Oscillatory convective instability in a superfluid ^{3}He-^{4}He mixture

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The linear stability analysis of a superfluid mixture subject to a vertical temperature gradient is presented. The main result obtained in this paper is the existence of an oscillatory instability in a superfluid mixture heated only from below. The criteria for onset of oscillatory instability and the corresponding neutral frequencies in a superfluid region of the ^{3}He-^{4}He phase diagram are treated. Depending on the magnitude of dissipation of superfluid motion, two different types of overstabilities exist. These have different physical origins. In the case of large dissipation of superfluid motion the oscillatory criterion and the neutral frequency are similar to those in a regular binary mixture with large abnormal thermal diffusion. In the case of small dissipation of superfluid motion a fundamentally new type of oscillatory convective instability is predicted. This instability is actually the undamped standing second-sound waves. We believe it should be possible to observe the predicted effect in the vicinity of the tricritical point.

I. INTRODUCTION

In the preceding paper, hereafter referred to as (I),[1] we discussed the stationary hydrodynamic instability of a horizontal layer of a superfluid ^{3}He-^{4}He mixture subject to a vertical temperature gradient. As shown in (I), there are several features common to a superfluid ^{3}He-^{4}He mixture and a regular binary mixture with an abnormally large thermal diffusion ($k_T > 0$). First of all, the concentration distribution is similar. Secondly, stationary convection commences when heated from above. Besides, as is well known,[2,3] oscillatory instability in a regular binary mixture with abnormal thermal diffusion occurs also when heated from below. In view of the common features, one would also expect the onset of the oscillatory instability in a superfluid mixture when heated from below. But in the case of a superfluid mixture two different physical mechanisms may be responsible for this instability. The first resembles the situation in a regular binary mixture. In this case, the instability is the result of a competition between a stabilizing effect (with a long relaxation time) and a destabilizing effect (with a short relaxation time).

Since the relaxation time of concentration perturbations $(Dk^2)^{-1}$ is usually long compared to the relaxation time of temperature perturbations $(\kappa k^2)^{-1}$, the stabilizing effect of concentration in a binary mixture with abnormal thermal diffusion heated from below can be eliminated while retaining the destabilizing effect of the temperature perturbations. Indeed, let us consider a fluid element with an upward velocity perturbation near the lower boundary. If its temperature relaxes to the bath faster than its concentration, it becomes richer on the heavier component compared with the surroundings. Then the restoring force causes the sinking of this fluid element. Obviously, while sinking it becomes poorer in the heavier component compared to the surroundings, for the same reason. Therefore, the direction of the restoring force changes again. These oscillations (overstability) will set in when the rate of a varying amount of kinetic energy and the rate of viscous dissipation and production of kinetic energy by the buoyancy force are synchronously balanced. Therefore, in this case the overstability onset is determined by the temperature gradient as well as by the ratio of the relaxation rates of the temperture and concentration fluctuations.

The second mechanism has a purely superfluid origin and is associated with an additional branch in the hydrodynamic perturbation spectrum. Any perturbations in the noncompressible superfluid ^{3}He-^{4}He mixture lead to the appearance of second-sound waves which decay rather rapidly.

When the second-sound wave velocity becomes rather low and comparable to the velocity of internal gravity waves,[4] one expects the rate of energy supply by the gravitational field to balance the rate of wave dissipation. Then the initiation of undamped second-sound waves, in fact, becomes possible. On the other hand, these steady waves are nothing but the neutral oscillations of the convective overstability.[5]

Temperature gradients and the relation between the characteristic time scales determine the onset of oscillatory instability in this case too.[5]

In both cases considered above, the neutral oscillations represent the undamped standing temperature (and concentration) waves. However, there are distinct features in these two cases. First of all, in the second case the neutral frequency is proportional to the second-sound wave velocity. In addition, the superfluid and normal component in these oscillations move in opposite directions as in the second-sound waves, so that the mass flux is identically zero. On the other hand, in the first case, mass transfer takes place inside each convective cell because both superfluid and normal components move together.

In Sec. II of this work, the criteria of the oscillatory instability in different limiting cases are obtained from the convection equations discussed in (I). We do not consider additional dissipation caused by the div V_n contribution in the Navier-Stokes equation. This contribution was treated in (I). As shown in (I), these terms correspond to relatively small "dissipation" lengths $(l_0/l)^3 < 1$. As in the stationary convection, two limiting cases with respect to the value of dissipation of superfluid motion are considered. In Sec. III, the results of criteria calculations in the superfluid range of temperatures and concentrations of the ^{3}He-^{4}He phase diagram are tabulated and analyzed. The asymptotic behavior of the oscillatory instability criteria near the λ line, near the tricritical point, and for dilute solutions are also described here. The results of this work are summarized in Sec. IV.

II. OVERSTABILITY ONSET CRITERIA

For the sake of simplicity we consider [as in (I)] a horizontal layer of superfluid ^{3}He-^{4}He mixture with free boundaries a distance l apart which are good heat conductors.

In order to obtain the oscillating instability criteria in a ^{3}He-^{4}He superfluid mixture, we used the complete set of the convection equations in the scaled variables[1] [all symbols are the same as in (I)]

$$\begin{aligned}
&\operatorname{div}\vec{j}=0,\quad \vec{j}=\vec{V}_n+\frac{\rho_s}{\rho_n}\vec{V}_s\,,\\
&\frac{\partial}{\partial t}(\nabla\operatorname{div}-\Delta)\vec{V}_n=(\nabla\operatorname{div}-\Delta)\vec{V}_n-(\nabla\operatorname{div}-\Delta)\vec{\gamma}\left[R_a\sigma+\left(\frac{l}{l_0}\right)^3L\mu_4\right],\\
&\frac{\partial}{\partial t}\operatorname{div}\vec{V}_n=\left[\left(\frac{l}{l_0}\right)^3\Delta\mu_4-m\Delta\operatorname{div}\vec{V}_n\right]\frac{\rho_s}{\rho_n}\,,\\
&P_T\frac{\partial\sigma}{\partial t}+\frac{nP_TL}{aR_a}\left(\frac{l}{l_0}\right)^3\frac{\partial\mu_4}{\partial t}+\vec{V}_n\vec{\gamma}+\left(\frac{l}{l_0}\right)^3\frac{\operatorname{div}\vec{V}_n}{aR_a}=\frac{a_1P_T}{aP_C}\Delta\sigma+\frac{n_1P_TL}{aP_CR_a}\left(\frac{l}{l_0}\right)^3\Delta\mu_4\,,\\
&P_T\frac{\partial\sigma}{\partial t}+\vec{V}_n\vec{\gamma}=\Delta\sigma+\frac{Ld}{R_a}\left(\frac{l}{l_0}\right)^3\Delta\mu_4\,,
\end{aligned}\tag{1}$$

where the following symbols are used:

