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Calculation of differential cross sections for proton-impact excitation of the $n=2$ levels of helium

S. K. Sur, Shyamal Datta, and S. C. Mukherjee
Department of Theoretical Physics, Indian Association for the Cultivation of Science, Jadavpur, Calcutta 700032, India
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Differential cross sections for the proton-impact $1^1S \to 2^1S$ and $1^1S \to 2^1P$ excitations of atomic helium are calculated at 25-, 50-, and 100-keV proton energies using the Glauber approximation. The sum of the results are compared with an earlier multistate eikonal calculation as well as with a very recent measurement. The angular dependence of the cross sections predicted by the Glauber theory is in reasonable agreement with the eikonal results and the observed data.

I. INTRODUCTION

Extensive investigations have so far been made about direct excitation of neutral atomic targets in fundamental ion-atom collision processes.[1] However, almost all the calculations and measurements concentrate on the determination of cross sections integrated over the scattering angles. Whereas the agreement between theory and experiment is more or less satisfactory in the high incident-energy region, this can hardly be said about intermediate-energy collisions.[1] Considering, for example, the case of H^+-He collisions, the calculated[2] and measured[3,4] total cross sections at intermediate proton energies show a spread over a wide range of absolute values which greatly exceeds the estimated uncertainty limits of the individual data. This calls for a more detailed study of the underlying physical processes. It is well known that a knowledge of the differential cross sections can furnish important information in this respect and can provide a better test of the theoretical models against experimental observations than do the total cross section data. However, the data of differential cross sections for excitation in fundamental ion-atom collisions in the intermediate energy region have so far been very scanty in general. The lack of experimental data is a testament to the severe problems of measurement of the angular differential cross sections at intermediate ion energies. On the other hand, the usually applied theoretical methods are based on the semiclassical impact-parameter formulation which cannot furnish exact quantal differential cross sections. For H^+-He collisions, the only calculation[5] which reports the differential cross sections for direct excitation of the $2s$, $2p$, $3s$, and $3p$ states of the target hydrogen employ the Glauber approximation.[6-9] Similarly, differential cross sections in H^+-He collisions have been calculated employing only the multistate eikonal[10] and the Glauber[11] theories. For both the systems, however, the differential cross sections predicted by the first Born approximation (FBA) have also been presented for comparison. In the absence of any experimental data, one could only note from these calculations the wide difference of the FBA cross sections from the Glauber[5,11] and the multistate eikonal predictions[10] except at very high incident energies.

However, of late, the situation has changed very much. Park *et al.*[12,13] have been able to overcome the difficulties of measurement of the angular differential cross sections for the scattering of intermediate-energy protons from atomic hydrogen[12] and helium.[13] They have presented the center-of-mass differential cross sections for excitation of the $n=2$ level of either target (without distinguishing the substates) by 25-, 50-, and 100-keV protons. For H^+-H collisions, the measurements of Park *et al.*[12] are in remarkable agreement with the predictions of the Glauber approximation[5] in respect to both absolute magnitude and angular dependence. The FBA differential cross sections differ appreciably from the measurement and the Glauber results except at the highest energy in the forward direction.

For excitation of atomic helium, although a number of Glauber calculations with electrons as projectiles are available (cf. Yates and Tenney,[14] Franco,[15] and Thomas and Chan[16]), proton-impact excitation of helium has been studied only by Chan and Chang[11] and by Sur and Mukherjee[2] using the Glauber method. However, whereas Chan and Chang present the total and differential Glauber cross sections for the $1^1S \to 2^1P$ excitations of He, Sur and Mukherjee report the total Glauber cross sections for $1^1S \to n^1S$ ($n=2,3,4$) transitions. The measurement by Park *et al.*[13] of the angular differential cross sections for proton-impact $n=2$ excitation of He, hence, cannot be compared with either of the above calculations.[2,11] The multistate eikonal calculation of Flannery and McCann[10] gives the proton-impact angular differential cross sections for $1\,^1S\to 2\,^1S$ and $1\,^1S\to 2\,^1P$ excitations of He. Since the contribution to the cross sections from the triplet state is negligible because of spin conservation, the eikonal results are readily comparable to the measurement.

