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Proposed optical test of metric gravitation theories

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An optical test of the metric theories of gravitation which employs a ring laser interferometer in an earthbound laboratory is suggested and analyzed. The proposed test would be sensitive to the presence of a preferred frame of the universe and to the geodetic and Lense-Thirring (frame-dragging) rotations of the local inertial frames relative to fixed stars. The frequency difference of the counterpropagating beams in a Sagnac-type experiment is determined within the framework of the parametrized post-Newtonian formalism. The precision with which the various parameters involved in the expression for the frequency difference can be measured is discussed.

I. INTRODUCTION

There has been a great deal of interest in recent years in high-precision experiments to test gravitational theories.[1-3] The theoretical framework of the parametrized post-Newtonian (PPN) formalism,[4] which provides a means for studying a very wide class of metric theories of gravitation in the weak-field and slow-motion setting of the solar system, has been developed to systematize the comparison between theory and experiment. In this paper we suggest and analyze, in the context of the PPN formalism, a new optical test[5] of metric theories of gravity employing a ring laser interferometer in an earthbound labortory. The experiment would be sensitive to the presence of a perferred frame in the universe and to the geodetic and Lense-Thirring (frame-dragging) rotations of local inertial frames relative to the fixed stars.[6] The proposed experiment provides but one example of the possible applications of quantum optics to the study of gravitation physics. In order to make the analysis clear and understandable to readers with backgrounds in quantum optics, we have avoided the use of esoteric techniques (e.g., Fermi-Walker transport) and have instead carried out an explicit general relativistic analysis in order to obtain the necessary representation of the metric in the ring interferometer laboratory.

Einstein's theory of general relativity predicts no preferred coordinate frame in the universe. Thus, if general relativity is correct, the physics we observe will be independent of the velocity of the laboratory frame in which we work. On the other hand, some theoretical alternatives to general relativity predict that the frame in which the universe is "at rest" is a preferred frame. As a result, it is of considerable interest to establish the presence or absence of physical effects that depend on our velocity relative to this frame and, therefore, the presence or absence of a cosomologically preferred frame.[4,7] This is especially interesting since one of the weakest points in the present empirical knowledge of gravitation physics is the limit on the value of the PPN preferred-frame parameter α_1. The present limit is $|\alpha_1| \leq 0.02$,[4] while general relativity predicts $\alpha_1 = 0$. The proposed experiment should be able to place tight new constraints on the magnitude of α_1.

Metric gravitation theories describe gravity in terms of a non-Euclidean "curved" geometry of space-time. These theories, however, differ in their prediction for the strength of this curvature. In the PPN formalism, the amount of space curvature which a unit rest mass produces in a given theory is measured by a parameter γ. Its value, in general relativity, is unity. Another important consequence of the metric gravitation theories is the dragging of inertial frames by a rotating body. This effect, called the Lense-Thirring effect, contributes to the precession of a gyroscope in a frame fixed on the earth because the earth's gravitational field induces a rotation of the gyroscope's proper reference frame with respect to the distant stars. The PPN parameters Δ_1 and Δ_2, which also have the value of one in general relativity, determine the magnitude of the frame-dragging rotations predicted by different metric theories. These effects predicted by general relativity have yet to be detected.

In the proposed experiment, a laser's output is split into two beams which are then injected into a passive cavity in opposite directions.[8] The frequencies of the beams are shifted by electro-optic means in order to "fix" the cavity modes of the counterpropagating beams. We show, in the next section, using the PPN formalism, that the beat note between the clockwise and counterclockwise laser beams are affected not only by the interferometer rotation and the earth rotation, but also by the off-diagonal gravitational contribution to the line element in the interferometer's frame. The α_1-dependent preferred-frame term, which is the largest gravitational correction to the frequency difference, can be separated from other effect and noise due to its periodic dependence on the sidereal time. We also discuss the possibility of separating and measuring the terms associated with the curvature parameter γ and the frame dragging parameters Δ_1 and Δ_2. In Sec. III, we discuss the precision to which we can measure the various quantities entering in the expression for the frequency difference.

II. SAGNAC FREQUENCY DIFFERENCE IN PPN FORMALISM

To determine the frequency difference between the two counterpropagating laser beams[9] we assume that the metric takes the form

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}\ , \tag{1}$$

where $\eta_{\mu\nu}$ is the metric of flat space-time and $h_{\mu\nu}$ is a small correction due to the earth's gravitational field and rotation of the ring. We can always construct coordinates with respect to which $h_{\mu\nu}=0$ at the center of the ring laser. In these coordinates $h_{\mu\nu}$ will be small throughout the region containing the interferometer.

