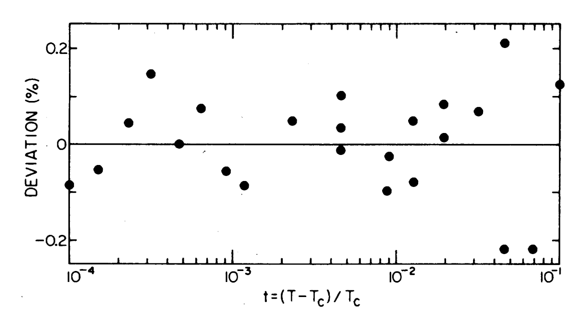

FIG. 3. Percent deviation between the dimensionless susceptibility χ^* and the theoretical expression $\chi^*=0.0577t^{-1.241}(1+1.29t^{\Delta}-1.55t^{2\Delta}+1.9t^{3\Delta})$, as a function of reduced temperature.

In all cases, the value of T_c found by the fitting procedure agreed to within 0.5 mK with the value observed by noting the temperature at which a meniscus formed upon cooling in small steps. This is just within the limit of accuracy with which the appearance can be determined. However, it should be noted that a shift of 0.5 mK in T_c amounts to a 2.5% change in the fitted value at $t=9.6\times10^{-5}$, which is a substantial effect. Thus, rather than actually reflecting a slight misdetermination of T_c, a shift in T_c can be used by the fitting program as a means of compensating for a systematic error which increases for the points nearest T_c.

Of course, the correction applied to the data for multiple scattering represents just such a systematic effect, and, in fact, it is the only uncertainty in the entire procedure which is not at the level of about 0.1%. However, the fact that the shape observed for the spatial distribution of the multiple scattering was the same at all temperatures, means that the fractional error made in determining the ratio of multiply to singly scattered intensity should be the same at all temperatures. Because of this condition, it is possible to quantitatively examine the effects of such an error on the final results obtained for the various parameters. When this was done, it was found that the only parameter sensitive to such an error was the adjusted value of

T_c. Even the effect of a 30% increase in the multiple-scattering correction could be compensated to within 0.1% at all data points leaving Γ^+, γ, a_1, a_2, and a_3 fixed, simply by shifting T_c by 0.5 mK. This is an extremely fortuitous result, because it greatly reduces the uncertainty in the important parameters Γ^+, γ, a_1, etc., which might have been caused by the large uncertainty in the multiple-scattering correction.

With confidence that the parameters obtained by fitting are, in fact, a true reflection of the critical properties of xenon, we now turn to the question of their meaning. In the case of Γ^+ and γ, it is clear that they represent the leading amplitude and exponent, and the data are certainly consistent with the best theoretical results for γ. However, the interpretation of the amplitude of the correction terms is much more difficult for the following reason. Theoretical calculations refer to the effect of irrelevant fields[13] on the critical behavior. In the case of pure fluids, the order parameter is not simply the density or density difference, because the coexistence curve does not exhibit complete symmetry. Thus, in general, it is not possible to identify *a priori* the relevant scaling fields; rather they must be considered[41,42] to be analytic functions of the physical variables μ and T. The consequences of such revised scaling have been worked out in detail by Ley-Koo and Green,[30] and incorporated into a parametric equation of state by Sengers and Sengers.[31] This equation of state has proved to give a very accurate description of *PVT* data for both steam[43] and ethene,[44] steam being a fluid with a particularly asymmetric coexistence curve. The effect of revised scaling on the general expression for the susceptibility $(\partial\rho^*/\partial\mu)_T$ is given[31] to the lowest order by

$$\chi^* = \Gamma^+ r^{-\gamma}[1+U_1(\theta)r^{\Delta}+V_0(\theta)r^{\gamma+\beta-1}] , \qquad (12)$$

where the parametric variables r and θ are defined so as to include to lowest order the mixing of μ and T in the relevant scaling fields, and $U_1(\theta)$ and $V_0(\theta)$ are polynomials defined in Ref. 31. Since for small θ, $r \simeq t$, the term in $r^{\gamma+\beta-1}$ may, in fact, be very similar to the first Wegner term r^{Δ}, since $\Delta \simeq 0.50$ and $\gamma+\beta-1 \simeq 0.57$. Consequently, data obtained along a path of constant θ would be very difficult to interpret so as to extract the amplitude of the leading Wegner term.

