Off-shell effects in one-photon free-free absorption

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One-photon free-free absorption amplitudes are derived for electrons scattering from a spinless spherically symmetric target in the dipole approximation. The exact amplitudes are compared to the usual on-shell approximation to the amplitudes. An effect analogous to the Fano effect in bound-free transitions is also found in free-free transitions if off-shell terms are retained in the amplitudes. Methods of detecting the off-shell effects experimentally are discussed.

I. INTRODUCTION

It has been suggested recently by one of the authors[1] (B.R.) that one-photon free-free absorption (electron scattering in an external electromagnetic field) can be used to resolve the unpolarized electron total cross section into its spin-nonflip and spin-flip components. The validity of this claim has been disputed by C. Jung.[2] A check of the derivation of the amplitudes in Ref. 1 by one of the authors (P.W.C.) uncovered an error in the original calculation. We agree with Jung that free-free transitions cannot be used to resolve unpolarized electron scattering cross sections into spin-nonflip and spin-flip components.

One of the conclusions in Ref. 1 is that a spin-polarization effect should be observed in free-free absorption analogous to the Fano effect[3] in bound-free absorption. Free-free absorption amplitudes are usually calculated by taking the initial and final electron energies to be equal. This on-shell approximation is made in Ref. 1 and in the final amplitude in Ref. 2. If the on-shell approximation is used, there is no analog of the Fano effect in free-free absorption.

The absence of such an effect is puzzling since it seems to be very plausible on physical grounds. For example, consider a circularly polarized laser beam with its photon spin along the incident electron beam direction. In the absence of the laser the polarization vector of the scattered electrons would not have a component parallel to the incident beam (assuming an unpolarized electron beam and a spinless target). This is also true in the presence of the laser if the on-shell approximation is used to calculate the scattering amplitudes. However, it seems likely that the scattered electrons will acquire a polarization component in the same direction as the laser polarization.

On-shell approximations have been used frequently in theoretical calculations[4] of free-free transition amplitudes. In this paper we will include off-shell terms in computing scattering amplitudes for one-photon free-free absorption in the dipole approximation.[5] We find that an analog of the Fano effect in free-free absorption does exist. The calculation is done in the weak-field limit where one-photon absorption is accurately described by the electric-dipole transition matrix elements of first-order perturbation theory.[6] We assume a spinless spherically symmetric target with no internal degrees of freedom. We suggest experiments which might be done to test the relative importance of the off-shell contributions to the one-photon absorption cross sections.

The formalism is outlined in the next section with the details left for an appendix. Some specific examples are examined in Sec. III. The results are discussed in the last section.

II. FORMALISM

In the dipole-velocity form of the radiative interaction the absorption cross section is given by

$$I_a=\frac{1}{2\pi}\left[\frac{I\alpha}{E_p^2}\right]a_0^4\left[\frac{m_e^2}{\hbar^3}\right]\frac{k'}{k}\times\frac{1}{2}\sum_{\mu\mu'}|\langle\psi^{(-)}_{\vec{k}',\mu'}(\vec{r})\,|\,\vec{\rho}\cdot(-i\vec{\nabla})\,|\,\psi^{(+)}_{\vec{k},\mu}(\vec{r})\rangle|^2\,. \tag{1}$$

I is the radiation intensity in W/cm^2, E_p is the photon energy in atomic units (hartrees), k (k′) is the initial (final) electron momentum in units of a_0^{-1}, and $\vec{\rho}$ is a unit vector in the direction of polarization of the radiation. α is the fine-structure constant, a_0 is the Bohr radius, and μ and μ' are used to label spin states of the electron. The electron wave functions can be obtained from the Lippmann-Schwinger equation:

$$\psi^{(\pm)}_{\vec{k}\mu}(\vec{r})=\chi_\mu e^{i\vec{k}\cdot\vec{r}}+\frac{1}{(2\pi)^3}\int d^3k'\frac{e^{i\vec{k}'\cdot\vec{r}}}{k^2-k'^2\pm i\epsilon}\langle e^{i\vec{k}'\cdot\vec{r}'}\mid U(\vec{r}',\vec{\sigma})\mid \psi^{(\pm)}_{\vec{k}\mu}(\vec{r}')\rangle\ . \tag{2}$$

χ_μ is a two-component spinor and it is understood that the integral is evaluated in the limit $\epsilon\to 0$ $(\epsilon>0)$.