$$\begin{aligned}
&R_a=\frac{1}{\rho_n}\left(\frac{\partial\rho}{\partial\sigma}\right)_{P,\mu_4}\frac{\rho_n g l^4}{\eta\kappa}\frac{d\sigma_0}{dz},\quad \nabla\sigma_0=\frac{d\sigma_0}{dz}\vec{\gamma},\quad l_0^3=\frac{\eta\kappa}{g\rho_n},\\
&m=\rho_n\frac{\zeta_4-\rho\zeta_3}{\eta},\quad \kappa=\frac{\chi_{\text{eff}}}{\rho CT}\left(\frac{\partial T}{\partial\sigma}\right)_{P,\mu_4},\quad P_T=\frac{\eta}{\rho_n\kappa},\quad P_C=\frac{\eta}{\rho_n D},\\
&a=\frac{\rho_n}{c}\frac{\left(\frac{\partial c}{\partial\sigma}\right)_{P,\mu_4}}{\left(\frac{\partial\rho}{\partial\sigma}\right)_{P,\mu_4}},\quad a_1=a\left[1+\frac{k_T}{T}\frac{\left(\frac{\partial T}{\partial\sigma}\right)_{P,\mu_4}}{\left(\frac{\partial c}{\partial\sigma}\right)_{P,\mu_4}}\right],\\
&n=\frac{\rho_n}{c}\frac{\left(\frac{\partial c}{\partial\mu_4}\right)_{P,\sigma}}{\left(\frac{\partial\rho}{\partial\mu_4}\right)_{P,\sigma}},\quad n_1=n\left[1+\frac{k_T}{T}\frac{\left(\frac{\partial T}{\partial\mu_4}\right)_{P,\sigma}}{\left(\frac{\partial c}{\partial\mu_4}\right)_{P,\sigma}}\right],\\
&d=\frac{\left(\frac{\partial T}{\partial\mu_4}\right)_{P,\sigma}}{\left(\frac{\partial T}{\partial\sigma}\right)_{P,\mu_4}}\frac{\left(\frac{\partial\rho}{\partial\sigma}\right)_{P,\mu_4}}{\left(\frac{\partial\rho}{\partial\mu_4}\right)_{P,\sigma}},\quad L=\frac{gl}{\rho_n}\left(\frac{\partial\rho}{\partial\mu_4}\right)_{P,\sigma}.
\end{aligned}\tag{1'}$$

In (I) we already noted that the set of equations (1) is non-self-adjoint. It is caused by taking into account the condition $\operatorname{div}V_n\neq 0$ in the Navier-Stokes equation. The resulting conclusions are investigated in (I). There we only discuss the case $\operatorname{div}V_n=0$ in the Navier-Stokes equation, valid when $(l_0/l)^3<1$ (I), i.e., when the dissipation length l_0 is small relative to the layer height l (small dissipation of a normal motion). This approximation is valid for $T > 0.5$ K as shown in (I). Thus in this paper we do not consider additional stabilizing effects caused by transferring the energy of superfluid motion to the normal one that dissipates rapidly when $(l_0/l)^3 > 1$.[1]

Regarding the stability analysis for the system (1), it remains rather complicated and, as in (I), we will investigate it in two limiting cases with respect to the parameter m.

1. $m \gg 1$. Large dissipation of superfluid motion. Since in this case the superfluid motion is insignificant, the fluctuations of the chemical potential μ_4 relax mostly diffusively as in a regular binary mixture. The difference between this case and a regular binary mixture shows up only in the relation between ΔT_0 and Δc_0 [see I (1)] (which is a result of superfluidity). Therefore in the superfluid case $\mu_4 \neq 0$ and $\text{div} V_n = 0$. Thus, there are two thermodynamic variables σ and μ_4, and we can express them and the vertical component of the normal velocity V_{nz} in the form

$$[V_{nz}, \sigma, \mu_4] = [v(z), \sigma(z), \xi(z)] e^{\lambda t} \cdot e^{i\vec{k}\cdot\vec{r}}. \tag{2}$$

Then the set of the convection equations can be obtained as

$$(\lambda - D)Dv - R_a k^2 \sigma - \left(\frac{l}{l_0}\right)^3 L k^2 \xi = 0 ,$$

$$v + \frac{P_T}{P_C}\left(\lambda P_C - \frac{a_1}{a} D\right)\sigma + \frac{nP_T L}{aP_C R_a}\left(\frac{l}{l_0}\right)^3 \left(\lambda P_C - \frac{n_1}{n} D\right)\xi = 0 ,$$

$$v + (\lambda P_T - D)\sigma - \frac{Ld}{R_a}\left(\frac{l}{l_0}\right)^3 D\xi = 0 ,$$

$$\left(D \equiv \frac{d^2}{dz^2} - k^2, \quad k^2 = k_x^2 + k_y^2\right), \tag{3}$$

with free boundary conditions

$$v = \frac{d^2 v}{dz^2} = \sigma = \xi = 0$$

at

$$z = \pm\tfrac{1}{2}.$$

As a result of the stability analysis (3) we obtain the following secular equation:

$$\lambda^3 + S_1\lambda^2 + S_2\lambda + S_3 = 0 , \tag{4}$$

where

$$S_1 = \frac{\pi^2 + k^2}{P_T}\left(1 + P_T + \frac{P_T}{P_C}\frac{n_1}{n} - \frac{ad}{n}\right),$$

$$S_2 = \frac{1}{P_T(\pi^2 + k^2)}\left[(\pi^2 + k^2)^3\left(1 + \frac{1 + P_T}{P_C}\frac{n_1}{n} - \frac{ad}{n} - \frac{a_1 d}{nP_C}\right) - R_a k^2\right],$$

$$S_3 = P_T^{-2}\left[(\pi^2 + k^2)^3 \frac{P_T}{P_C}\left(\frac{n_1}{n} - \frac{a_1 d}{n}\right) - R_a k^2\left(\frac{P_T}{P_C}\frac{n_1 - a_1}{n} + \frac{a(1-d)}{n}\right)\right].$$

The resulting condition for stationary instability following (4) is $S_3 = 0$, and the corresponding criterion appears as

$$\tilde{R}_a^s = 27\pi^4/4 \quad \text{at} \quad k_c^2 = \pi^2/2 ,$$
$$\tilde{R}_a^s = R_a\psi_1, \quad \psi_1 = \frac{P_T(n_1 - a_1) - aP_C(d-1)}{P_T(n_1 - a_1 d)}. \tag{5}$$

This is identical to expression (23) in (I). The criterion for oscillatory instability and the corresponding frequency of overstable motion are determined by Orlando's relations[6]

$$S_1 S_2 - S_3 = 0, \quad S_3 > 0 \quad \text{and} \quad \omega^2 = S_2. \tag{6}$$

From (6) it follows that

$$\tilde{R}_a^0 = \frac{27\pi^4}{4} \quad \text{at} \quad k_c^2 = \frac{\pi^2}{2}, \quad \tilde{R}_a^0 = R_a\psi_2 , \tag{7}$$

$$\psi_2 = nP_C \frac{nP_C(1 + P_T) + a_1 P_T - aP_C}{[(n - ad)P_C + n_1 P_T][(nP_C + n_1)(1 + P_T) - d(a_1 + aP_C)]},$$

$$\omega^2 = \frac{9\pi^4}{4P_T}\left(1 + \frac{n_1 - a_1 d}{nP_C} + \frac{n_1 P_T}{nP_C} - \frac{ad}{n} - \psi_2^{-1}\right),$$

and the criterion and the characteristic frequency are similar to those for a regular binary mixture.[2,3]

2. $m \ll 1$. Small dissipation of superfluid motion. In this case, the superfluid motion is essential, and any perturbations of the chemical potential relax with the second-sound velocity u, if the second-sound velocity is the largest one in the system.