In view of the remarkable success of the Glauber theory in predicting the observed differential cross sections for excitation in H^+-H collisions[12] as mentioned above, we undertake in the present work a study of the Glauber-predicted angular differential cross sections for $1\,^1S\to 2\,^1S$ and $1\,^1S\to 2\,^1P$ excitations of He by the impact of intermediate energy protons and compare the sum with the measurement of Park *et al.*[13] The method of calculating the Glauber cross sections for $1\,^1S\to n\,^1P$ excitations is an extension of our earlier work for studying the $1\,^1S\to n\,^1S$ excitations[2,17] and has been described elsewhere.[18]

II. THEORY

For the excitation of a helium atom from an initial state $\Psi_i(\vec{r}_1,\vec{r}_2)$ to a final state $\Psi_f(\vec{r}_1,\vec{r}_2)$ due to the impact of a particle of charge Z_i and relative velocity $\vec{v}_i$, the Glauber scattering amplitude $F(\vec{q})$ (Refs. 6 and 7) in the center-of-mass (c.m.) system can be expressed as[2,17] (using atomic units throughout)

$$F(i\to f;\vec{q})=F_1(\vec{q})+F_2(\vec{q})\,, \tag{1}$$

where $F_1(\vec{q})$ and $F_2(\vec{q})$ have been termed[19] the single-scattering and double-scattering amplitudes, respectively, and are given by

$$F_1(\vec{q})=\frac{ik_i}{\pi}\int \Psi_f^*\Gamma(\vec{b},\vec{r}_1)\Psi_i e^{i\vec{q}\cdot\vec{b}}d^2b\,d\vec{r}_1 d\vec{r}_2\,, \tag{2}$$

$$F_2(\vec{q})=-\frac{ik_i}{2\pi}\times\int \Psi_f^*\Gamma(\vec{b},\vec{r}_1)\Gamma(\vec{b},\vec{r}_2)\Psi_i e^{i\vec{q}\cdot\vec{b}}d^2b\,d\vec{r}_1 d\vec{r}_2\,. \tag{3}$$

Here $\vec{q}(=\vec{k}_i-\vec{k}_f)$ is the momentum transfer, $\vec{k}_i$ and $\vec{k}_f$ are the incident and final wave vectors in the c.m. system, respectively, and

$$\Gamma(\vec{b},\vec{r}_j)=1-(|\vec{b}-\vec{s}_j|/b)^{2i\eta}\,, \tag{4}$$

with $\eta=-Z_i/v_i$ and $\vec{s}_j$ being the projection of $\vec{r}_j$ onto the plane of $\vec{b}$.

A. $1\,^1S\to n\,^1S$ transitions

The usual type of approximate He wave functions of the product form without correlation terms can be expressed as

$$\Psi^*_{n^1S}\Psi_{1\,^1S}=\sum_j C_j[\chi_j(\vec{r}_1,\vec{r}_2)+\chi_j(\vec{r}_2,\vec{r}_1)]\,, \tag{5}$$

with

$$\chi_j(\vec{u},\vec{v})=u^{t_j}v^{p_j}e^{-\alpha_j u-\beta_j v}\,, \tag{6}$$

where α_j and β_j involve wave-function parameters, and t_j and p_j take integral values.

On substitution for $\Psi_f^*\Psi_i$ from Eq. (5) in (2), we obtain the single-scattering amplitude for $1\,^1S\to n\,^1S$ transitions as

$$F_1^S(\vec{q})=\frac{ik_i}{\pi}\sum_j C_j F_{1j}^S(\vec{q})\,, \tag{7}$$

where

$$F_{1j}^S(\vec{q})=\int[\chi_j(\vec{r}_1,\vec{r}_2)+\chi_j(\vec{r}_2,\vec{r}_1)]\Gamma(\vec{b},\vec{r}_1)e^{i\vec{q}\cdot\vec{b}}d^2b\,d\vec{r}_1 d\vec{r}_2\,. \tag{8}$$

Using the expression (6) in Eq. (8), we can express $F_{1j}^S(\vec{q})$ as

$$F_{1j}^S(\vec{q})=A(\beta_j,p_j)D(\vec{q},\alpha_j,t_j)+A(\alpha_j,t_j)D(\vec{q},\beta_j,p_j)\,, \tag{9}$$

where

$$A(x,y)=\int e^{-xr}x^y d\vec{r}=4\pi(y+2)!/x^{y+3} \tag{10}$$

and

$$D(\vec{q},x,y)=\int e^{i\vec{q}\cdot\vec{b}}e^{-xr}r^y\Gamma(\vec{b},\vec{r})d^2b\,d\vec{r}\,. \tag{11}$$