We will define

$$M(\vec{k}'\mu';\vec{k}\mu)=\langle \psi^{(-)}_{\vec{k}'\mu'}(\vec{r})\mid \vec{\rho}\cdot(-i\vec{\nabla})\mid \psi^{(+)}_{\vec{k},\mu}(\vec{r})\rangle \tag{3}$$

and

$$F(\vec{k}'\mu';\vec{k}\mu)=-\frac{1}{4\pi}\langle \chi_{\mu}\cdot e^{i\vec{k}'\cdot\vec{r}}\mid U(\vec{r},\vec{\sigma})\mid \psi^{(+)}_{\vec{k},\mu}(\vec{r})\rangle\ . \tag{4}$$

If $U(\vec{r},\vec{\sigma})$ is invariant under time reversal and parity, then

$$F(\vec{k}-\mu;\vec{k}'-\mu')=-\frac{1}{4\pi}(-1)^{1-\mu-\mu'}\langle \psi^{(-)}_{\vec{k}'\mu'}(\vec{r})\mid U(\vec{r},\vec{\sigma})\mid e^{i\vec{k}\cdot\vec{r}}\chi_\mu\rangle\ . \tag{5}$$

If we use Eqs. (2), (4), and (5) to evaluate $M(\vec{k}'\mu';\vec{k}\mu)$ we obtain the result[7]

$$\begin{aligned}M(\vec{k}'\mu';\vec{k}\mu)=&\frac{2\pi a_0^2\vec{\rho}}{Ep}\cdot[\vec{k}'F(\vec{k}'\mu';\vec{k}\mu)-\vec{k}(-1)^{1-\mu-\mu'}F(\vec{k}-\mu;\vec{k}'-\mu')]\\&+\frac{2}{\pi}\sum_{\mu''}(-1)^{1-\mu'-\mu''}\int d^3k''\vec{\rho}\cdot\vec{k}''\frac{F(\vec{k}''-\mu'';\vec{k}'-\mu')F(\vec{k}''\mu'';\vec{k}\mu)}{(k'^2-k''^2+i\epsilon)(k^2-k''^2+i\epsilon)}\ .\end{aligned} \tag{6}$$

We have used the fact that $k\neq k'$ for single photon absorption to eliminate one term in Eq. (6). It can also be shown that

$$\begin{aligned}F(k\mu;k'\mu')=&(-1)^{1-\mu-\mu'}F(\vec{k}'-\mu';\vec{k}-\mu)\\&+\frac{k'^2-k^2}{2\pi^2}\sum_{\mu''}(-1)^{1-\mu-\mu''}\int d^3k''\frac{F(\vec{k}''-\mu'';\vec{k}-\mu)F(\vec{k}''\mu'';k'\mu')}{(k^2-k''^2+i\epsilon)(k'^2-k''^2+i\epsilon)}\ ,\end{aligned} \tag{7}$$

if $U(\vec{r},\vec{\sigma})$ is invariant under time reversal and parity. Equation (6) can be rewritten as

$$\begin{aligned}M(k'\mu';k\mu)=&\frac{2\pi a_0^2}{Ep}\vec{\rho}\cdot(\vec{k}'-\vec{k})F(\vec{k}'\mu';\vec{k}\mu)\\&+\frac{2}{\pi}\sum_{\mu''}(-1)^{1-\mu'-\mu''}\int d^3k''\vec{\rho}\cdot(\vec{k}''-\vec{k})\frac{F(\vec{k}''-\mu'';\vec{k}'-\mu')F(\vec{k}''\mu'';\vec{k}\mu)}{(k^2-k''^2+i\epsilon)(k'^2-k''^2+i\epsilon)}\ .\end{aligned} \tag{8}$$

Only the first term in Eq. (8) is retained in making the on-shell approximation.

We will use Eq. (8) to investigate the exact form of the scattering amplitudes for one-photon free-free absorption in the dipole approximation. We will assume that the electron-atom interaction can be written as

$$U(\vec{r},\vec{\sigma})=v(r)+\frac{\vec{\sigma}\cdot\vec{l}}{\hbar}w(\vec{r})\ , \tag{9}$$

where $\vec{l}$ is the orbital angular momentum operator for the electron.

We will now write the expressions we obtain for the various $M(\vec{k}'\mu';\vec{k}\mu)$. We will use $+(-)$ to represent $\mu=+\frac{1}{2}(-\frac{1}{2})$. It is convenient to write the polarization vector in its spherical representation

$$\begin{aligned}&\rho_\pm=\mp\rho_x-i\rho_y\ ,\\&\rho_0=\rho_z\ .\end{aligned} \tag{10}$$

We will choose the z axis in the direction of $\vec{k}$ and use (θ,ϕ) for the spherical angles representing the direction of $\vec{k}'$. The coordinate system is shown in Fig. 1. Then

$$M(\vec{k}';\vec{k}+)=\frac{a_0^2}{2E_p}\left[\rho\cdot(\vec{k}'-\vec{k})F(\theta)+\vec{\rho}\cdot\vec{k}\sum_l D_l Y_{l0}(\theta,\phi)+\sum_l\left[\frac{\rho_+}{\sqrt{2}}A_l Y_{l,-1}(\theta,\phi)+\frac{\rho_-}{\sqrt{2}}B_l Y_{l,1}(\theta,\phi)+\rho_0 C_l Y_{l0}(\theta,\phi)\right]\right], \tag{11}$$