Assuming that the variable fluctuations have the same form as in (2), from the condition $\mathrm{div}\vec{V}_n = 0$ in the Navier-Stokes equation we obtain in this case the following set of convection equations:

$$(\lambda - D)Dv - R_a k^2\sigma - \left(\frac{l}{l_0}\right)^3 L k^2 \xi = 0 ,$$
$$\lambda\left(1 - \frac{a_1 P_T}{a P_C}\right)D\sigma + \frac{n P_T L}{a R_a}\left(\frac{l}{l_0}\right)^3\left[\lambda^2 + \lambda P_T^{-1}\left(\frac{n_1 P_T}{n P_C} - \frac{ad}{n}\right) - \omega_0^2 D\right]\xi = 0 , \tag{8}$$
$$v + (\lambda P_T - D)\sigma - \frac{Ld}{R_a}\left(\frac{l}{l_0}\right)^3 D\xi = 0 ,$$

where $\omega_0^2 = -(1/nP_T L)(\rho_s/\rho_n)(l/l_0)^3$ and we assume the same boundary conditions as in (3). The appropriate secular equation has the form

$$\lambda^4 + S_1\lambda^3 + S_2\lambda^2 + S_3\lambda + S_4 = 0 , \tag{9}$$

where

$$S_1 = \frac{\pi^2 + k^2}{P_T}\left(1 + P_T + \frac{n_1 P_T}{n P_C} - \frac{ad}{n}\right),$$
$$S_2 = \frac{1}{P_T(\pi^2 + k^2)}\left[(\pi^2 + k^2)^3\left(1 + \frac{n_1 - a_1 d}{n P_C} + \frac{n_1 P_T}{n P_C} - \frac{ad}{n}\right) + P_T\omega_0^2(\pi^2 + k^2)^2 - R_a k^2\right] ,$$
$$S_3 = P_T^{-2}\left[(\pi^2 + k^2)^3 P_T \frac{n_1 - a_1 d}{n P_C} + \omega_0^2 P_T(1 + P_T)(\pi^2 + k^2)^2 - \left(\frac{a}{n} - \frac{a_1}{n}\frac{P_T}{P_C} + \frac{n_1 P_T}{n P_C} - \frac{ad}{n}\right)R_a k^2\right],$$
$$S_4 = \omega_0^2 P_T^{-1}[(\pi^2 + k^2)^3 - R_a k^2] .$$

Comparing to the first case [see Eq. (4)], we see that the λ^4 term in the secular equation (9) manifests the additional mode in the perturbation spectrum. Indeed, as in a regular binary mixture, in the first case three modes in the perturbation spectrum are obtained. There are hydrodynamic, thermal, and diffusive modes. As is well known, the additional mode in the superfluid mixture perturbation spectrum is the second-sound mode.

Let us analyze Eq. (9). The condition for stationary stability following from (9) is $S_4 = 0$. That corresponds to the criterion of stationary instability

$$R_a^s = 27\pi^4/4 \quad \text{at} \quad k_c^2 = \pi^2/2 . \tag{10}$$

This result was obtained in (I) at $\delta = 0$. Using Orlando's formula,[6] one can obtain the following condition for oscillatory instability:

$$S_3(S_1 S_2 - S_3) - S_1^2 S_4 = 0 . \tag{11}$$

Then the corresponding neutral frequency is determined by

$$\omega^2 = S_3/S_1 . \tag{12}$$

From the relation (11) the quadratic equation for the oscillatory instability criterion follows:

$$\nu_0 R_a^2 - [\nu_1(\pi^2 + k^2) + \nu_2 P_T\omega_0^2]\frac{\pi^2 + k^2}{k^2}R_a + [\nu_3(\pi^2 + k^2)^2 + \nu_4 P_T\omega_0^2(\pi^2 + k^2) + \nu_5 P_T^2\omega_0^4]\frac{(\pi^2 + k^2)^4}{k^4} = 0 , \tag{13}$$

where ν_i are the cumbersome functions of parameters P_T, P_C, a, a_1, n, n_1, and d from Eq. (1). Thus, from Eq. (13) two oscillatory instability branches occur which consist of the interaction between usual hydrodynamic modes and the second-sound mode. Clearly, in this case, the criteria of oscillatory instability will be very complicated functions of all coefficients ν_i and it is impossible to express them in analytical form. Hence, let us consider two limiting cases. The convenient parameter for this consideration is the relation $\omega_0^2 P_T/(\pi^2 + k^2)$ (all coefficients ν_i should be of order unity). In the first limiting case when $\omega_0^2 P_T/(\pi^2 + k^2) < 1$, the preceding results are obtained. It should be noted here that one of the solutions is the oscillatory instability criterion (7), but the second solution is the stationary instability criterion (5) because the neutral frequency for this solution is identically zero.

The physical reason for this mode of the oscillatory instability when $m < 1$ and $\omega_0^2 P_T/(\pi^2+k^2) < 1$ is clear. As long as the characteristic time of the second-sound wave propagation ω_0^{-1} is large compared to the characteristic times of relaxation processes $(Dk^2)^{-1}$, $(\kappa k^2)^{-1}$, and $(\eta k^2)^{-1}$, the latter becomes the dominating factor in the fluctuation dynamics. Then the fluctuations of chemical potential relax diffusively, and the competition between the different relaxation modes determines the mechanical stability of the system, as mentioned above.

In the second limiting case when $\omega_0^2 P_T/(\pi^2+k^2) > 1$, a new type of overstability is obtained from Eq. (13). Two branches of oscillatory instability appear here and as a result the following expressions for the criteria and the neutral frequencies, are, respectively (after minimizing k and substituting k_c in the expression for the neutral frequency),

$$\tilde{R}_{a_i}^0 = 4\pi^2$$

at

$$k_c = \pi \quad (i=1,2)$$

$$\tilde{R}_{a_1}^0 = -R_a P_T^{-1}\omega_0^{-2}\left(1 - \frac{a_1 P_T - aP_C}{n_1 P_T - adP_C}\right),$$

$$\omega_1^2 = 2\pi^2\omega_0^2 , \tag{14}$$

and

$$\tilde{R}_{a_2}^0 = -R_a P_T^{-1}\omega_0^{-2}\left(1 + \frac{a_1 P_T - aP_C}{nP_C(1+P_T)}\right),$$

$$\omega_2^2 = 2\pi^2\omega_0^2\left(1 + \frac{a_1 P_T - aP_C}{nP_C(1+P_T)}\right)^{-1}. \tag{15}$$

Let us show now that ω_0 is the frequency of the standing second-sound waves. Indeed, the velocity of the second-sound wave in an incompressible superfluid mixture (in the thermodynamic variables P, σ, and μ_4) is

$$u^2 = -\frac{\rho_s}{\rho_n} c\left(\frac{\partial \mu_4}{\partial c}\right)_{P,\sigma}. \tag{16}$$

Therefore, the characteristic frequency of the standing second-sound wave is given in dimensional form by

$$\omega_*^2 = u^2 k^2 = -\frac{\pi^2}{l^2}\frac{\rho_s}{\rho_n} c\left(\frac{\partial \mu_4}{\partial c}\right)_{P,\sigma}. \tag{17}$$

The expression (17) is the same as the dimensional form of ω_0^2 from (8).

