PHYSICAL REVIEW A VOLUME 24, NUMBER 5 NOVEMBER 1981

Enhanced magnetostatic modes in a nonuniform plasma

P. K. Shukla, H. U. Rahman,* and M. Y. Yu
Institut für Theoretische Physik, Ruhr-Universität Bochum, 4630 Bochum 1, Federal Republic of Germany

R. K. Varma
Physical Research Laboratory, Ahmedabad 38009, India
(Received 3 March 1981)

Parametric excitation of the magnetostatic mode by finite-amplitude drift waves is considered. Both decay and modulational instabilities are analyzed. Expressions for the growth rates are obtained analytically. Application of our results to electron cross-field diffusion in tokamak devices is also discussed.

I. INTRODUCTION

The aperiodic, long-lived magnetostatic mode[1-4] has received considerable attention in the last few years. In this mode, the wave magnetic field is perpendicular to the external field $B_0\hat{z}$ and the long-wavelength oscillations damp out slowly owing to electron collisions or gyroviscosity. Thus, in a plasma at thermal equilibrium, a test electron moving freely with velocity v_z along $B_0\hat{z}$ achieves a perpendicular drift velocity $v_z\vec{\nabla}A_z\times\hat{z}/B_0$, where A_z is the parallel component of the vector potential of the magnetostatic mode. Since v_z is randomly distributed, this drift leads to a cross-field diffusion, which turns out to have a Bohm-like scaling.[1-4]

In the presence of external sources, such as currents, waves, or nonuniformities, the magnetostatic mode may be enhanced to above the thermal level. The enhanced magnetic field fluctuations can give rise to the formation of large scale magnetic islands,[2,3] as well as produce anomalous heat[5] and particle loss from the plasma.

Recently, we[6] have shown that magnetostatic modes can be driven unstable by kinetic Alfvén waves in a uniform plasma. In this paper, we consider parametric drive of magnetostatic modes by mode coupling of drift waves in a nonuniform plasma. It is found that the ponderomotive force $\langle\vec{v}^d_{e\perp}\cdot\vec{\nabla}v^d_{ez}\rangle$, where the superscript d denotes drift wave, can drive magnetostatic fluctuations. A coupled set of differential equations governing this parametric process is derived. We then obtain a general dispersion relation. Growth rates for both decay and modulational instabilities are derived analytically. We found that the two instabilities have similar growth rates, and that a small real frequency appears in each case.

II. FORMULATION

We consider the interaction of drift waves with magnetostatic modes in a low-β (β = particle pressure/magnetic pressure $\ll m_e/m_i$) plasma. Since the parallel phase velocity of the drift waves lies between the electron and ion thermal velocities ($v_{ti}\ll\omega_0/k_{0z}\ll v_{te}$), the electrons thermalize along the external magnetic field $B_0\hat{z}$. Then the electron density perturbation follows from the z component of the momentum equation. One obtains

$$n_e=n_0\exp(e\phi/T_e)\,, \tag{1}$$

where n_0 is the average plasma density, T_e is temperature of the parallel electron motion, and ϕ is the electrostatic potential of the drift waves.

The ion density perturbation for drift waves is determined from the ion continuity equation,

$$\frac{\partial n_i}{\partial t}+n_0\vec{\nabla}\cdot\vec{v}_i+\vec{v}_{i\perp}\cdot\vec{\nabla}n_0+\vec{\nabla}\cdot(n_i\vec{v}^m_i)=0\,, \tag{2}$$

where the superscript m denotes magnetostatic mode. For $\omega_0\ll\Omega_i$, we obtain the perpendicular ion velocity in the drift approximation:

$$\vec{v}_{i\perp}=-\frac{c}{B_0}\vec{\nabla}\phi\times\hat{z}-\frac{c}{B_0\Omega_i}\left(\frac{\partial}{\partial t}+\vec{v}^m_i\cdot\vec{\nabla}\right)\vec{\nabla}\phi+\frac{B^m_\perp}{B_0}v_{iz}+\frac{c}{B_0\Omega_i}(\vec{\nabla}\phi\times\hat{z}\cdot\vec{\nabla})\vec{v}^m_{i\perp}\times\hat{z}\,, \tag{3}$$

where $\Omega_i=eB_0/m_ic$ is the gyrofrequency. Note that the drift-wave quantities are without subscript or superscript.