$$M(\vec{k}'-;\vec{k}'-)=\frac{a_0^2}{2E_p}\left[\rho\cdot(\vec{k}'-\vec{k})F(\theta)+\vec{\rho}\cdot\vec{k}\sum_l D_l Y_{l0}(\theta,\phi)+\sum_l\left[\frac{\rho_+}{\sqrt{2}}B_l Y_{l-1}(\theta,\phi)+\frac{\rho_-}{\sqrt{2}}A_l Y_{l,1}(\theta,\phi)+\rho_0 C_l Y_{l0}(\theta,\phi)\right]\right], \tag{12}$$

$$M(\vec{k}'-;\vec{k}+)=-\frac{a_0^2}{2E_p}\left[\vec{\rho}\cdot(\vec{k}'-\vec{k})G(\theta)e^{i\phi}+\vec{\rho}\cdot\vec{k}\sum_l \delta_l Y_{l1}(\theta,\phi)-\sum_l\left[\frac{\rho_+}{\sqrt{2}}\alpha_l Y_{l0}(\theta,\phi)+\frac{\rho_-}{\sqrt{2}}\beta_l Y_{l2}(\theta,\phi)+\rho_0\gamma_l Y_{l1}(\theta,\phi)\right]\right], \tag{13}$$

$$M(\vec{k}'+;\vec{k}-)=\frac{a_0^2}{2E_p}\left[\vec{\rho}\cdot(\vec{k}'-\vec{k})G(\theta)e^{-i\phi}+\vec{\rho}\cdot\vec{k}\sum_l \delta_l Y_{l,-1}(\theta,\phi)-\sum_l\left[\frac{\rho_+}{\sqrt{2}}\beta_l Y_{l,-2}(\theta,\phi)+\frac{\rho_-}{\sqrt{2}}\alpha_l Y_{l0}(\theta,\phi)+\rho_0\gamma_l Y_{l,-1}(\theta,\phi)\right]\right]. \tag{14}$$

Explicit expressions for A_l, B_l, etc., will be given in the Appendix.

We will use these equations for the amplitudes to discuss some particular examples in the next section. The polarization of the final electrons can be obtained from

$$P_x=2\frac{\mathrm{Re}(M_{++}M^*_{-+}+M_{+-}M^*_{--})}{\sum_{\mu\mu'}|M_{\mu\mu'}|^2},$$

$$P_y=-2\frac{\mathrm{Im}(M_{++}M^*_{-+}+M_{+-}M^*_{--})}{\sum_{\mu\mu'}|M_{\mu\mu'}|^2}, \tag{15}$$

$$P_z=\frac{|M_{++}|^2+|M_{+-}|^2-|M_{--}|^2-|M_{-+}|^2}{\sum_{\mu\mu'}|M_{\mu\mu'}|^2}.$$

We have used M_{++} to represent $M(\vec{k}'+;\vec{k}+)$, etc. We note that there is no polarization component along the incident beam if $|M_{++}|=|M_{--}|$ and $|M_{+-}|=|M_{-+}|$.

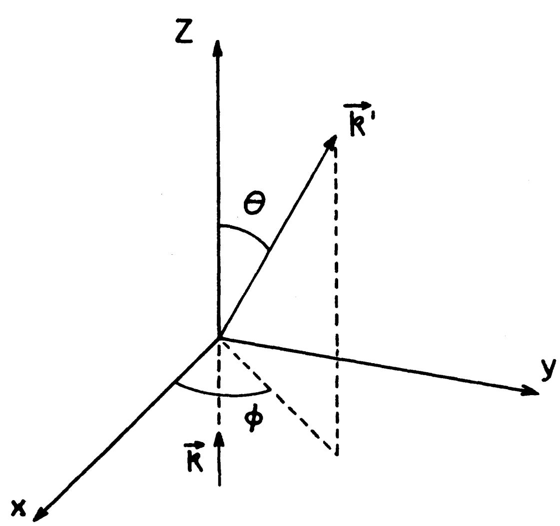

FIG. 1. Coordinate system orientation. The incident electron beam has momentum $\vec{k}$ along the z axis. The scattered electron has momentum $\vec{k}'$ with direction specified by the spherical angles (θ,ϕ).

These conditions are satisfied if the on-shell approximation is made.

III. APPLICATIONS

The on-shell amplitudes are given by

$$M_{0-s}(\vec{k}'\pm;\vec{k}\pm)=\frac{a_0^2}{2E_p}\vec{\rho}\cdot(\vec{k}'-\vec{k})F(\theta)\ , \tag{16}$$

$$M_{0-s}(\vec{k}'\mp;\vec{k}\pm)=\mp\frac{a_0^2}{2E_p}\vec{\rho}\cdot(\vec{k}'-\vec{k})G(\theta)e^{\pm i\phi}\ , \tag{17}$$

where we set $k=k'$. We will now compare the predictions of the on-shell amplitudes with the predictions of the exact amplitudes for some specific cases.