In addition, we would like to emphasize the features of criteria (14) and (15). First of all, it is easy to show that both expressions are proportional to l^2 [unlike the usual dependence of the Rayleigh number on a layer height ($\sim l^4$)]. Both criteria are proportional to the square of ratio of the frequencies

$$\tilde{R}_{a_i}^0 \sim -(\omega_g/\omega_d)^2 . \tag{18}$$

Here, $\omega_g^2 = (g/\rho_n)(\partial\rho/\partial\sigma)_{P,\mu_4}(d\sigma_0/dz)$ is the internal gravity wave frequency[4] and $\omega_d^2 = \omega_0^2\eta^2/\rho_n^2 l^4$ is the characteristic frequency of the standing second-sound wave in dimensional form ($\omega_d^2 \sim \omega_*^2$). This expression is very different from the usual one for the Rayleigh number (that is, derived from the relation between the buoyancy force and stabilizing dissipation factor). But criteria (14) and (15) can also be described as resulting from competition between two mechanisms with different characteristic times $(\rho_n l^4/\eta\kappa)$, and ω_d^{-1}. Therefore, as in the preceding case, the overstability onset here is determined by the entropy gradient as well as by the relation between the two time scales. Indeed, both criteria (14) and (15) can be written as

$$\tilde{R}_{a_i}^0 \sim -R_a P_T^{-1}\omega_0^{-2} = -R_a\frac{\eta\kappa}{\rho_n\omega_d^2 l^4} = -\left(\frac{\omega_g}{\omega_d}\right)^2 , \tag{19}$$

which coincides with (18).

III. NUMERICAL ESTIMATES; EXPERIMENTAL DETECTABILITY

(1) Let us write the obtained results in convenient thermodynamic variables and estimate the value of the critical temperature gradient and its sign in each limiting case. From (7), (14), and (15), the expressions for the criteria and neutral frequencies are functions of five parameters a, a_1, n, n_1, and d. But they can be written in a form where these parameters are expressed as functions of three parameters only (e.g., φ, φ_0, and ad/n). The parameter φ is the separation[1,2,3] and defines the significance of thermal diffusion for convection instability. The remaining parameters are purely thermodynamic. Thus from (7), (14), and (15) we get

$$\tilde{R}_a^0 = R_a\psi_2 ,$$

$$R_a = -\frac{\alpha_T g\rho^2\left(C_{P,c} + K^2 T\,\dfrac{\partial(z/\rho)}{\partial c}\right)}{\eta\chi_{\text{eff}}}(1+\varphi_0)l^4\frac{dT_0}{dz} ,$$

$$\psi_2 = \frac{1 + P_T + \dfrac{a}{n}\left(\dfrac{a_1 P_T}{aP_C} - 1\right)}{\left(1 - \dfrac{ad}{n} + \dfrac{n_1 P_T}{nP_C}\right)\left[(1+P_C^{-1})(1+P_T) - \dfrac{ad}{n}\left(1 + \dfrac{a_1}{a}P_C^{-1}\right)\right]},$$

$$\omega^2 = \frac{9\pi^4}{4P_T}\left[\left(1 - \frac{ad}{n}\right)(1+P_C^{-1}) + \frac{n_1 P_T}{nP_C} - \psi_2^{-1}\right], \tag{20a}$$

$$\tilde{R}_{a_1}^0 = -R_a P_T^{-1}\omega_0^{-2}\left(1 + \frac{a}{n}\,\frac{1 - a_1 P_T/aP_C}{n_1 P_T/nP_C - ad/n}\right),$$

$$\omega_0^2 = -\frac{c\left(\dfrac{\partial\mu_4}{\partial c}\right)_{P,T}}{glP_T}\left(1 - \frac{ad}{n}\right)\left(\frac{l}{l_0}\right)^3\frac{\rho_s}{\rho_n} , \quad \omega_1^2 = 2\pi^2\omega_0^2 ; \tag{20b}$$

$$\tilde{R}^0_{a_2} = -R_a P_T^{-1} \omega_0^{-2} \left(1 - \frac{a}{n} \frac{1 - a_1 P_T/aP_C}{1 + P_T}\right),$$

$$\omega_2^2 = 2\pi^2 \omega_0^2 \left(1 - \frac{a}{n} \frac{1 - a_1 P_T/aP_C}{1 + P_T}\right)^{-1}, \tag{20c}$$

where

$$\frac{a_1}{a} = 1 - \frac{\varphi}{\varphi_0}, \quad \frac{a}{n} = \frac{\varphi_0 + ad/n}{1 + \varphi_0}, \quad \frac{n_1}{n} = 1 - \frac{\varphi}{\varphi_0} \frac{ad}{n},$$

$$\varphi = -\frac{\beta}{\alpha_T} \frac{k_T}{T}, \quad \varphi_0 = -\frac{\beta}{\alpha_T} \frac{\left(\frac{\partial \mu_4}{\partial T}\right)_{P,c}}{\left(\frac{\partial \mu_4}{\partial c}\right)_{P,T}},$$

$$\frac{ad}{n} = \frac{\left(\frac{\partial \mu_4}{\partial T}\right)_{P,c} \left(\frac{\partial \sigma}{\partial c}\right)_{P,T}}{\left(\frac{\partial \mu_4}{\partial c}\right)_{P,T} \left(\frac{\partial \sigma}{\partial T}\right)_{P,c}}.$$

In Table I of (I) typical values of the parameters affecting the onset of oscillatory convection for several values of concentrations and temperatures below the λ line and on the left of the coexistence separation line on the ^{3}He-^{4}He phase diagram are tabulated. The sign of the critical temperature gradients causing the overstability and appropriate neutral frequencies can be determined without calculations.

First of all, when the overstability shows up as the undamped second-sound wave, the characteristic frequency $\omega_0^2 > 0$. Indeed, as seen from expression (20b) the sign of ω_0^2 is determined by the thermodynamic parameter ad/n and the derivative $(\partial \mu_4/\partial c)_{P,T}$ which should always be negative from the condition of thermodynamic stability of binary mixtures. On the other hand, it is possible to show that the expression enclosed in brackets in (20b) is always positive. For this purpose let us write this expression as a function of the parameters φ, φ_0, and ad/n as

$$1 + \frac{a}{n} \frac{1 - a_1 P_T/aP_C}{n_1 P_T/nP_C - ad/n} = \left[\left(1 - \frac{\varphi}{1+\varphi_0} \frac{ad}{n}\right) \frac{P_T}{P_C} + \frac{\varphi_0}{1+\varphi_0} - \frac{ad}{n}\right] \times \left[\left(1 - \frac{\varphi}{\varphi_0} \frac{ad}{n}\right) \frac{P_T}{P_C} - \frac{ad}{n}\right]^{-1}. \tag{21}$$

The sign of the parameters φ and φ_0 is determined by the sign of α_T only, but both have the same sign in the superfluid mixtures. Besides, a superfluid mixture is similar to a regular one with large abnormal thermal diffusion; it means that both parameters $|\varphi|$ and $|\varphi_0|$ should be at least larger than unity. It is then easy to see that expression (21) is positive. Therefore, the sign of the criterion $\tilde{R}^0_{a_1}$ is opposite to the sign of R_a; i.e., in this case the oscillatory instability occurs in superfluid mixtures heated from below. From (20c) the sign of the criterion $\tilde{R}^0_{a_2}$ is also opposite to the sign of R_a when oscillatory convection takes place, i.e., when $\omega_2^2 > 0$. Thus, if this oscillatory instability branch is realized, it occurs also in a system heated from below.