The covariant form of Maxwell's equations for the vector potential $A_\sigma(X^\mu)$ is

$$\partial_\mu[\sqrt{-g}\,g^{\mu\rho}g^{\nu\rho}(\partial_\sigma A_\rho-\partial_\rho A_\sigma)]=0, \tag{2}$$

where $g=\det g_{\mu\nu}$ and the contravariant metric tensor is the matrix inverse of $g_{\mu\nu}$. The smallness of the laser wavelength relative to the size of the ring cavity and the scale on which the gravitational field varies, means that the high-frequency limit of Maxwell equations suffices to determine A_μ.[10] We write the electromagnetic potential as

$$A_\mu=\mathscr{A}_\mu e^{iS}\ , \tag{3}$$

where $\mathscr{A}_\mu$ is a slowly varying amplitude function and S is a rapidly varying phase function. The Maxwell equations then reduce to the eikonal equations

$$g^{\mu\nu}S_{,\mu}S_{,\nu}=0\ . \tag{4}$$

We solve Eq. (4) to first order in $h_{\mu\nu}$.

The phase function S can be expressed as $S=S_0+\Delta S$ where $S_0=\omega t-\vec{k}\cdot\vec{X}$ is the zeroth-order solution of Eq. (4) where $k\equiv(\omega/c,\vec{k})$ is an arbitrary "Minkowskian" null vector, i.e.,

$$\eta_{\mu\nu}k^\mu k^\nu=\frac{\omega^2}{c^2}-k^2=0\ . \tag{5}$$

It then follows from Eq. (4) that the first-order equation in h_μ is

$$\eta^{\mu\nu}S_{0,\mu}\Delta S_{,\nu}=-\tfrac{1}{2}S_{0,\mu}S_{0,\nu}\,h^{\mu\nu} \tag{6}$$

since

$$k^\nu=\eta^{\mu\nu}S_{0,\mu}\ . \tag{7}$$

Equation (6) can be written in the form

$$k^\mu\Delta S_{,\nu}=-\tfrac{1}{2}k^\mu k^\nu h_{\mu\nu}\ . \tag{8}$$

It is apparent that the lhs is just ω times the derivative of ΔS along the unperturbed ray path. If we parametrize the ray by the time for propagation, we obtain the following integral expression for the phase shift induced by $h_{\mu\nu}$ during the propagation from the point at t_0 to another point at t_1:

$$S=-\frac{\omega}{2}\int_{t_0}^{t_1}h_{\mu\nu}n^\mu n^\nu dt\ , \tag{9}$$

where $n^\mu=ck^\mu/\omega$ is the unit tangent to the traversed path.

We choose to call one direction of circulation around the ring positive, the other negative. We denote the frequencies of waves circulating in these directions by ω_+ and ω_-. The change of phase of a wave during a single positive circuit is

$$\Delta S^+=-\frac{\omega^+}{2}\int_0^T dt\,(h_{00}+2h_{0i}n^i+h_{ij}n^in^j)\ , \tag{10}$$

where T is the circulation time such that $P=cT$ is the perimeter of the cavity. In a similar manner, we obtain, for the wave propagating in the opposite direction,

$$\Delta S^-=-\frac{\omega_-}{2}\int_0^T dt\,(h_{00}-2h_{0i}n^i+h_{ij}n^in^j)\ . \tag{11}$$

Since, in the proposed experiment, the cavity modes are the same for the two beams, the resonance condition

$$S^{+}=S^{-} \tag{12}$$

is obeyed. It then follows from Eqs. (10) and (11) that the frequency difference $\Delta\omega=\omega_{+}-\omega_{-}$ is given by

$$\Delta\omega=\frac{\omega_{+}+\omega_{-}}{P}\int_0^T h_{0i}n^i dt=\frac{2}{\bar{\lambda}P}\oint dl^i h_{0i}\ , \tag{13}$$

where $\bar{\lambda}=2c/(\omega_{+}+\omega_{-})$ is the reduced mean wavelength. The line integral in Eq. (13) is around the positively oriented path. The Stokes's theorem can be applied to obtain the frequency difference between the counter running waves

$$\Delta\omega=\frac{2}{\bar{\lambda}P}\int_{\Sigma}\int(\vec{\nabla}\times\vec{h})\cdot d\vec{a}\simeq\frac{2A}{\lambda P}(\vec{\nabla}\times\vec{h})\cdot\hat{a}\ , \tag{14}$$

where $(\vec{h})_i\equiv h_{0i}$, A is the area of the planar surface bounded by the ray path of the interferometer, and $\hat{a}$ is a unit vector normal to the plane of the device. The surface Σ is bounded by the ray path and the curl is evaluated at the center of the interferometer. In deriving Eq. (14), we have retained only the term linear in $h_{\mu\nu}$. When the interferometer spins on its axis with angular rate Ω in the absence of gravity, Eq. (14) becomes $\Delta\omega=4A\Omega/\bar{\lambda}P$, which gives the usual Sagnac effect.