A. Linear polarization parallel to the incident beam

In this case we choose $\rho_x=\rho_y=0$, $\rho_z=1$. The on-shell amplitudes are

$$M_{0-s}(\vec{k}\pm:\vec{k}\pm)=\frac{a_0^2}{2E_p}k(\cos\theta-1)F(\theta)\ , \tag{18}$$

$$M_{0-s}(\vec{k}\pm;\vec{k}\pm)=\mp\frac{a_0^2}{2E_p}k(\cos\theta-1)G(\theta)e^{\pm i\phi}\ . \tag{19}$$

Thus the on-shell absorption cross section has no azimuthal angle dependence, there is no final polarization along the z axis, and the magnitude of the final polarization has no azimuthal dependence. There is no reason to expect any azimuthal dependence here because there is nothing in the scattering process to define any direction other than the incident beam direction. The final polarization is perpendicular to the scattering plane.

For the exact amplitudes,[8]

$$M(\vec{k}'\pm;\vec{k}\pm)=\frac{a_0^2}{2E_p}(k'\cos\theta-k)F(\theta)+\sum_l(C_l+kD_l)Y_{l0}(\theta,\phi)\ , \tag{20}$$

$$M(\vec{k}\mp;\vec{k}\pm)=\mp\frac{a_0^2}{2E_p}(k'\cos\theta-k)G(\theta)e^{\pm i\phi}+\sum_l(-\gamma_l+k\delta_l)Y_{l\pm1}(\theta,\phi)\ . \tag{21}$$

The exact amplitudes also predict no polarization parallel to the incident beam and no ϕ dependence of the absorption cross section or the magnitude of the polarization. The final polarization is again perpendicular to the scattering plane.

B. Circular polarization

We will now consider the case where $\rho_z=\rho_-=0$ and $\rho_+=-\sqrt{2}$. This corresponds to having the photon spin along the negative z axis ($\rho_x=1/\sqrt{2}$, $\rho_y=-i/\sqrt{2}$). If the laser beam is propagating in the positive z direction, it is left-circularly polarized. The on-shell amplitudes are

$$M_{0-s}(\vec{k}'\pm;\vec{k}\pm)=\frac{a_0^2}{2E_p}\frac{k}{\sqrt{2}}\sin\theta F(\theta)e^{-i\phi}\ , \tag{22}$$

$$M_{0-s}(\vec{k}'\pm;\vec{k}\pm)=\mp\frac{a_0^2}{2E_p}\frac{k}{\sqrt{2}}\sin\theta G(\theta)e^{\pm i\phi}e^{-i\phi}\ . \tag{23}$$

There is no final polarization along the beam direction. The absorption cross section and the magnitude of the final polarization have no ϕ dependence and the polarization is perpendicular to the scattering plane.

The exact amplitudes are

$$M(\vec{k}'+;\vec{k}+)=\frac{a_0^2}{2E_p}\frac{k'}{\sqrt{2}}\sin\theta F(\theta)e^{-i\phi}-A(\theta)e^{-i\phi}\ , \tag{24}$$

$$M(\vec{k}'-;\vec{k}-)=\frac{a_0^2}{2E_p}\frac{k'}{\sqrt{2}}\sin\theta F(\theta)e^{-i\phi}-B(\theta)e^{-i\phi}\ , \tag{25}$$

$$M(\vec{k}'-;\vec{k}+)=\frac{-a_0^2}{2E_p}\frac{k'}{\sqrt{2}}\sin\theta G(\theta)-\alpha(\theta)\ , \tag{26}$$

$$M(\vec{k}'+;\vec{k}-)=\frac{a_0^2}{2E_p}\frac{k'}{\sqrt{2}}\sin\theta G(\theta)e^{-2i\phi}-\beta(\theta)e^{-2i\phi}\ , \tag{27}$$

where

$$A(\theta)e^{-i\phi}=\sum_l A_l Y_{l-1}(\theta,\phi)\ , \tag{28}$$

$$B(\theta)e^{-i\phi}=\sum_l B_l Y_{l-1}(\theta,\phi)\ , \tag{29}$$

$$\alpha(\theta)=\sum_l \alpha_l Y_{l0}(\theta,\phi)\ , \tag{30}$$

$$\beta(\theta)e^{-2i\phi}=\sum_l \beta_l Y_{l-2}(\theta,\phi)\ . \tag{31}$$

We see that $|M(\vec{k}'+;\vec{k}+)|\neq|M(\vec{k}'-;\vec{k}-)|$ and $|M(\vec{k}'-;\vec{k}+)|\neq|M(\vec{k}'+;\vec{k}-)|$, which means that we do expect some polarization along the incident beam direction. The absorption cross section and the magnitude of the final polarization are independent of ϕ. This is to be expected since there is nothing to define a preferred direction perpendicular to the incident beam direction. The existence of the final-state polarization along the beam direction is the free-free absorption analog of the Fano effect that we expected. The final polarization is not perpendicular to the scattering plane.