In the second limiting case, when the usual hydrodynamic branch of oscillatory instability appears, the sign of the neutral frequency (20a) is determined by the value and sign of the parameter ψ_2 only (all other terms are positive). In any case, when $\psi_2 < 0$ the neutral frequency $\omega^2 > 0$, and, therefore, the sign of the criterion $\tilde{R}^0_a$ is opposite to that of R_a, i.e., in this case too the overstability occurs when the mixture is heated from below.[7] Thus, the oscillatory convection appears in a superfluid mixture heated from below only, unlike the stationary convection that appears when heated from above only.[1]

Since the value of the parameter m in the different regions of the ^{3}He-^{4}He phase diagram was previously discussed in detail and depicted in Fig. 1 (I), here we determine the value of the second parameter $\omega_0^2 P_T/(\pi^2 + k^2)$. As seen from Table I the parameter is always more than one in every part of the phase diagram where $(l_0/l)^3 < 1$.

Therefore, there are actually two limiting cases in accordance with the value of the parameter m.

(1) $m \gg 1$, where the oscillatory instability criterion (20a) is similar to the one in a regular binary mixture with large abnormal thermal diffusion.[2,3]

(2) $m \ll 1$ and $\omega_0^2 P_T/(\pi^2 + k^2) > 1$, where the overstability appears as the undamped second-sound wave.

The results of the oscillatory instability criteria and the neutral frequencies calculations are shown in Table I.

As is apparent from Table I in the region of the ^{3}He-^{4}He phase diagram, where $m \gg 1$ [at $T > 0.8$ K (Ref. 1)], there is a narrow temperature interval near 1 K at low concentrations which widen up to 1.5 K (between 10% and 30% ^{3}He), where the neutral frequency $\omega^2 > 0$. Therefore, the corresponding overstability branch occurs in this temperature and concentration range. The concentration dependence of both the appropriate critical temperature gradient and neutral frequency at temperature 1 and 1.5 K is presented in Fig. 1 (at layer height $l = 1$ cm).

As can be seen from Fig. 1, the mechanical stability of a superfluid mixture heated from below rises with reduced concentration. Therefore, the critical temperature gradient increases by a

TABLE I. The values of the oscillatory instability criteria and the neutral frequencies at different temperatures and concentrations (in % mol^{3}He): (a) $x=0.01$ (b) $x=0.1$ (c) $x=1$ (d) $x=10$ (e) $x=30$ (f) $x=50$.

(a)

$x=0.01$

T (K)	$\omega_0^2 P_T l^{-2}$ (cm^{-2})	$\frac{ad}{n}$	α_T (10^{-3}K^{-1})	φ	φ_0	ψ_2	$R_a / l^4\frac{dT_0}{dz}$ (cm^{-3}K^{-1})	$\tilde{R}_a^0 / l^4\frac{dT_0}{dz}$ (cm^{-3}K^{-1})	$\omega_d^2 l^4$ (cm^{4}sec^{-2})	$\tilde{R}_{a_1}^0 / l^2\frac{dT_0}{dz}$ (cm^{-1}K^{-1})	$\omega_{1d}^2 l^2$ (cm^{2}sec^{-2})	$\tilde{R}_{a_2}^0 / l^2\frac{dT_0}{dz}$ (cm^{-1}K^{-1})	$\omega_{2d}^2 l^2$ (cm^{2}sec^{-2})
0.5	2×10^4	−4.4	0.13	−1.4	−2.5	−0.02	0.2	-4×10^{-3}	5.4×10^7	-1.8×10^{-5}	1.5×10^9	2.7×10^{-6}	-5.6×10^9
1	1.4×10^{10}	−32.4	0.46	−3.9	−7	−0.0012	1.8×10^5	−216	94	-1.4×10^{-5}	1.1×10^8	7×10^{-6}	-2.10^8
1.5	1.3×10^{13}	−224.5	−2	7.2	9.6	8.6×10^{-5}	2.5×10^8	1.5×10^4	−1.0	-1.9×10^{-5}	1.1×10^8	-1.2×10^{-4}	1.7×10^7
2	1.75×10^{14}	−1103	−12	4.2	5.7	3.8×10^{-5}	4.45×10^9	1.1×10^5	−0.09	-2.5×10^{-5}	6.3×10^7	-8.2×10^{-4}	2×10^6

(b)

$x=0.1$

T (K)	$\omega_0^2 P_T l^{-2}$ (cm^{-2})	$\frac{ad}{n}$	α_T (10^{-3}K^{-1})	φ	φ_0	ψ_2	$R_a / l^4\frac{dT_0}{dz}$ (cm^{-3}K^{-1})	$\tilde{R}_a^0 / l^4\frac{dT_0}{dz}$ (cm^{-3}K^{-1})	$\omega_d^2 l^4$ (cm^{4}sec^{-2})	$\tilde{R}_{a_1}^0 / l^2\frac{dT_0}{dz}$ (cm^{-1}K^{-1})	$\omega_{1d}^2 l^2$ (cm^{2}sec^{-2})	$\tilde{R}_{a_2}^0 / l^2\frac{dT_0}{dz}$ (cm^{-1}K^{-1})	$\omega_{2d}^2 l^2$ (cm^{2}sec^{-2})
0.5	8.3×10^6	−2.4	0.13	−3	−5	−0.0014	40	−0.056	7.6×10^3	-6.4×10^{-6}	7×10^8	1.5×10^{-6}	-2.2×10^9
1	1.9×10^9	−2	0.46	−1.8	−4.4	−0.053	3.3×10^5	-1.75×10^4	16.5	-3.2×10^{-4}	1.2×10^8	1.1×10^{-4}	-1.8×10^8
1.5	10^{12}	−18	−2	2.4	7.3	3.3×10^{-3}	3.3×10^6	10^4	−0.28	-3×10^{-6}	10^8	-7.3×10^{-6}	4×10^7
2	1.6×10^{13}	−83	−12	1.4	4.2	1.1×10^{-3}	4.6×10^7	5×10^4	−0.035	-2.4×10^{-6}	0.8×10^8	-4.4×10^{-5}	0.5×10^7

(c)