For measurements made along the critical isochore, the variable θ, while small, is not constant. Thus, it was necessary to analyze in detail the behavior of the various terms introduced by revised scaling. This was done numerically using the parameters appropriate to steam which should be more seriously affected than xenon. It was found that to within 0.6% the function $U_1(\theta)=1$ for the entire range of t, while $V_0(\theta)=-1.24\theta$, and that the maximum value of θ encountered was 0.065 occurring at $t=10^{-1}$. The amplitude of the term involving $r^{\gamma+\beta-1}$ was only 1% of the Wegner term for $t=10^{-4}$ but rose to 12.5% at $t=10^{-2}$ and to 42% at $t=10^{-1}$. Thus, it might be confused with, and affect the results for a_1 especially when data for large t were included. This is not the case, however, because along the thermodynamic path $\rho=\rho_C$ the variable θ is a function of t, and when analyzed in detail the functional dependence was found to be $\theta \sim t^{0.52}$ over the entire range $10^{-4} < t < 10^{-1}$. Since for small θ, $t=r$ quite accurately, the entire effect of the lowest-order revised scaling term is to appear as $t^{\gamma+\beta-1+0.52} \simeq t^{1.1}$ and not as $t^{0.57}$. This result, of course, applies only to susceptibility measurements, and then only to measurements made at the critical density. Nevertheless, it is sufficient to show that the parameter a_1 derived from fitting the data may be identified as the amplitude of the first Wegner correction term even when allowing for the effects of revised scaling. The analysis also shows clearly that the terms in t and $t^{1.5}$ used in the fitting procedure are actually being used as an approximation to a very complicated mixture of Wegner corrections and revised scaling effects. That these two terms are, in fact, adequate, is supported by the fact that the results for Γ^+, γ, and a_1 were all quite insensitive to the range over which the data were analyzed.

One final point which must be considered is that of the actual thermodynamic path followed in the experiment. Since the mean density of the cell contents had been adjusted to equal ρ_c to within 0.1%, the measurements were for the most part made along the path $\rho=\rho_c$. However, for values of $t \lesssim 3.5\times10^{-4}$, the vertical position of the cell was adjusted slightly ($\lesssim 0.3$ mm) relative to the laser beam and collection optics so as to maximize the measured susceptibility. Thus, in the range $t \leq 3.5\times10^{-4}$, the measurements were made along a path which could have deviated from the critical isochore. However, over this range, the maximum susceptibility and the susceptibility at $\rho=\rho_c$ are identical to within much less than 0.1% even in the case of steam. For this reason, the measurements may be regarded as having been made along the critical isochore.

In summary, the main results of the data analysis applied to the susceptibility are as summarized in Table II. The parameters Γ^+ and γ may be interpreted as the amplitude and exponent of the leading singularity, respectively, and a_1 should equal the amplitude of the leading Wegner correction term for the susceptibility. The parameters a_2 and a_3 do not have a simple interpretation, and include effects both from revised scaling and corrections to scaling.

V. DISCUSSION

In this section we wish to compare our result for the leading amplitude Γ^+ to that obtained by Hocken and Moldover,[24] and to explore the extent to which existing data can be used to deduce values for other leading amplitudes, when all exponents are fixed at the theoretical values. The results of this analysis are used to calculate various amplitude ratios for comparison to theory. Since our data also yield an accurate value for ξ_0, the leading amplitude for the correlation range singularity, we are also able to test the extent to which two-scale-factor universality is supported by existing data. Finally, we consider the situation with regard to the amplitude of the first Wegner term for properties other than the susceptibility so as to test as far as possible theoretical predictions[9,28] concerning the ratios of such amplitudes.