C. Linear polarization perpendicular to the incident beam

Finally, consider the case $\rho_y=\rho_z=0$, $\rho_x=1$. Thus the x axis is defined by the direction of polarization. The on-shell amplitudes are

$$M_{0-s}(\vec{k}'\pm;\vec{k}\pm)=\frac{a_0^2}{2E_p}k\sin\theta F(\theta)\cos\phi\ , \tag{32}$$

$$M_{0-s}(\vec{k}'\mp;\vec{k}\pm)=\mp\frac{a_0^2}{2E_p}k\sin\theta G(\theta)\cos\phi e^{\pm i\phi}\ . \tag{33}$$

The absorption cross section is proportional to $\cos^2\phi$ and should be zero for scattering in the y-z plane. There is no final polarization along the incident beam direction and the magnitude of the polarization has no ϕ dependence. The final polarization is again predicted to be perpendicular to the scattering plane.

The exact amplitudes in this case are

$$M(\vec{k}'+;\vec{k}+)=\frac{a_0^2}{2E_p}k'\sin\theta F(\theta)\cos\phi-\frac{1}{\sqrt{2}}A(\theta)e^{-i\phi}-\frac{1}{\sqrt{2}}B(\theta)e^{i\phi}\ , \tag{34}$$

$$M(\vec{k}'-;\vec{k}-)=\frac{a_0^2}{2E_p}k'\sin\theta F(\theta)\cos\phi-\frac{1}{\sqrt{2}}B(\theta)e^{-i\phi}-\frac{1}{\sqrt{2}}A(\theta)e^{i\phi}\ , \tag{35}$$

$$M(\vec{k}'-;\vec{k}+)=-\frac{a_0^2}{2E_p}k'\sin\theta G(\theta)\cos\phi e^{i\phi}+\frac{1}{\sqrt{2}}\alpha(\theta)-\frac{1}{\sqrt{2}}\beta(\theta)e^{2i\phi}\ , \tag{36}$$

$$M(\vec{k}'+;\vec{k}-)=\frac{a_0^2}{2E_p}k'\sin\theta G(\theta)\cos\phi e^{-i\phi}+\frac{1}{\sqrt{2}}\beta(\theta)e^{-2i\phi}-\frac{1}{\sqrt{2}}\alpha(\theta)\ . \tag{37}$$

We can rewrite these equations in a more useful form by defining

$$a(\theta)=\frac{a_0^2}{2E_p}k'\sin\theta F(\theta)-\frac{A(\theta)}{\sqrt{2}}-\frac{B(\theta)}{\sqrt{2}}\ , \tag{38}$$

$$b(\theta)=\frac{i}{\sqrt{2}}[A(\theta)-B(\theta)]\ , \tag{39}$$

$$c(\theta)=\frac{-a_0^2}{2E_p}k'\sin\theta G(\theta)+\frac{\alpha(\theta)}{\sqrt{2}}-\frac{\beta(\theta)}{\sqrt{2}}\ , \tag{40}$$

$$d(\theta)=\frac{-i}{\sqrt{2}}[\alpha(\theta)+\beta(\theta)]\ . \tag{41}$$

Then

$$M(\vec{k}'+;\vec{k}+)=a(\theta)\cos\phi+b(\theta)\sin\phi\ , \tag{42}$$

$$M(\vec{k}'-;\vec{k}-)=a(\theta)\cos\phi-b(\theta)\sin\phi\ , \tag{43}$$

$$M(\vec{k}'-;\vec{k}+)=[c(\theta)\cos\phi+d(\theta)\sin\phi]e^{i\phi}\ , \tag{44}$$

$$M(\vec{k}'+;\vec{k}-)=-[c(\theta)\cos\phi-d(\theta)\sin\phi]e^{-i\phi}\ . \tag{45}$$

The absorption cross section depends on

$$\frac{1}{2}\sum_{\mu\mu'} |M(\vec{k}'\mu';\vec{k}\mu)|^2 = (|a(\theta)|^2+|c(\theta)|^2)\cos^2\phi+(|b(\theta)|^2+|d(\theta)|^2)\sin^2\phi \ . \tag{46}$$

The cross section is not zero in the y-z plane, but it will be small there if the off-shell effects are small as they are assumed to be if the photon energy is much less than the electron energy.[4] However, cases have been studied[5] in which off-shell effects have been expected to be quite significant.

The polarization of the final electrons is given by

$$P_x = 2\{\mathrm{Im}[a(\theta)c^*(\theta)\cos^2\phi+b(\theta)d^*(\theta)\sin^2\phi]+\mathrm{Re}[a(\theta)d^*(\theta)+b(\theta)c^*(\theta)]\cos^2\phi\}\sin\phi \times \left[\frac{1}{2}\sum_{\mu\mu'}|M(\vec{k}'\mu';\vec{k}\mu)|^2\right]^{-1}, \tag{47}$$

$$P_y = -2\{\mathrm{Im}[a(\theta)c^*(\theta)\cos^2\phi+b(\theta)d^*(\theta)\sin^2\phi]-\mathrm{Re}[a(\theta)d^*(\theta)+b(\theta)c^*(\theta)]\sin^2\phi\}\cos\phi \times \left[\frac{1}{2}\sum_{\mu\mu'}|M(\vec{k}'\mu';\vec{k}\mu)|^2\right]^{-1}, \tag{48}$$

$$P_z = \mathrm{Re}[a(\theta)b^*(\theta)-c(\theta)d^*(\theta)]\sin 2\phi \left[\frac{1}{2}\sum_{\mu\mu'}|M(\vec{k}'\mu';\vec{k}\mu)|^2\right]^{-1}. \tag{49}$$

The final polarization is not perpendicular to the scattering plane and its magnitude depends on ϕ.