$x=1$

T (K)	$\omega_0^2 P_T l^{-2}$ (cm^{-2})	$\frac{ad}{n}$	α_T (10^{-3}K^{-1})	φ	φ_0	ψ_2	$R_a / l^4\frac{dT_0}{dz}$ (cm^{-3}K^{-1})	$\tilde{R}_a^0 / l^4\frac{dT_0}{dz}$ (cm^{-3}K^{-1})	$\omega_d^2 l^4$ (cm^{4}sec^{-2})	$\tilde{R}_{a_1}^0 / l^2\frac{dT_0}{dz}$ (cm^{-1}K^{-1})	$\omega_{1d}^2 l^2$ (cm^{2}sec^{-2})	$\tilde{R}_{a_2}^0 / l^2\frac{dT_0}{dz}$ (cm^{-1}K^{-1})	$\omega_{2d}^2 l^2$ (cm^{2}sec^{-2})
0.5	1.3×10^{10}	−4.8	0.13	−110.8	−36	4×10^{-5}	2.2×10^4	0.88	-2.5×10^4	-2×10^{-6}	9×10^8	1.4×10^{-4}	-10^7
1	1.25×10^{11}	−1.1	0.58	−17	−7	−0.24	1.7×10^6	-4×10^5	0.5	-2.5×10^{-5}	4.4×10^8	9×10^{-6}	-6.6×10^8
1.5	1.25×10^{13}	−2.4	−1.7	14	9.3	1.3×10^{-2}	2.4×10^8	3×10^6	-10^{-3}	-2.2×10^{-5}	1.8×10^8	-7×10^{-6}	4.8×10^8
2	1.5×10^{14}	−9	−11.4	6.3	4.5	1.4×10^{-3}	6.4×10^9	9.3×10^6	-3.5×10^{-3}	-4×10^{-5}	7.4×10^7	-6.85×10^{-5}	4.6×10^7

TABLE I. (*Continued.*)

(d)

$x=10$

T (K)	$\omega_0^2 P_T l^{-2}$ (cm^{-2})	$\frac{ad}{n}$	α_T ($10^{-3}K^{-1}$)	φ	φ_0	ψ_2	$R_a \Big/ l^4 \frac{dT_0}{dz}$ ($cm^{-3}K^{-1}$)	$\tilde{R}_a^0 \Big/ l^4 \frac{dT_0}{dz}$ ($cm^{-3}K^{-1}$)	$\omega_d^2 l^4$ ($cm^4 sec^{-2}$)	$\tilde{R}_{a_1}^0 \Big/ l^2 \frac{dT_0}{dz}$ ($cm^{-1}K^{-1}$)	$\omega_{1d}^2 l^2$ ($cm^2 sec^{-2}$)	$\tilde{R}_{a_2}^0 \Big/ l^2 \frac{dT_0}{dz}$ ($cm^{-1}K^{-1}$)	$\omega_{2d}^2 l^2$ ($cm^2 sec^{-2}$)
0.5	3.3×10^{12}	−3.4	0.12	−1750	−862	1.5×10^{-3}	2.5×10^{8}	4×10^{5}	−1.5	-9×10^{-5}	10^{9}	3.5×10^{-5}	-2×10^{9}
1	6.2×10^{12}	−1.7	1.7	−35.3	−20.5	−0.017	6×10^{7}	-10^{6}	0.05	-1.6×10^{-5}	4×10^{8}	10^{-6}	-4×10^{9}
1.5	2.2×10^{13}	−0.8	1	−63	−36	-6×10^{-3}	2.4×10^{8}	-1.4×10^{6}	0.023	-2.4×10^{-5}	8.9×10^{8}	4.6×10^{-7}	-7×10^{8}
2	10^{12}	−0.93	−5.6	17.7	12.5	0.05	1.65×10^{9}	8.7×10^{7}	-10^{-3}	-3×10^{-3}	10^{6}	-3×10^{-4}	6×10^{6}

(e)

$x=30$

T (K)	$\omega_0^2 P_T l^{-2}$ (cm^{-2})	$\frac{ad}{n}$	α_T ($10^{-3}K^{-1}$)	φ	φ_0	ψ_2	$R_a \Big/ l^4 \frac{dT_0}{dz}$ ($cm^{-3}K^{-1}$)	$\tilde{R}_a^0 \Big/ l^4 \frac{dT_0}{dz}$ ($cm^{-3}K^{-1}$)	$\omega_d^2 l^4$ ($cm^4 sec^{-2}$)	$\tilde{R}_{a_1}^0 \Big/ l^2 \frac{dT_0}{dz}$ ($cm^{-1}K^{-1}$)	$\omega_{1d}^2 l^2$ ($cm^2 sec^{-2}$)	$\tilde{R}_{a_2}^0 \Big/ l^2 \frac{dT_0}{dz}$ ($cm^{-1}K^{-1}$)	$\omega_{2d}^2 l^2$ ($cm^2 sec^{-2}$)
1	10^{10}	−2.5	4.5	−66	−38.5	-4.6×10^{-3}	1.1×10^{9}	-5.06×10^{6}	0.02	−0.15	7.5×10^{4}	5.6×10^{-3}	-1.5×10^{6}
1.5	10^{10}	−0.8	8	−19.5	−11.5	−0.028	0.65×10^{9}	-1.8×10^{7}	1.6×10^{-3}	−0.15	3.7×10^{4}	7×10^{-3}	-3.4×10^{5}

(f)

$x=50$

T (K)	$\omega_0^2 P_T l^{2}$ (cm^{-2})	$\frac{ad}{n}$	α_T ($10^{-3}K^{-1}$)	φ	φ_0	ψ_2	$R_a \Big/ l^4 \frac{dT_0}{dz}$ ($cm^{-3}K^{-1}$)	$\tilde{R}_a^0 \Big/ l^4 \frac{dT_0}{dz}$ ($cm^{-3}K^{-1}$)	$\omega_d^2 l^4$ ($cm^4 sec^{-2}$)	$\tilde{R}_{a_1}^0 \Big/ l^2 \frac{dT_0}{dz}$ ($cm^{-1}K^{-1}$)	$\omega_{1d}^2 l^2$ ($cm^2 sec^{-2}$)	$\tilde{R}_{a_2}^0 \Big/ l^2 \frac{dT_0}{dz}$ ($cm^{-1}K^{-1}$)	$\omega_{2d}^2 l^2$ ($cm^2 sec^{-2}$)
1	1.25×10^{14}	−3	7.7	−56.5	−48.7	2.8×10^{-3}	3.1×10^{9}	8.6×10^{6}	-8×10^{-3}	-3.3×10^{-5}	2.6×10^{8}	-1.5×10^{-6}	4.2×10^{9}

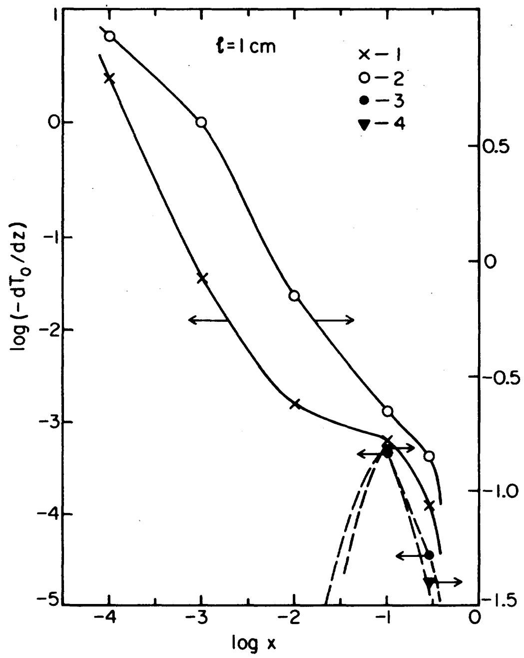

FIG. 1. Concentration dependence of the critical temperature gradient and the neutral frequency at $m > 1$ for two different temperatures. The critical temperature gradient (K/cm): 1—1 K, 3—1.5 K; the neutral frequency ($\sec^{-1}$): 2—1 K, 4—1.5 K.

factor of ~10^5 and the neutral frequency by ~100 when the concentration of ^{3}He atoms changes from 30 to 0.01% at 1 K. The second limiting case ($m \ll 1$, $\omega_0^2 P_T/(\pi^2+k^2) > 1$, may show up at low temperatures (less than 0.8 K). As seen from the calculations, the actual values of the critical temperature gradient are so large in this case that they practically cannot be realized for all possible values of layer height. It follows from further analysis that this instability branch may be observed in the vicinity of the tricritical point.