Our results may be compared directly with those of Hocken and Moldover who measured the gravitationally induced refractive index gradient in a very limited region of density and temperature quite close to the critical point. They obtained $\gamma=1.23$, while we have 1.246. However, both data sets are quite compatible with the theoretical value 1.241, and their data have been analyzed[38] subject to that constraint. Since the amplitude and exponent corresponding to a given data set are very highly correlated, it is essential that any comparison of amplitudes be made using the same exponent values. With $\gamma=1.241$, their data yield (Ref. 38) $\Gamma^+=0.0578$, while our best value is 0.0577, with all fits yielding values within 1% of this. We find this level of agreement notable, because the two experiments rely on completely different effects, and there is no region of overlap between the two data sets.

In order to test the universal amplitude ratios A^+/A^-, $\Gamma^+DB^{\delta-1}$, and $A^+\Gamma^+/B^2$, we wish to consider existing data with all exponents fixed at the values predicted by renormalization-group calculations,[5-7] provided the data are consistent with such a constraint. We begin with the heat-capacity data,[45] which possibly suffer from experimental problems related to thermal relaxation times and nonagreement of T_c values with those obtained by meniscus observation. The data as analyzed by the authors are compatible with the expression

$$C^{\pm}=(A^{\pm}/\alpha)t^{-\alpha}+C_B^{\pm} \ ,$$

with values for α ranging from 0.065 to 0.125, which includes the theoretical result $\alpha=0.109$. Over the entire range for which the data were originally analyzed, we find that the expressions

$$C^+=47.4\,|\,t\,|^{-0.109}(1+0.23\,|\,t\,|^{0.5})-46$$

and

$$C^-=74.8\,|\,t\,|^{-0.109}(1+0.23\,|\,t\,|^{0.5})-17$$

(measured in J/mole K) are indistinguishable from the original fits. Thus, we find upon reduction to dimensionless form[31] $A^+=2.15$ and $A^-=3.39$.

In order to obtain values for B and D we may perform a similar analysis on the experimental results of Hocken and Moldover[24] who obtained $B=1.48$, and $\Gamma^+D=0.367$, with $\beta=0.329$, $\gamma=1.23$, and $\delta=4.74$. With $\beta=0.325$, $\gamma=1.241$, and $\delta=4.815$, the same data will be very accurately described with $B=1.415$ and $\Gamma^+D=0.418$. Since we have $\Gamma^+=0.0577$ from our results, we obtain $D=7.24$. Clearly it would be more desirable to actually fit the data with fixed exponent values, but this has been done only in the context of a given equation of state,[38] which automatically imposes certain values for the amplitude ratios. We thus feel that our analysis provides the best *independent* estimate of the amplitudes, short of a detailed fitting.

The values of the amplitude ratios are summarized in Table III, which also gives the theoretical results[25] for both high-temperature-series studies of the Ising model and renormalization-group calculations for the Landau-Ginzburg-Wilson Hamiltonian. With the exception of the heat-capacity ratio A^+/A^-, the agreement is remarkable. As the table also shows, the data are completely consistent with two-scale-factor universality, whether the test is made directly, or in terms of the amplitudes B^2 and Γ^+ assuming the theoretical result for $A^+\Gamma^+/B^2$. We thus conclude that the best existing data for xenon are in close accord with the theoretical results.

The final question which we wish to consider is the extent to which any other existing data are ca-

TABLE III. Amplitude ratios obtained from data for xenon.

Ratio	Experiment	Theory[a]	Theory[b]
A^+/A^-	0.63	0.55	0.51
$\Gamma^+ DB^{\delta-1}$	1.57	1.6	1.75
$A^+\Gamma^+/B^2$	0.062	0.066	0.059
$(A^+P_c/k_BT_c)^{1/3}\xi_0$	0.27	0.27	0.25
$(B^2P_c/\Gamma^+k_BT_c)^{1/3}\xi_0$	0.68	0.67	0.65

[a]From ϵ expansion of renormalization-group results.
[b]From series expansions for the Ising model.