Equation (2) describes the coupling of drift waves with the magnetostatic modes. On the other hand, the parallel ion motion is given by

$$\frac{\partial v_{iz}}{\partial t}+\vec{v}_{i\perp}\cdot\vec{\nabla}v^m_{iz}+\vec{v}^m_i\cdot\vec{\nabla}v_{iz}=-\frac{e}{m_i}\frac{\partial\phi}{\partial z}+\frac{e}{m_ic}(\vec{v}_{i\perp}\times\vec{B}^m_\perp)_z\,. \tag{4}$$

For the magnetostatic modes, the dynamics perpendicular to $B_0\hat{z}$ is not important, and the corresponding density perturbations are neglected. Thus, combining Eqs. (1)–(4), and using the charge neutrality condition ($n_e \approx n_i$, a good assumption for $\omega_{pi}^2/\Omega_i^2 = v_A^2/c^2 \ll 1$), we readily obtain for the drift waves,

$$\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial t^2}(1-\rho_s^2\vec{\nabla}_\perp^2)\phi - D\vec{\nabla}\frac{\partial\phi}{\partial t}\times\hat{z}\cdot\nabla\ln n_0 - c_s^2\frac{\partial^2\phi}{\partial z^2}\right) = \frac{c_s^2}{B_0}\left[\vec{\nabla}_\perp A_z\times\hat{z}\cdot\vec{\nabla}\frac{\partial^2\phi}{\partial t\partial z} - \frac{1}{\Omega_i}\vec{\nabla}_\perp\left(A_z\vec{\nabla}_\perp\frac{\partial^3\phi}{\partial t^2\partial z}\right)\right] + \frac{\Omega_i}{B_0}A_z\frac{\partial}{\partial z}\left(\frac{\partial^2\phi}{\partial t^2} + c_s^2\frac{\partial^2\phi}{\partial z^2}\right), \quad (5)$$

where $\rho_s^2 = c_s^2/\Omega_i^2$ and $D = cT_e/eB_0$. Here, c_s is the sound speed, and the drift-wave electric potential is normalized by T_e/e. Also we have used $v_{iz}^m = eA_z/m_ic$, and $\vec{B}_\perp^m = \vec{\nabla}\times A_z\hat{z} = \vec{\nabla}A_z\times\hat{z}$, with A_z being the vector potential of the magnetostatic modes. The ions are assumed to be cold, and only the lowest-order nonlinearities have been retained.

It is well known that the nonlinear coupling of the drift waves may give rise to a low-frequency ponderomotive force. The latter can enhance the magnetostatic modes. The driven magnetostatic mode dynamics is governed by[1]

$$\frac{\partial v_{ez}^m}{\partial t} + \langle\vec{v}_{e\perp}\cdot\vec{\nabla}_\perp v_{ez}\rangle + \left\langle v_{ez}\frac{\partial}{\partial z}v_{ez}\right\rangle = -\frac{e}{m_e}E_z^m + (\mu\vec{\nabla}^2 - \nu)v_{ez}^m. \quad (6)$$

The last two terms on the left-hand side represent the ponderomotive force. The angular brackets denote averaging over the period of the drift waves.

Recalling that

$$E_z^m = -\frac{1}{c}\frac{\partial A_z}{\partial t}, \quad (7)$$

$$\vec{\nabla}\times\vec{B}_\perp^m = \frac{4\pi}{c}\vec{j} \approx -\frac{4\pi en_0}{c}v_{ez}^m\hat{z}, \quad (8)$$

we can write Eq. (6) as

$$\left[\frac{\partial}{\partial t}\left(\vec{\nabla}^2 - \frac{\omega_{pe}^2}{c^2}\right) - (\mu\vec{\nabla}^2 - \nu)\right]A_z = -\frac{4\pi en_0}{c}\left\langle\vec{v}_{e\perp}\cdot\vec{\nabla}_\perp v_{ez} + v_{ez}\frac{\partial}{\partial z}v_{ez}\right\rangle, \quad (9)$$

where ω_{pe} is the electron plasma frequency. We note that in (8) the displacement current is neglected due to the low frequency. Furthermore, the ion current is neglected because $j_{ze}^m/j_{zi}^m \approx m_i/m_e$.