(2) We will now determine the asymptotic behavior of the oscillatory instability criteria and neutral frequencies when the concentration of ^{3}He atoms approaches zero.

Using the relations for infinite dilute solutions [see (I) Appendix III], it is easy to derive the following expressions for the criterion and the neutral frequency in the case of large dissipation of superfluid motion ($m \gg 1$):

$$\tilde{R}_a^0 = R_a \psi_2, \quad R_a = \left(\frac{l}{l_0}\right)^3 \frac{k_T}{T} l \frac{dT_0}{dz}$$

[from (29) (I)],

$$\psi_2 = \frac{M_4 R C_{40}}{M_3(M_3+M_4)S_{40}^2(1+\varphi)} c\,, \tag{22a}$$

$$\omega^2 = -\frac{9\pi^4}{4P_T}\frac{M_3 S_{40}^2}{RC_{40}c}\left(1+\frac{M_3}{M_4}\right)\varphi\,,$$

$$\varphi = \varphi_0\,, \quad \varphi = -\frac{M_3 S_{40}}{3\alpha_T TR}\,, \tag{22b}$$

where R is the gas constant (in J/mol K), S_{40} is the ^{4}He entropy per gram, and C_{40} is the ^{4}He specific heat per gram. As is well known in the temperature and concentration ranges of the phase diagram considered, α_T changes sign in the ^{4}He and ^{3}He-^{4}He mixture.[8] For dilute ^{3}He-^{4}He solutions $\alpha_T > 0$ at $T \leq 1.1$ K.[8] Therefore, for temperatures near 1 K ($0.8 < T \leq 1.1$ K), we have $m > 1$ and from (22a) and (22b) $\varphi < 0$, $\omega^2 > 0$, and $\tilde{R}_a^0 < 0$ (because $|\varphi| > 1$ and $R_a > 0$). Thus, in this case, the overstability occurs in dilute ^{3}He-^{4}He super-fluid solutions heated from below (Table II). Therewith both the critical temperature gradient and the neutral frequency increases when the concentration tends to zero, i.e., the superfluid mixture becomes mechanically stable. However, as the concentration approaches zero, the critical temperature gradient must approach zero in pure He II. This contradiction may be explained in the same manner as in the case of the stationary instability (I). As may be seen from Fig. 1 (I), there exists a narrow temperature interval below 0.8 K for dilute solutions where $m < 1$ and $\omega_0^2 P_T/(\pi^2+k^2) > 1$, and the second limiting case may be observed. Indeed, using the relations for dilute superfluid solutions one obtains

$$\tilde{R}_{a_1}^0 = -\frac{gC_{40}}{RS_{40}T^2}\frac{\rho}{\rho_s} l^2 \frac{dT_0}{dz}\frac{M_3+(M_3+M_4)\varphi}{(M_3+M_4)(1+\varphi)}\,,$$
$$\omega_0^2 = \frac{\rho_n^2 l^2}{\eta^2}\frac{TS_{40}^2}{C_{40}}\frac{\rho_s}{\rho_n}\,, \tag{23a}$$
$$\omega_{1d}^2 = \frac{2\pi^2}{l^2}\frac{S_{40}^2 T}{C_{40}}\frac{\rho_s}{\rho_n}\,,$$

and

$$\tilde{R}_{a_2}^0 = -\frac{gS_{40}M_3^2}{R^2T^2(1+P_T)(1+\varphi)c}\frac{\rho}{\rho_s} l^2 \frac{dT_0}{dz}\,,$$
$$\omega_{2d}^2 = \frac{2\pi^2}{l^2}\frac{(1+P_T)RP_T(1+\varphi)}{M_3}\frac{\rho_s}{\rho_n}c\,. \tag{23b}$$

Since $\alpha_T > 0$ at $T < 1.1$ K, the separation parameter φ is negative and $|\varphi| > 1$. Therefore, from (23b) we have $\omega_{2d}^2 < 0$, and only one branch of the oscillatory instability (23a) can take place.[9] However, numerical estimates for $\tilde{R}_{a_1}^0$ show that for all realistic values of layer height, the corresponding critical temperature gradient is so large that it is experimentally impossible to realize this instability branch.

Thus, for dilute superfluid solutions the oscillatory instability may occur in a narrow temperature interval near 1 K ($0.8 < T \leq 1.1$ K) when heated from below, and this instability is similar to the one that appears in a regular binary mixture with large abnormal thermal diffusion.

Since near the λ line $m \sim \epsilon_\lambda^{-1}$, as shown in (I), there is the limiting case $m > 1$. It is easy to show that the system is stable when heated from below because the appropriate asymptotic value of the neutral frequency is negative.

(3) As calculations indicate (see Table I) there is no region in the ^3He-^4He phase diagram where the second branch of oscillatory instability (20b) and (20c) can appear. The only possibility of observing the undamped standing second-sound waves is the region in the vicinity of the tricritical point (T_t, X_t).

As mentioned in (I), since both temperature and concentration gradients relax with the same time constant [which indicates strong coupling via k_T (Ref. 10)], in the tricritical region the chemical potential fluctuations are insignificant, and the limiting case with $m \ll 1$ pertains. Also in this region the condition $\omega_0^2 P_T/(\pi^2 + k^2) > 1$ is fulfilled.

According to Refs. 10 and 11, the following singularities of thermodynamic and kinetic properties exist near the tricritical point

$$\left[\frac{\partial(Z/\rho)}{\partial c}\right]_{P,T}^{-1} \sim \epsilon_t^{-1}, \quad \left(\frac{\partial \mu_4}{\partial c}\right)_{P,T}^{-1} \sim \epsilon_t^{-1},$$

$$k_T \sim \epsilon_t^{-1}, \quad D \sim \epsilon_t^{1/3},$$

and

$$\rho_s/\rho \sim \epsilon_t,$$

where

$$\epsilon_t = (T_t - T)/T_t .$$

The effective thermal conductivity shows only a weak variation with temperature for the tricritical mixture.[10]

Using these singularities one can easily find the asymptotic temperature behavior of the oscillatory instability criteria and the neutral frequencies from (20b) and (20c):

$$l_0^3 \sim \epsilon_t, \quad R_a \sim \epsilon_t^{-2}, \quad \omega_0^2 \sim \epsilon_t,$$

$$\tilde{R}_{a_1}^0 \sim \epsilon_t^{-2}, \quad \omega_1^2 \sim \epsilon_t,$$

$$\tilde{R}_{a_2}^0 \sim \epsilon_t^{-2}, \quad \omega_2^2 \sim \epsilon_t .$$

Then the critical temperature gradient tends to zero as ϵ_t^2 for both criteria.