IV. DISCUSSION

We have derived general expressions for one-photon free-free absorption scattering amplitudes for nonrelativistic electrons scattering from a spinless target with no internal degrees of freedom in the dipole approximation. We have compared the predictions of these scattering amplitudes to those of the on-shell amplitudes and found some notable differences. The importance of these differences depend on the relative sizes of the on-shell and off-shell contributions to the scattering amplitudes. We are currently doing calculations to determine the importance of the off-shell contributions. We will now discuss some ways to determine the size of the off-shell contributions experimentally.

In the case of linear polarization parallel to the incident beam, there is not much difference between the on-shell and exact amplitudes. The only significant difference is that the on-shell amplitudes are all zero in the forward direction while the exact spin-nonflip amplitudes are nonzero in the forward direction. A measurement of the forward scattering cross reaction could serve to determine the validity of the on-shell approximation in this case.

The cases of circular polarization and linear polarization perpendicular to the beam direction provide more direct means of measuring the off-shell contributions. The off-shell terms for these cases are defined by Eqs. (28)–(31). We note that $A(\theta)$, $B(\theta)$, and $\beta(\theta)$ are all zero for forward scattering whereas $\alpha(\theta)$ is nonzero for forward scattering. In the case of circular polarization (B) the photon spin is in the negative z direction. All of the forward scattering amplitudes are zero with the exception of $M(\vec{k}'-;\vec{k}+)$. This amplitude is nonzero (see Fig. 2) since the electron can absorb one photon and continue on in the forward direc-

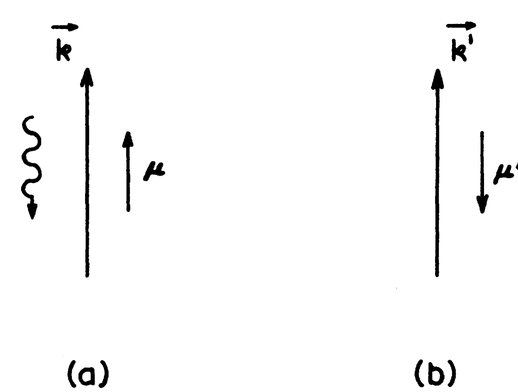

FIG. 2. Forward scattering in one-photon absorption. (a) represents the initial state consisting of the incident electron with spin oriented in the beam direction and a photon (wary line) with spin oriented in the opposite direction. (b) represents the final state where the electron is still traveling along the incident beam direction with its spin oriented in the opposite direction. The photon has been absorbed so that the angular momentum is the same in (a) and (b).

tion if its spin flips to conserve the z component of angular momentum. This nonzero forward scattering amplitude is not predicted by the on-shell amplitudes. An experimental measurement of the forward scattering one-photon absorption cross section should give an indication of the size of the off-shell terms in the scattering amplitudes. The experiment would be a direct determination of an isolated off-shell term and should provide a stringent test of the validity of making the usual on-shell approximations.

The forward scattering cross section is also nonzero in the case of linear polarization perpendicular to the incident beam direction. In this case the final polarization also exhibits a dependence on azimuthal angle which depends on the off-shell contributions to the scattering amplitudes. The off-shell contributions can be determined by detailed measurements of the polarization of the scattered electrons.[9]

To estimate the polarization effects that might be observed, we will use Eq. (49) to calculate the component of the polarization along the incident beam direction. The spin-nonflip amplitude $a(\theta)$ is generally larger in magnitude than the spin-flip amplitude $c(\theta)$, although they have comparable magnitudes for some values of θ.[10] In Fig. 3 we plot the z component of the polarization for fixed θ assuming that the off-shell contributions to the amplitude is a 10% effect. We assume that $|c|=0.25|a|$ and $|b|=0.1|a|$,

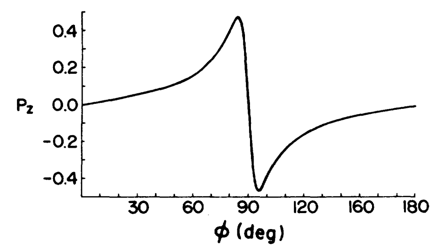

FIG. 3. Polarization component along incident beam direction as a function of azimuthal angle. The laser polarization is perpendicular to the incident beam. It has been assumed that $|c|=0.25|a|$, $|b|=0.1|a|$, $|d|=0.1|c|$, and $\mathrm{Re}(ab-cd)-0.05|a|^2$.