In the absence of the nonlinear forces, Eq. (9) gives the linear dispersion relation of the magnetostatic mode

$$\omega = -i\frac{(\mu k_\perp^2 + \nu)k_\perp^2c^2}{c^2k_\perp^2 + \omega_{pe}^2} \equiv -i\Gamma, \quad (10)$$

where we have assumed that the electron motion in the perpendicular direction remains cold. The perpendicular electron velocity appearing in (9) is then given by the high-frequency polarization drift

$$\vec{v}_{e\perp} \approx \frac{c}{B_0\Omega_e}\frac{\partial}{\partial t}(\vec{\nabla}_\perp\phi), \quad (11)$$

where $\Omega_e = eB_0/m_ec$ is the electron gyrofrequency. All the other drifts do not enter the coupling due to the averaging process. On the other hand, the parallel electron velocity can be obtained from the condition of conservation of current density $\vec{j}$, namely,

$$\vec{\nabla}_\perp\cdot\vec{j}_\perp = -\partial_z j_z, \quad (12)$$

where $n_e = n_i$ has been assumed. Using Eqs. (3) and (4), we find

$$\frac{\partial^2 v_{ez}}{\partial z\partial t} = -\frac{e}{m_i}\frac{\partial^2\phi}{\partial z^2} - \frac{c}{B_0\Omega_i}\frac{\partial^2}{\partial t^2}(\vec{\nabla}_\perp^2\phi). \quad (13)$$

Equations (5), (9), (11), and (13) constitute a coupled set of equations describing the interaction of drift waves and magnetostatic modes.

III. STABILITY ANALYSIS

Here, we consider the parametric instabilities of a finite-amplitude drift wave as a pump in the presence of magnetostatic modes. For this purpose, we decompose ϕ into three parts, the pump ϕ_0 and the two satellites $\phi_\pm$,

$$\phi = \phi_0\exp(-i\omega_0t + i\vec{k}_0\cdot\vec{x}) + \text{c.c.} + \phi_+\exp(-i\omega_+t + i\vec{k}_+\cdot\vec{x}) + \phi_-\exp(-i\omega_-t + i\vec{k}_-\cdot\vec{x}), \quad (14)$$

where $\omega_\pm = \omega \pm \omega_0$, and $\vec{k}_\pm = \vec{k} \pm \vec{k}_0$.

Letting $A_z = \exp(-i\omega t + i\vec{k}\cdot\vec{x})$, we can perform the usual linear analysis of the coupled equations. One obtains

$$\epsilon_\pm\phi_\pm = \alpha_\pm A_z\begin{pmatrix}\phi_0\\ \phi_0^*\end{pmatrix}, \quad (15)$$

$$(\omega + i\Gamma)A_z = \beta_+\phi_+\phi_0^* + \beta_-\phi_-\phi_0^*, \quad (16)$$

where

$$\epsilon_\pm = i\omega_\pm[\omega_\pm^2(1 + \rho_s^2k_{\pm\perp}^2) + \omega_\pm\omega_\pm^* - c_s^2k_{0z}^2],$$

$$\alpha_\pm = \mp\frac{c_s^2k_{0z}}{B_0}\left(\omega_0(\vec{k}_{0\perp}\cdot\vec{k}\times\hat{z}) \pm i\frac{\omega_0^2}{\Omega_i}(\vec{k}_{0\perp}\cdot\vec{k}_{\pm\perp}) + i\frac{\Omega_i}{c_s^2}(\omega_0^2 + c_s^2k_{0z}^2)\right),$$

$$\beta_{\pm} = -\frac{c^2\omega_{pi}^2(\vec{k}_{0\perp}\cdot\vec{k}_{\pm\perp})}{B_0\Omega_e k_{0z}(\omega_{pe}^2+c^2k^2)}\left(\frac{\omega_0^2}{\Omega_i^2}(k_{0\perp}^2+k_{\pm\perp}^2)+2k_{0z}^2\right),$$

and $\omega_{\pm}^{*}=D\vec{k}_{\pm\perp}\times\hat{z}\cdot\vec{\nabla}\ln n_0$ is the drift-wave frequency. Since the magnetostatic modes do not have a finite k_z, the parallel wavelengths of the drift waves must be the same. Combining (15) and (16), we obtain the dispersion relation

$$\omega+i\Gamma=\left(\frac{\alpha_+\beta_+}{\epsilon_+}+\frac{\alpha_-\beta_-}{\epsilon_-}\right)|\phi_0|^2. \quad (17)$$

For $\omega\ll\omega_0$ and $|\vec{k}_\perp|\ll|\vec{k}_{0\perp}|$, we have

$$\epsilon_{\pm}\approx i\omega_0(\omega_0-c_s^2k_{0z}^2/v_Dk_{0y})(\omega\pm\delta\omega), \quad (18)$$

where $v_D=-Dd\ln n_0/dx$, $\delta\omega=\omega_0-\omega_{k0}^{*}$, and

$$\omega_{k0}^{*}=\frac{k_{0y}v_D}{1+k_{0\perp}^2\rho_s^2}+\frac{k_{0z}^2c_s^2}{k_{0y}v_D}.$$