Let us now clarify which criterion ($\tilde{R}_{a_1}^0$ or $\tilde{R}_{a_2}^0$) determines the mechanical stability of superfluid mixture heated from below in the tricritical region. Since near the tricritical point $\alpha_T > 0$ and therefore $\varphi < 0$, $\varphi_0 < 0$, and $\varphi/\varphi_0 > 1$,

$$\tilde{R}_{a_1}^0 \sim \epsilon_t^{-2}\left[1 - \left(1 - \frac{\varphi}{\varphi_0}\right)\epsilon_t\right]$$

and (24)

$$\tilde{R}_{a_2}^0 \sim \epsilon_t^{-2}\left[1 + \left(1 - \frac{\varphi}{\varphi_0}\right)\epsilon_t^{1/3}\right] .$$

TABLE II. Typical values of parameters affecting the onset of oscillatory convection and corresponding values of the criteria and the neutral frequencies in dilute superfluid ^3He-^4He solutions at different temperatures.

T	$\omega_0^2 P_T l^{-2}$	α_T	C_{40}	S_{40}	ρ_n/ρ	η	D	P_c	φ	ψ_2	$R_a \Big/ l^4 \frac{dT_0}{dz}$	$\tilde{R}_a^0 \Big/ l^4 \frac{dT_0}{dz}$	$\omega_d^2 l^4$
(K)	(cm^{-2})	($10^{-3}K^{-1}$)	$\left(\frac{J}{mol\,K}\right)$	$\left(\frac{J}{mol\,K}\right)$		(μP)	($cm^2 sec^{-1}$)				($cm^{-3}K^{-1}$)	($cm^{-3}K^{-1}$)	($cm^4 sec^{-2}$)
1	2.5×10^4	0.46	0.4	0.067	0.01	40	10^{-2}	2.8	−4.35	$-169.4c$	1.5×10^4	$-2.5\times10^6 c$	$2.6\times10^{-3}c^{-1}$
1.5	10^{10}	−2	4.52	0.8	0.12	13	10^{-3}	0.75	∞	$5c$	4.8×10^7	$2.4\times10^8 c$	$-1.6\times10^{-4}c^{-1}$
2	5.5×10^{12}	−12	20.7	3.76	0.55	14	2×10^{-4}	0.9	4.7	$1.6c$	3.4×10^9	$5.4\times10^9 c$	$-2.9\times10^{-5}c^{-1}$
$T_{\lambda 0}$	7.7×10^{13}	$-\epsilon_t^{-\alpha}$	25	6.24	1	18.6	10^{-4}	1.3	1.25	$1.8c$	1.4×10^{10}	$2.5\times10^{10} c$	$-1.7\times10^{-5}c^{-1}$

Thus we have $\tilde{R}^0_{a_1} > \tilde{R}^0_{a_2}$ in the asymptotic limit and the critical temperature gradient that corresponds to $\tilde{R}^0_{a_1}$ defines the stability.

Using the numerical estimates of the criterion $\tilde{R}^0_{a_1}$ and the neutral frequency ω_{1d} beyond the tricritical point, one can obtain the following expressions:

$$\tilde{R}^0_{a_1} = -10^{-4} l^2 \frac{dT_0}{dz} \epsilon_t^{-2} , \quad \omega_{1d}^2 = 4\times 10^8 l^{-2} \epsilon_t . \qquad (25)$$

For layer height $l = 10$ cm and $\epsilon_t = 0.3\times 10^{-4}$, we obtain the realistic values for the critical temperature gradient and the neutral frequency, respectively:

$$\frac{dT_0}{dz} = -3\times 10^{-6} \frac{\mathrm{K}}{\mathrm{cm}} , \quad \omega_{1d} \simeq 10 \ \mathrm{sec}^{-1} . \qquad (26)$$

Thus, the estimates presented above indicate the possibility of observing the predicted effect in the vicinity of the tricritical point of the ^{3}He-^{4}He superfluid mixture.

IV. SUMMARY

The main result obtained in this paper is the existence of the oscillatory instability in a superfluid mixture heated from below only. In different regions of the ^{3}He-^{4}He phase diagram, the different types of overstability exist and have different physical origins. Two parameters, m and $\omega_0^2 P_T/(\pi^2 + k^2)$, define the areas of application of the criteria (20a), (20b), and (20c). Since here we consider only the case of small "dissipation" length l_0 so that the condition $(l_0/l)^3 < 1$ should be fulfilled, the condition $\omega_0^2 P_T/(\pi^2 + k^2) > 1$ is also fulfilled in all considered regions of the phase diagram. Due to that, there are only two limiting cases in accordance with the value of the parameter m: (1) $m \gg 1$, which means large dissipation of superfluid motion, and (2) $m \ll 1$, which means small dissipation of superfluid motion. The condition $m \gg 1$ is fulfilled for $T > 0.8$ K in a major part of the superfluid ^{3}He-^{4}He phase diagram [see Fig. 1 (I)]. The oscillatory instability criteria and the neutral frequency (20a) are similar, in this case, to the corresponding conditions in a regular binary mixture with large abnormal thermal diffusion. The calculations show that this type of convection instability may be observed in a fairly narrow temperature range between curves $m \simeq 1$ and $\alpha_T = 0$ (in the region where $\alpha_T > 0$).

The fundamentally new type of the oscillatory convective instability (that, in fact, is the undamped standing second-sound waves) takes place when the condition $m < 1$ is fulfilled. This condition is fulfilled either for low temperatures $T < 0.8$ K or in the vicinity of the tricritical point. The calculations show that for low temperatures the corresponding critical temperature gradient is so large for all possible values of layer height that it is impossible to observe this effect experimentally. The only possibility of observing the undamped standing second-sound waves is in the region of the tricritical point.

In conclusion, it should be noted that the main features of oscillatory instability in ^{3}He-^{4}He superfluid mixture remain the same for more realistic boundary conditions too.

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[7] As seen from Table I, $\psi_2 < 0$ and $\omega^2 > 0$ only when $\alpha_T > 0$. This corresponds to a negative separation parameter ($\varphi < 0$). As shown in Refs. 2 and 3, the oscillatory instability occurs in a regular binary mixture also when the separation parameter φ is negative.
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[9] For the same reason $\tilde{R}^0_{a1} < 0$ in (23a).
[10] H. Meyer, J. Ruppeiner, and M. Ryschewitsch, *Dynamical Critical Phenomena and Related Topics*, edited by Charles P. Eez (Springer, Berlin, 1979), p. 172.
[11] G. Ahlers, *The Physics of Liquid and Solid Helium*, edited by J. B. Keterson and K.H. Benneman (Wiley, New York, 1976), Vol. I, Ch. 2.