$|d|=0.1|c|$. We also assume that $\mathrm{Re}(ab-cd)=0.05|a|^2$. The polarization along the beam direction becomes fairly large in the region $\phi=80°$, but the one-photon absorption cross section is relatively small in this region.

Off-shell effects should be large if the incident electron energy is near a resonance (shape resonance here since the internal degrees of freedom of the target are neglected). This can be seen from the expansion of the wave function in the Appendix in terms of radial and angular functions. In particular, we note that $F(\vec{k}'\mu';\vec{k}\mu)$ can differ significantly from $F(\vec{k}-\mu;\vec{k}'-\mu')$ if the radial wave functions vary rapidly with energy [see Eqs. (A3)–(A5) in the Appendix]. Thus in Eq. (8) we expect the term

$$-\frac{2}{\pi}\sum_{\mu''}\vec{\rho}\cdot\vec{k}(-1)^{1-\mu'-\mu''}\int d^3k''\frac{F(\vec{k}''-\mu'';\vec{k}'-\mu')F(\vec{k}''\mu'';\vec{k}\mu)}{(k^2-k''^2+i\epsilon)(k'^2-k''^2+i\epsilon)}$$

to make a nonnegligible contribution to the total amplitude near a resonance. It seems reasonable to expect the remaining off-shell term to be equally important.

Although one-photon free-free absorption cannot be used to determine the separate spin-nonflip and spin-flip components of the electron-atom scattering amplitudes, we see that some of the effects predicted in Ref. 1 do occur because of the presence of off-shell terms. In particular we have shown that an analog of the Fano effect in free-free absorption does exist. We expect off-shell effects to be especially important near a resonance.

APPENDIX

The solutions of Eq. (2) with a potential defined by Eq. (9) can be written generally as

$$\psi^{(+)}_{\vec{k}\mu}(\vec{r})=\chi_\mu 4\pi\sum_{l,m}i^l\eta_{lm\mu}(k,r)Y^*_{lm}(\hat{k})Y_{lm}(\hat{r})+\chi_{-\mu}4\pi\sum_{l,m}i^l\alpha_{lm\mu}(k,r)Y^*_{lm-2\mu}(\hat{k})Y_{lm}(\hat{r}) , \tag{A1}$$

where $\hat{k}(\hat{r})$ is a unit vector in the direction of $\vec{k}(\vec{r})$. We use $Y_{lm}(\hat{k})$ to stand for $Y_{lm}(\theta_k,\phi_k)$. The η's and α's satisfy the symmetry relations

$$\begin{aligned}&\eta_{l-m-\mu}(k,r)=\eta_{lm\mu}(k,r)\,,\\&\alpha_{l-m-\mu}(k,r)=\alpha_{lm\mu}(k,r)\,.\end{aligned}\tag{A2}$$

The $F(\vec{k}'\mu';\vec{k}\mu)$ can be written as

$$F(\vec{k}'\mu';\vec{k}\mu)=-\delta_{\mu\mu'}4\pi\sum_{l,m}v_{lm\mu}(k'k)Y_{lm}(\hat{k}')Y^*_{lm}(\hat{k})-\delta_{\mu-\mu'}4\pi\sum_{lm}w_{lm\mu}(k'k)Y_{l,m+2\mu}(\hat{k}')Y^*_{lm}(\hat{k})\,,\tag{A3}$$

where

$$v_{lm\mu}(k'k)=\int r^2dr\,j_l(k'r)\{[v(r)+2m\mu w(r)]\eta_{lm\mu}(k,r)+\sqrt{l(l+1)-m(m+2\mu)}w(r)\alpha_{l,m+2\mu}(kr)\}\tag{A4}$$

and

$$w_{lm\mu}(k'k)=\int r^2dr\,j_l(k'r)\{[v(r)-2\mu(m+2\mu)w(r)]\alpha_{l,m+2\mu,\mu}(k,r)+\sqrt{l(l+1)-m(m+2\mu)}w(r)\eta_{lm\mu}(kr)\}\,,\tag{A5}$$

where $j_l(kr)$ is a spherical Bessel function of the first kind. Using Eq. (A2) we note that

$$\begin{aligned}&v_{l-m-\mu}(k'k)=v_{lm\mu}(k'k)\,,\\&w_{l-m-\mu}(k'k)=w_{lm\mu}(k'k)\,.\end{aligned}\tag{A6}$$