pable of providing values for the amplitudes of the first Wegner correction for properties other than the susceptibility. So far as a direct estimate is concerned, the only property which has been measured with sufficient accuracy to reveal departures from pure power-law behavior is the density difference between liquid and vapor. Garland and Thoen[46] find that their data are incompatible with theory, in that $\beta=0.356$, and β does not decrease significantly as $T\rightarrow T_c$. This conflicts with the result obtained by Hocken and Moldover very near T_c, and also with the experiment of Estler *et al.*,[18] who found β to decrease as $T\rightarrow T_c$. Consequently, we can only consider the data of Estler *et al.* They found their data to be described to within 1% over the range $5\times10^{-5}\leq|t|\leq10^{-2}$ by the function $1.52|t|^{0.332}+0.46|t|^{0.61}$ with the deviations being systematic. However, we find that the expression $1.45|t|^{0.325}(1+1.62|t|^{0.5}-5.88|t|)$ deviates by less than 0.2% over the same range. Thus, their data are very consistent with theoretical results, and provide an estimate for a_M. The only other data for $\Delta\rho$ of which we are aware are those of Hayes and Carr[19] obtained using NMR techniques. In the same fashion, we find their NMR frequency to be represented to within 0.2% by the expression $4782|t|^{0.325}(1+2.08|t|^{0.5}-5.87|t|)$ over nearly the same temperature range. We thus find $a_M/a_\chi^+\approx1.4$ using our value of 1.3 for a_χ^+.

This same ratio has been estimated for a number of fluids by Aharony and Ahlers,[9] using the differences between the theoretical values for the exponents and the effective exponents β_{eff} and γ_{eff}, obtained from previous fits[8] to *PVT* data, in the range $|t|\leq3\times10^{-2}$. They found $A_M/A_\chi^+\simeq0.6$ on the average. We suspect the primary cause of the difference is the use of *PVT* data which tend to give low values for γ_{eff} as compared to light scattering, because the *PVT* results are easily affected by gravity. For example, with $\beta_{\text{eff}}=0.355$ and $\gamma_{\text{eff}}=1.19$, one obtains $(\beta_{\text{eff}}-\beta)/(\gamma-\gamma_{\text{eff}}) = 0.59$, but over the same range, where $\beta_{\text{eff}}=0.355$, light-scattering results[22,23] yield $\gamma_{\text{eff}}=1.22$, which should be much more accurate than the *PVT* result. This gives a value of 1.43 for the ratio, in perfect accord with the value obtained by actually fitting the data using correction terms. It is thus possible that the ratio A_M/A_χ^+ may be unity or larger, while the best available theoretical value[28] is $A_M/A_\chi^+\simeq0.85$.

The ratio A_M/A_χ^+ has also been estimated for ^{4}He near the gas-liquid critical point. There the coexistence curve has been measured[21] accurately enough to determine $A_M\simeq0.9$, while the susceptibility showed no deviation from simple power-law behavior, with $\gamma_{\text{eff}}=1.19$. However, the susceptibility data were also consistent with $\gamma=1.240$, provided at least one correction term was involved. In this way the estimate $A_M/A_\chi^+=0.4\pm0.2$ was obtained.[9]

The situation with regard to SF_6 is quite similar to that of ^{4}He in that A_M has been determined to be 0.82 by direct fit[30] to the coexistence curve,[20] which definitely deviates from simple power-law form, while the susceptibility[23] is known to within a few percent, and does not deviate significantly from a simple power law with $\gamma_{\text{eff}}=1.22$. We have refit the reduced susceptibility data, and find them to be fit to within 0.9% rms deviation by the expression $0.0425t^{-1.241}(1+1.14t^{0.5}-3.04t)$, which is as good a description as the original single power-law fit. We thus estimate $A_M/A_\chi^+\simeq0.7$ for SF_6.

Clearly it is very difficult at the present to obtain reliable experimental values for the ratios of correction amplitudes. This is not surprising because such amplitudes are measures of deviations at the level of a few percent. At present xenon is the only system in any universality class for which two properties have been measured with sufficient accuracy to directly reveal such deviations, and it would be very desirable to have other measurements of similar accuracy on the same system.

As a final minor point we note that the experience we have had in "forcing" data to fit the current theoretical framework involving correction terms has made it clear to us that almost any reasonable data set can be brought into very good agreement with theory, primarily because of the tremendous flexibility afforded by the correction terms. When one couples this flexibility with the additional flexibility and complexity generated by revisions to scaling, it is clear that theory is at present in an almost unassailable position. Consequently, any theoretical progress which would reduce the number of effectively free parameters would be extremely valuable.

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