We now introduce the generating function

$$I_1^S(\vec{q},x)=\int e^{i\vec{q}\cdot\vec{b}}\Gamma(\vec{b},\vec{r})\frac{e^{-xr}}{r}d^2b\,d\vec{r}\,, \tag{12}$$

in terms of which $D(\vec{q},x,y)$ becomes

$$D(\vec{q},x,y)=\left(\frac{-\partial}{\partial x}\right)^{y+1}I_1^S(\vec{q},x)\,. \tag{13}$$

The function $I_1^S(\vec{q},x)$ can be reduced to a closed form following the procedure employed by Thomas and Gerjuoy[20] for $1s$-ns transitions in hydrogen giving

$$I_1^S(\vec{q},x)=-16\pi^2 i\eta\Gamma(1+i\eta)\Gamma(1-i\eta)q^{2i\eta-2}x^{-2i\eta-2}\times{}_2F_1(1-i\eta,1-i\eta;1;-x^2/q^2)\,. \tag{14}$$

Combining Eqs. (7), (9), and (13), we obtain for the single-scattering amplitude for $1\,^1S\to n\,^1S$ transitions in He

$$F_1^S(\vec{q})=\frac{ik_i}{\pi}\sum_j C_j\left[A(\beta_j,p_j)\left(-\frac{\partial}{\partial\alpha_j}\right)^{t_j+1}I_1^S(\vec{q},\alpha_j)+A(\alpha_j,t_j)\left(-\frac{\partial}{\partial\beta_j}\right)^{p_j+1}I_1^S(\vec{q},\beta_j)\right]\,, \tag{15}$$

$A(x,y)$ and $I_1^S(\vec{q},x)$ being given by Eqs. (10) and (14), respectively.

For evaluating the double scattering amplitude $F_2^S(\vec{q})$ for $1\,^1S\to n\,^1S$ transitions, we substitute from Eq. (5) in Eq. (3), obtaining

$$F_2^S(\vec{q})=-\frac{ik_i}{\pi}\sum_j C_j F_{2j}^S(\vec{q})\,, \tag{16}$$

where

$$F^S_{2j}(\vec{q}) = \int \chi_j(\vec{r}_1, \vec{r}_2)\Gamma(\vec{b}, \vec{r}_1)\Gamma(\vec{b}, \vec{r}_2)e^{i\vec{q}\cdot\vec{b}}d^2b\, d\vec{r}_1 d\vec{r}_2 . \tag{17}$$

Performing the azimuthal integration on the $\vec{b}$ plane, we have

$$F^S_{2j}(\vec{q}) = 2\pi \int_0^\infty db\, bJ_0(qb)R_0(\alpha_j, t_j, b)R_0(\beta_j, p_j, b) , \tag{18}$$

where

$$R_0(x, y, b) = \int e^{-xr}r^y\Gamma(\vec{b}, \vec{r})d\vec{r} . \tag{19}$$

We now introduce the generating function

$$\Gamma_0(x, \eta, b) = \int \frac{e^{-xr}}{r}\Gamma(\vec{b}, \vec{r})d\vec{r} , \tag{20}$$

in terms of which R_0 becomes

$$R_0(x, y, b) = \left(\frac{-\partial}{\partial x}\right)^{y+1}\Gamma_0(x, \eta, b) . \tag{21}$$

The function Γ_0 can be reduced as in Ref. 16 to give

$$\Gamma_0(x, \eta, b) = -16\pi\eta^2 K_0(x, \eta, b) , \tag{22}$$

where K_0 is given by (cf. Refs. 2 and 17)

$$K_0(x, \eta, b) = -2^{2i\eta-1}\frac{\Gamma(i\eta)}{\Gamma(1 - i\eta)}x^{-2}\int_0^\infty dt \frac{t^{-2i\eta+1}J_0(t)}{t^2 + x^2b^2} . \tag{23}$$

The double-scattering amplitude for $1\,^1S \to n\,^1S$ transitions obtained by combining Eqs. (16), (18), and (21)–(23) can be written as

$$\begin{aligned} F^S_2(\vec{q}) = -512i\pi^2\eta^4 k_i & \\ \times \int_0^\infty db\, bJ_0(qb) \sum_j C_j & \left[\left(-\frac{\partial}{\partial \alpha_j}\right)^{t_j+1} K_0(\alpha_j, \eta, b)\right] \\ & \times \left[\left(-\frac{\partial}{\partial \beta_j}\right)^{p_j+1} K_0(\beta_j, \eta, b)\right] . \end{aligned} \tag{24}$$

The method of evaluation of $K_0(x, \eta, b)$ and its various derivatives with respect to x has been discussed in detail in Refs. 2 and 17, whence the integral in Eq. (24) can be evaluated numerically. The full Glauber scattering amplitude for $1\,^1S \to n\,^1S$ transitions is then obtained from Eq. (1) as

$$F(1\,^1S \to n\,^1S; \vec{q}) = F^S_1(\vec{q}) + F^S_2(\vec{q}) , \tag{25}$$

F^S_1 and F^S_2 being given by Eqs. (15) and (24), respectively.