We first consider the three wave decay interaction. Accordingly, we take ϵ_+ to be off-resonant. Dropping the first term on the right-hand side of (17), using (18), we obtain

$$(\omega+i\Gamma)(\omega-\delta\omega)=\frac{P}{Q}(R-iS)|\phi_0|^2, \quad (19)$$

where

$$P=\frac{2c^2k_{0\perp}^2(k_{0z}^2+\omega_0^2k_{0\perp}^2/\Omega_i^2)}{B_0^2\Omega_e^2(1+c^2k^2/\omega_{pe}^2)},$$

$$Q=\omega_0(\omega_0-c_s^2k_{0z}^2/v_Dk_{0y}),$$

$$R=\Omega_i^2[\omega_0^2(1+\rho_s^2k_{0\perp}^2)+c_s^2k_{0z}^2],$$

$$S=\omega_0\Omega_ic_s^2(\vec{k}_{0\perp}\cdot\vec{k}\times\hat{z}).$$

The growth rate $\gamma_D=\mathrm{Im}\,\omega$ for the decay instability is

$$\gamma_D=\left(\frac{P^2}{Q^2}(R^2+S^2)\right)^{1/4}\left|\sin\left(\frac{1}{2}\tan^{-1}\frac{S}{R}\right)\phi_0\right|, \quad (20)$$

where $\gamma_D\gg\Gamma$, $\delta\omega$ has been assumed.

Next, we consider the problem of modulational instability. Here, both of the satellites are not normal modes of the system. We must thus retain both the Stokes and anti-Stokes drift satellites. The dispersion relation for this case becomes

$$(\omega+i\Gamma)(\omega^2-\delta\omega^2)=2(P\omega/Q)(R-iS)|\phi_0|^2. \quad (21)$$

For the limit γ_m (growth rate of the modulational instability) $\ll\Gamma,\delta\omega$, we obtain

$$\gamma_m=2^{1/2}\gamma_D. \quad (22)$$

When the plasma β becomes larger than m_e/m_i (but $\beta<1$) then the drift wave acquires an electromagnetic character, usually refered to as the drift-Alfvén wave.[7] In this case, it is possible to consider the parametric drive of magnetostatic modes by drift-Alfvén waves. The essential formulation of our work remains unchanged except that one must include some new nonlinear terms owing to the magnetic fields of the high-frequency waves.

IV. DISCUSSION

In this paper, we have shown that the magnetostatic mode can be enhanced to above the thermal level in a nonuniform plasma. Universally unstable drift waves in a low-β plasma can pump energy into the magnetic fluctuations via either the decay or modulational instability of the drift waves. We found that the growth rates of these instabilities are approximately

$$\gamma/\omega_0\sim 2k_0\lambda_D(\omega_{pe}/\Omega_e)^2|E_0^2/8\pi n_0T_e|^{1/2}, \quad (23)$$

where λ_D is the electron Debye length. For a typical plasma ($e\phi/T_e\sim10^{-2}$, $k_{0\perp}c_s/\Omega_i\sim1$, $B_0\sim10^4$ Gauss, $\rho_s/L\sim0.1$, $\omega_{pe}\sim\Omega_e$), one obtains a growth of the order of 10^2/sec. Thus, the instabilities discussed here are of importance in steady-state devices.

The two-dimensional magnetic fluctuations perpendicular to the external magnetic field can give rise to an electron cross-field diffusion given by[1]

$$D=\frac{T}{m_eB_0^2}\sum_k\int_0^\infty\langle B^2\rangle_k\exp(-Dk^2\tau)d\tau, \quad (24)$$

where $T=m_e\langle v_z^2\rangle$ is the parallel electron temperature. This diffusion is due to the $v_z\vec{B}_k/B_0$ drift of the electrons. It is not possible at present to estimate the diffusion coefficient because the saturation mechanism of the instability is not yet well understood, so that the spectrum $\langle B^2\rangle_k$ cannot be calculated self-consistently. However, since the $v_z\vec{B}_k/B_0$ drift corresponds to a two-dimensional incompressible motion, one might speculate that the usual dual-cascade behavior of a two-dimensional fluid is also realized here, although an external current in the z direction is necessary for the mode coupling to occur. In this case, we expect an eventual quasi-steady state consisting of large scale magnetic islands.[3,8]

ACKNOWLEDGMENTS

This work has been performed under the auspices of the Sonderforschungsbereich 162 "Plasmaphysik Bochum/Jülich" and "DFVLR/ISRO exchange program." One of us (H.U.R.) is grateful to DAAD for the award of a fellowship.

*Permanent address: Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan.

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