We can now give explicit expressions for the A_l, B_l, etc., in Eqs. (11)–(14) in terms of the $v_{lm\mu}$ and $w_{lm\mu}$. We will define

$$D(k,k',k'')=(k^2-k''^2+i\epsilon)(k^2-k''^2+i\epsilon)\,.$$

Then

$$A_l=8\sqrt{4\pi(2l+1)}\int dk''\frac{k''^3}{D(k,k',k'')}\sum_{l'}(2l'+1)\begin{pmatrix}1&l&l'\\0&0&0\end{pmatrix}\times\left[v_{l'0-}(k'',k)v_{l-1+}(k''k')\begin{pmatrix}1&l&l'\\1&-1&0\end{pmatrix}-w_{l'0-}(k''k)w_{l-1+}(k''k')\begin{pmatrix}1&l&l'\\1&0&-1\end{pmatrix}\right],\tag{A7}$$

$$B_l=8\sqrt{4\pi(2l+1)}\int dk''\frac{k''^3}{D(k,k',k'')}\sum_{l'}(2l'+1)\begin{pmatrix}1&l&l'\\0&0&0\end{pmatrix}\times\left[v_{l'0-}(k'',k)v_{l1+}(k''k')\begin{pmatrix}1&l&l'\\-1&1&0\end{pmatrix}-w_{l'0-}(k''k)w_{l1+}(k''k')\begin{pmatrix}1&l&l'\\-1&2&-1\end{pmatrix}\right],\tag{A8}$$

$$C_l=8\sqrt{4\pi(2l+1)}\int dk''\frac{k''^3}{D(k,k',k'')}\sum_{l'}(2l'+1)\begin{pmatrix}1&l&l'\\0&0&0\end{pmatrix}\times\left[v_{l'0-}(k''k)v_{l0+}(k''k')\begin{pmatrix}1&l&l'\\0&0&0\end{pmatrix}-w_{l'0-}(k''k)w_{l0+}(k''k')\begin{pmatrix}1&l&l'\\0&1&-1\end{pmatrix}\right],\tag{A9}$$

$$D_l=-8\sqrt{4\pi(2l+1)}\int dk''\frac{k''^2}{D(k,k',k'')}[v_{l0-}(k''k)v_{l0+}(k''k')+w_{l0-}(k''k)w_{l0+}(k''k')]\,, \tag{A10}$$

$$\alpha_l=-8\sqrt{4\pi(2l+1)}\int dk''\frac{k''^3}{D(k,k',k'')}\sum_{l'}(2l'+1)\begin{bmatrix}1 & l & l'\\ 0 & 0 & 0\end{bmatrix}\left[v_{l'0-}(k''k)w_{l0-}(k''k')\begin{bmatrix}1 & l & l'\\ 1 & -1 & 0\end{bmatrix}\right.$$
$$\left.-w_{l'0-}(k''k)v_{l0-}(k''k')\begin{bmatrix}1 & l & l'\\ 1 & 0 & -1\end{bmatrix}\right]\,, \tag{A11}$$

$$\beta_l=-8\sqrt{4\pi(2l+1)}\int dk''\frac{k''^3}{D(k,k',k''}\sum_{l'}(2l'+1)\begin{bmatrix}1 & l & l'\\ 0 & 0 & 0\end{bmatrix}\left[v_{l'0-}(k''k)w_{l2-}(k''k')\begin{bmatrix}1 & l & l'\\ -1 & 1 & 0\end{bmatrix}\right.$$
$$\left.-w_{l'0-}(k''k)v_{l2-}(k''k')\begin{bmatrix}1 & l & l'\\ -1 & 2 & -1\end{bmatrix}\right]\,, \tag{A12}$$

$$\gamma_l=-8\sqrt{4\pi(2l+1)}\int dk''\frac{k''^3}{D(k,k',k'')}\sum_{l'}(2l'+1)\begin{bmatrix}1 & l & l'\\ 0 & 0 & 0\end{bmatrix}$$
$$\times\left[v_{l'0-}(k''k)w_{l1-}(k''k')\begin{bmatrix}1 & l & l'\\ 0 & 0 & 0\end{bmatrix}\right.$$
$$\left.-w_{l'0-}(k''k)v_{l1-}(k''k')\begin{bmatrix}1 & l & l'\\ 0 & 1 & -1\end{bmatrix}\right]\,, \tag{A13}$$

$$\delta_l=-8\sqrt{4\pi(2l+1)}\int dk''\frac{k''^2}{D(k,k'k'')}[v_{l0-}(k''k)w_{l1-}(k''k')+w_{l0-}(k''k)v_{l1-}(k''k')]\,. \tag{A14}$$

We have also defined

$$F(\theta)=(4\pi)^{3/2}\sum_l\sqrt{(2l+1)}v_{l0+}(k'k)Y_{l0}(\hat{k}')\,,$$
$$G(\theta)e^{i\phi}=(4\pi)^{3/2}\sum_l\sqrt{(2l+1)}w_{l0+}(k'k)Y_{l1}(\hat{k}')\,. \tag{A15}$$

In the Born approximation,

$$v_{lm\mu}(k'k)=v_l(k',k)+2\mu m w_l(k'k)\,, \tag{A16}$$

$$w_{lm\mu}(k'k)=\sqrt{l(l+1)-m(m+2\mu)}w_l(k'k)\,, \tag{A17}$$

where

$$v_l(k',k)=\int r^2dr\, j_l(k'r)v(r)j_l(kr)\,, \tag{A18}$$

$$w_l(k',k)=\int r^2dr\, j_l(k'r)w(r)j_l(kr)\,. \tag{A19}$$

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