B. $1\,^1S \to n\,^1P$ transitions

Reduction of the single-scattering amplitude $[F^P_1(\vec{q})]$ and the double-scattering amplitude $[F^P_2(\vec{q})]$ for $1\,^1S \to n\,^1P$ transitions in helium to give the full Glauber amplitude

$$F(1\,^1S \to n\,^1P; \vec{q}) = F^P_1(\vec{q}) + F^P_2(\vec{q}) \tag{26}$$

proceeds in a manner similar to that for $1\,^1S \to n\,^1S$ transitions described above.

We take the usual type of approximate He wave functions to write

$$\Psi^*_{n^1P_m}\Psi_{1^1S} = \sum_j C_j[\chi_j(\vec{r}_1, \vec{r}_2) + \chi_j(\vec{r}_2, \vec{r}_1)] , \tag{27}$$

with

$$\chi_j(\vec{u}, \vec{v}) = u^{t_j}v^{p_j}e^{-\alpha_j u - \beta_j v}P^m_1(\cos\theta_u)e^{-im\varphi_u} , \tag{28}$$

where P^m_1 represents the usual Legendre polynomial of order $(1 - m)$, other notations being the same as before.

Substitution from Eqs. (27) and (28) in Eq. (2) for the single-scattering amplitude makes the integral involving the term $\chi_j(\vec{r}_2, \vec{r}_1)$ vanish and gives

$$F^P_1(\vec{q}) = \frac{ik_i}{\pi}\sum_j C_j F^P_{1j}(\vec{q}) , \tag{29}$$

where

$$F^P_{1j}(\vec{q}) = A(\beta_j, p_j)G(\vec{q}, \alpha_j, t_j) \tag{30}$$

and

$$G(\vec{q}, x, y) = \int e^{-xr}r^y\Gamma(\vec{b}, \vec{r})e^{i\vec{q}\cdot\vec{b}}P^m_1(\cos\theta)e^{-im\phi}d^2b\, d\vec{r} , \tag{31}$$

$A(x, y)$ being given by Eq. (10). As before, $G(\vec{q}, x, y)$ can be expressed in terms of a generating function

$$I^P_1(\vec{q}, x) = \int \frac{e^{-xr}}{r} r\Gamma(\vec{b}, \vec{r})e^{i\vec{q}\cdot\vec{b}}P^m_1(\cos\theta)e^{-im\varphi}d^2b\, d\vec{r} \tag{32}$$

as

$$G(\vec{q}, x, y) = \left(\frac{-\partial}{\partial x}\right)^y I^P_1(\vec{q}, x) . \tag{33}$$

The single-scattering amplitude (29) for $1\,^1S \to n\,^1P$ transitions hence becomes

$$F^P_1(\vec{q}) = \frac{ik_i}{\pi}\sum_j C_j A(\beta_j, p_j)\left(\frac{-\partial}{\partial \alpha_j}\right)^{t_j} I^P_1(\vec{q}, \alpha_j) , \tag{34}$$

where I^P_1 can be reduced along the lines of Ref. 20 to give

$$\begin{aligned} I^P_1(\vec{q}, x) = {} & -32\pi^2\eta e^{-im\Phi}q^{-3+2i\eta}x^{-2-2i\eta} \\ & \times\Gamma(1 + i\eta)\Gamma(2 - i\eta) \\ & \times[{}_2F_1(2 - i\eta, 1 - i\eta; 2; -x^2/q^2) \\ & \quad - (1 - i\eta){}_2F_1(2 - i\eta, 2 - i\eta; 2; -x^2/q^2)] . \end{aligned} \tag{35}$$

The double-scattering amplitude is obtained by substitution from Eqs. (27) and (28) in Eq. (3) and subsequent reduction, as

$$F_2^P(\vec{q}) = -\frac{ik_i}{\pi} \sum_j C_j F_{2j}^P(\vec{q}), \tag{36}$$

where

$$F_{2j}^P(\vec{q}) = 2\pi i e^{-im\Phi_q} \int_0^\infty db\, bJ_1(qb) R_0(\beta_j, p_j, b) R_1(\alpha_j, t_j, b), \tag{37}$$

with

$$R_1(x, y, b) = \int e^{-xr} r^y \Gamma(\vec{b}, \vec{r}) P_1^m(\cos\theta) e^{-im\Phi} d\vec{r}. \tag{38}$$

Here R_1 can be put in the form

$$R_1(x, y, b) = \left(\frac{-\partial}{\partial x}\right)^y \Gamma_1(x, \eta, b), \tag{39}$$

where

$$\Gamma_1(x, \eta, b) = \int \frac{e^{-xr}}{r} r\Gamma(\vec{b}, \vec{r}) P_1^m(\cos\theta) e^{-im\Phi} d\vec{r}, \tag{40}$$

and can be reduced to give

$$\Gamma_1(x, \eta, b) = -16\pi\eta[(1+i\eta)K_1(x, \eta, b) + \eta K_0(x, \eta, b)], \tag{41}$$

with

$$K_1(x, \eta, b) = x^{-3} b^{-1} (ixb)^{-2i\eta} \mathfrak{L}_{2i\eta, 1}(ixb). \tag{42}$$

The integral for K_1 is given by

$$K_1(x, \eta, b) = 2^{2i\eta} i \frac{\Gamma(i\eta)}{\Gamma(-i\eta)} x^{-2} \int_0^\infty dt\, t^{-2i\eta} \frac{J_1(t)}{t^2 + x^2 b^2}. \tag{43}$$

The double-scattering amplitude for $1\,^1S \rightarrow n\,^1P$ transitions are obtained by combining Eqs. (36), (37), (39), and (41):

$$F_2^P(\vec{q}) = 512\pi^2\eta^3 k_i e^{-im\Phi_q} \int_0^\infty db\, bJ_1(qb) \sum_j C_j \left[\left(-\frac{\partial}{\partial \beta_j}\right)^{p_j+1} K_0(\beta_j, \eta, b)\right] \times \left[\left(-\frac{\partial}{\partial \alpha_j}\right)^{t_j} [(1+i\eta)K_1(\alpha_j, \eta, b) + \eta K_0(\alpha_j, \eta, b)]\right]. \tag{44}$$

We have discussed earlier (cf. Sur and Mukherjee[18]) the method of evaluating $K_1(x, \eta, b)$ and its various derivatives with respect to x. The integral in Eq. (44) can then be calculated numerically.

III. RESULTS AND DISCUSSION

A. $1\,^1S \rightarrow 2\,^1S$ excitations

Our results of the center-of-mass differential cross sections in the Glauber approximation for $1\,^1S - 2\,^1S$ excitations of He at 25- and 100-keV proton energies are plotted in Figs. 1 and 2, respectively. The excited state He wave functions used in our calculation are those given by Winter and Lin,[21] while the ground state wave function is due to Green *et al.*[22] We have also included our FBA calculations in these figures. The only other available calculation is due to Flannery and McCann[10] who have used multistate eikonal approximations and the FBA method in their work. These results are also compared in Figs. 1 and 2 with our FBA and Glauber results.

From a comparison of the curves for the present full Glauber (G) and the corresponding single-scattering Glauber (SSG) results of Figs. 1 and 2, it is seen that the relative contribution of the double-scattering term (F_2) in the Glauber scattering amplitude (F) is appreciable at low energies and inclusion of this term lowers the G cross sections significantly at small scattering angles from the

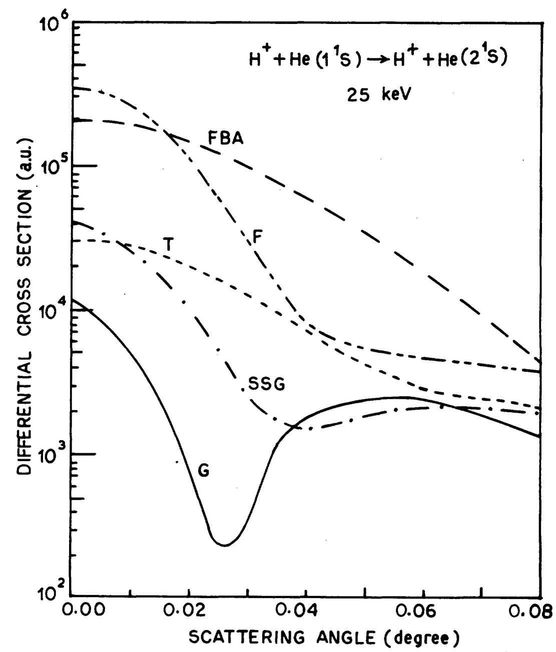

FIG. 1. Angular differential cross sections (in the center-of-mass system) for $2\,^1S$ excitation of He by the impact of 25-keV (laboratory energy) protons. Present calculation: (—), Glauber (G); (-•-), single scattering Glauber (SSG); (— —), first Born approximation (FBA). Calculation of Flannery and McCann (Ref. 10): (- - -), two-state eikonal (T); (— -- —), four-state eikonal (F).

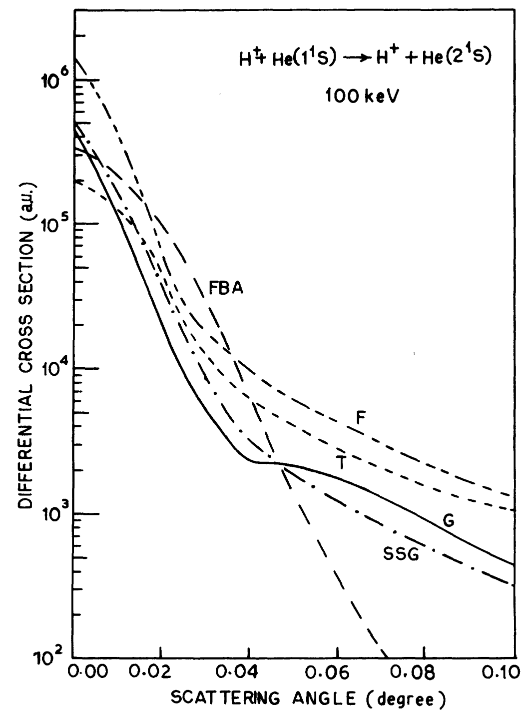

FIG. 2. Same as Fig. 1, but by the impact of 100-keV protons.

corresponding SSG results. Since the contribution to the integrated Glauber cross sections comes predominately from the small-angle region, this causes a consequent reduction of the total Glauber cross sections from the corresponding single-scattering results. With the increase of energy, the G and SSG curves of Figs. 1 and 2, however, approach each other in the forward direction, explaining the agreement of the integrated G and SSG results at high energies as already observed by Sur and Mukherjee.[2] The common agreement of both of these Glauber results also with the integrated FBA results in the high-energy region can be similarly explained. At intermediate energies, however, the behaviors of the differential FBA cross sections are completely different from those of the G or SSG results, as may be seen from Fig. 1.

An interesting feature of the differential (G) cross sections at 25 keV is the minimum occurring at a scattering angle of 0.026° (Fig. 1). At a proton energy of 50 keV also, this minimum in the differential cross sections (not presented here) is observable and occurs at 0.0344°. With increasing energy, however, this minimum flattens and the G curve at 100 keV (Fig. 2) shows no such minimum. The overall nature of the Glauber differential cross sections of Figs. 1 and 2, especially the minima in cross-section-scattering-angle curves at lower energies, resemble the corresponding trends obtained in the H^+-H differential Glauber cross sections for $1s$-ns excitations.[5,23] The SSG curve of Fig. 1 also shows a broad minimum near 0.04°.

As regards the relative behavior of our present Glauber differential cross sections and the multistate eikonal results of Flannery and McCann, Figs. 1 and 2 show that there is appreciable difference between the predictions of the two calculations for small-angle scattering. In particular, the small-angle minima occurring in the Glauber curves of Fig. 1 being absent from the eikonal curves, the absolute values of the two sets of results differ by orders of magnitude near the scattering angles concerned. In the forward direction also, the absolute values of the Glauber and the multistate eikonal cross sections show a large difference. It should be noted, however, that the two eikonal calculations themselves appreciably differ from each other in absolute values in the forward direction—the four-state (F) results overestimating the two-state (T) ones almost by an order. The agreement between the results of the present Glauber and the multistate-eikonal calculations improves with the increase of incident proton energy when, as mentioned already, the minima in the Glauber curves gradually flatten. The angular dependence of the Glauber and the eikonal cross sections becomes similar also with the increase of the scattering angle at any given proton energy. We would like to mention here that although the wave functions used in the present work are different from those employed in the eikonal calculations,[10] our present FBA cross sections agree closely with those of Flan-

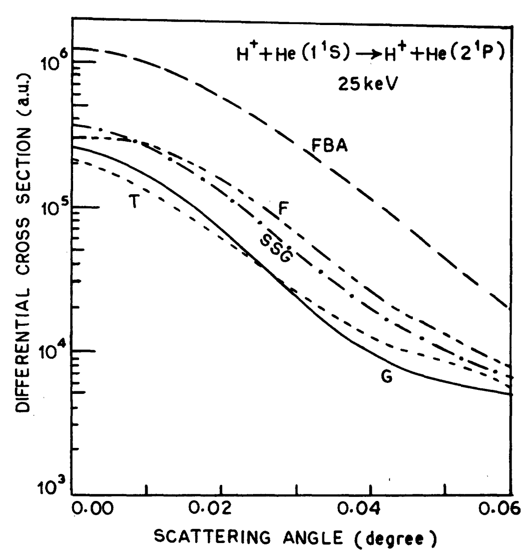

FIG. 3. Same as Fig. 1, but for 2^1P excitation of He.

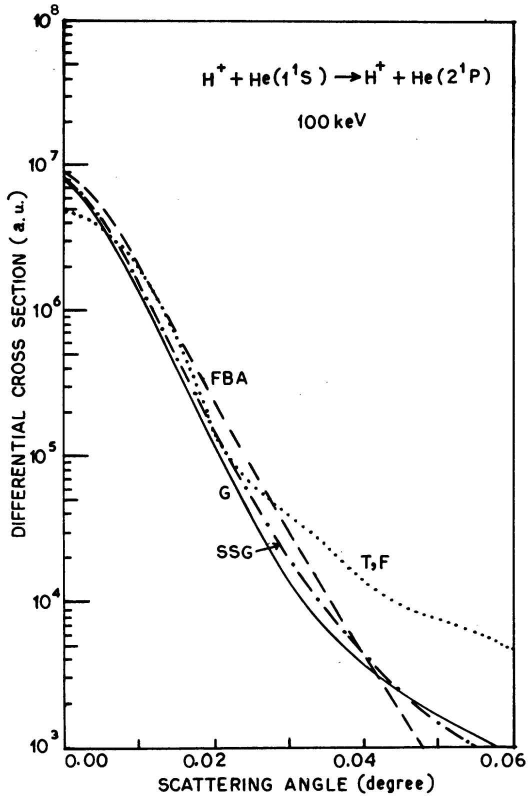

FIG. 4. Same as Fig. 1, but for 2^1P excitation of He by the impact of 100-keV protons. Calculation of Flannery and McCann (Ref. 10): (···), two- and four-state eikonal (T, F).

nery and McCann.[10] This indicates that the difference in absolute values as well as in the angular dependence between the two sets of results as discussed above may not be attributed only to the difference in the wave functions employed. Hence the full elucidation of the matter should await an absolute measurement of the $1\,^1S \rightarrow 2\,^1S$ differential cross sections.

B. $1\,^1S \rightarrow 2\,^1P$ excitations

In Figs. 3 and 4, we present our differential Glauber (G and SSG) as also the FBA cross sections for $1\,^1S \rightarrow 2\,^1P$ excitations of He by 25- and 100-keV protons at various scattering angles and compare the results with the calculations of Flannery and McCann.[10] It is worth mentioning that our present results agree closely with the earlier Glauber calculations of Chan and Chang[11] employing a different set of He wave functions.

Again, the effect of the double scattering term (F_2) on the full Glauber amplitude (F) is appreciable at the intermediate-energy region and is found to lower the G cross sections from the corresponding SSG results at small scattering angles. With increasing energy, as before, each of the G and SSG curves approaches the FBA curve in the forward direction (cf. Fig. 4).

At 25 keV, although the Glauber (G and SSG) and the multistate eikonal (T and F) differential cross sections show some spread in absolute values, the angular dependence of these results in the range of scattering angles shown in Fig. 3 are in reasonable agreement. The FBA cross sections, however, considerably overestimate the other theoretical predictions. At 100 keV, on the other hand, the behavior of the Glauber and the eikonal curves of Fig. 4 (the T and F curves are now coincident) appreciably differ at larger scattering angles. The FBA results are now in better absolute agreement with the Glauber results in the forward direction, but fall much more quickly in comparison with other theoretical results as the scattering angle increases.

C. Excitation of the $n=2$ levels

As mentioned earlier, the contribution from the triplet states in the excitation of $n=2$ levels of He is negligible because of spin conservation. Hence we compare the sum of our calculated $1\,^1S \rightarrow 2\,^1S$ and $1\,^1S \rightarrow 2\,^1P$ differential cross sections with the

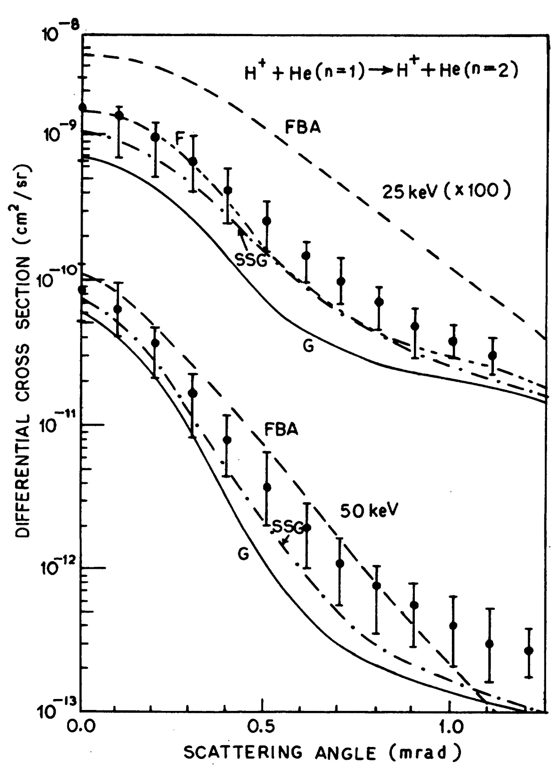

FIG. 5. Same as Fig. 1, but for $(2\,^1S + 2\,^1P)$ excitation of He by 25- and 50-keV protons. Experiment: (●), Park *et al.* (Ref. 13).

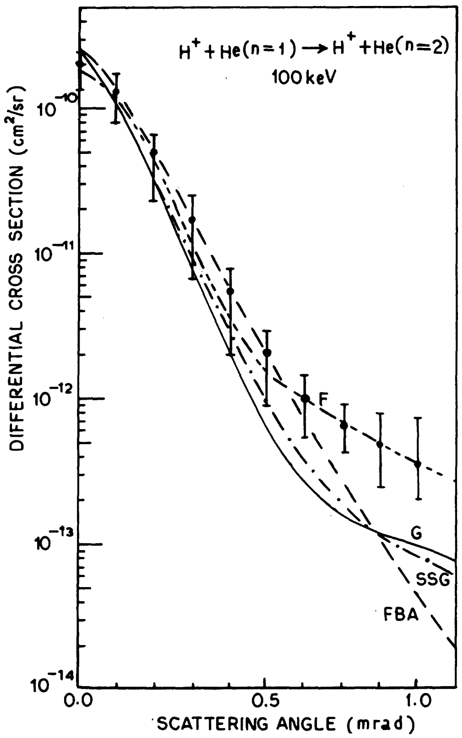

FIG. 6. Same as Fig. 5, but by 100-keV protons.

corresponding results of Flannery and McCann,[10] and with the measurements of Park *et al.*[13] at 25- and 100-keV proton energies. At 50 keV, the multistate eikonal results are not available.

In the range of scattering angles considered, the present G and SSG curves of the differential cross sections displayed in Figs. 5 and 6 do not differ much in shape and both agree qualitatively with the observed angular distributions.

The present Glauber results in general underestimate the observed cross sections. But the absolute agreement between the two results in the forward direction improves at higher energies and is within the accuracy of measurement at 50 and 100 keV. On the other hand, although the present FBA results show some agreement with the measurement in the forward direction at high energies, they fail to predict the observed angular distributions. The four-state results (F) of Flannery and McCann[10] are in excellent agreement with the experiment in both curve shape and absolute magnitude.

It is worth noting here that Park *et al.*[13] make no estimate of the possible systematic errors in their measurement, which may arise chiefly from the data-analysis method and also from the absolute measurement of the interaction length and pressure. As a result, the shapes of the curves are more reliable, according to the aughors,[13] than the absolute magnitude and the authors comment that the excellent agreement of their measured cross sections with the four-state eikonal results[10] is perhaps fortuitous, especially with respect to magnitude. We may also note from the work of Park *et al.*[13] that their integrated results which correspond to the measured angular differential cross sections overestimate the earlier calculations and measurements of the total cross sections.[24] In view of these facts, the underestimation shown by our differential Glauber (G) results of Figs. 5 and 6, in comparison with the eikonal calculations and the measurement, is probably not serious.

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[24]For details see Fig. 6 of Ref. 13 and the references cited therein.