In order to determine the value of $\Delta\omega$, we now turn to the PPN formalism. We construct an explicit transformation from the PPN coordinate system centered on the earth (and rotationally tied to the fixed stars) to a system in which the interferometer is at rest and in which the metric has $h_{\mu\nu}=g_{\mu\nu}-\eta_{\mu\nu}=0$ at the center of the ring. We begin with the rectangular PPN coordinate system $(T,\vec{X})$. The line element has the form

$$ds^2=(1-2U)c^2dT^2-(1+2\gamma U)(dX^l)^2+2cg_l^{\rm PPN}dX^l dT\ , \tag{15}$$

with

$$g_l^{\rm PPN}=\tfrac{7}{2}\Delta_1 V^l+\tfrac{1}{2}\Delta_2 W^l+\left[\frac{\alpha_1}{2}-\alpha_2\right]w^l U+\alpha_2 w^j U^{jl}\ . \tag{16}$$

Here γ, Δ_1, Δ_2, α_1, and α_2 are the PPN parameters; as mentioned before, measured space curvature produced by unit rest mass, Δ_1 and Δ_2 are the parameters associated with the dragging of inertial frames, and α_1 and α_2 measure the extext of and nature of preferred-frame effects. The Newtonian potential U and the potentials V^l, W^l, and U^{jl} are determined by the density $\rho_0(X)$ and velocity distribution $\vec{v}(X)$ of the matter in the earth through the following expressions:

$$U=\int\frac{\rho_0(\vec{X}')d^3X'}{|\vec{X}-\vec{X}'|}\ , \tag{17a}$$

$$V^l=\int\frac{\rho_0(\vec{X}')v^l(\vec{X}')}{|\vec{X}-\vec{X}'|}d^3X'\ , \tag{17b}$$

$$W^l=\int\frac{\rho_0(\vec{X}')[(\vec{X}-\vec{X}')\cdot\vec{v}(X')](X^l-X'^l)}{|\vec{X}-\vec{X}'|^3}d^3X'\ , \tag{17c}$$

$$U^{jl}=\int\frac{\rho_0(\vec{X}')(X^j-X'^j)(X^l-x'^l)}{|\vec{X}-\vec{X}'|^3 d^3X'}\ , \tag{17d}$$

$\vec{w}$ is the velocity of the earth relative to fixed stars, i.e., relative to the frame in which the cosmic background appears isotropic. In Eqs. (15) and (16) the velocity $\vec{v}$ is $O(|\vec{w}|)$ and the potential U is $O(|\vec{w}|^2)$, where the symbol O is chosen to denote the order of smallness. The line element is obtained by retaining the terms in g_{00} to $O(|\vec{w}|^4)$, in $g_{0\alpha}$ to $O(|\vec{w}|^3)$, and in $g_{\alpha\beta}$ to $O(|\vec{w}|^2)$.

We now transform the line element (15) to a system in which the interferometer is at rest and in which the metric has $h_{\mu\nu}=0$ at the center of the ring. The explicit transformation to the new coordinate system is carried out in the Appendix. The resulting expression for the line element is given by Eqs. (A6) and (A7). We have retained, as before, the terms in g_{00} to $O(|\vec{w}|^4)$, in $g_{0\alpha}$ to $O(|\vec{w}|^3)$, and in $g_{\alpha\beta}$ to $O(|\vec{w}|^2)$.

It can be shown, using Eqs. (A5) and (A6), that

$$\vec{\nabla}\times\vec{h}=2\left[1+U+\frac{\bar{\Omega}^2 r_\oplus^2}{c^2}\cos^2 l\right]\bar{\Omega}\hat{z}+2(\gamma+1)\bar{\Omega}r_\oplus\cos l\times(U_{,x}\hat{z}-U_{,z}\hat{x})-\vec{\nabla}\times\vec{g}^{\rm PPN}\ , \tag{18}$$

where $r_\oplus$ is the radius of the earth. The laboratory's velocity through the PPN coordinates is $\vec{v}=(o,\bar{\Omega}r_\oplus\cos l,0)$ and its PPN coordinate acceleration is $\vec{A}=\vec{a}+\vec{\nabla}U=(-\Omega^{-2}r_\oplus\cos l,0,0)$, where $\vec{a}$ is the acceleration of the laboratory relative to the freely falling frames. It is easy to verify that $\bar{\Omega}^3 r_{\oplus}^2 \cos l \hat{z} = \vec{v} \times \vec{A}$. Furthermore, the combination $(1+U+\frac{1}{2}v^2/c^2)\bar{\Omega}=\Omega_{\oplus}$ is simply the rate of rotation of the earth as measured in the laboratory. The factor $(1+U+\frac{1}{2}v^2/c^2)$ accounts for the corrections arising due to redshift and the time dilation of laboratory clocks relative to the clocks at rest in the PPN coordinates far from the earth. In terms of these quantities, Eq. (18) becomes[12]

$$\vec{\nabla}\times\vec{h}=2[\Omega_{\oplus}\hat{z}-(\gamma+1)\vec{v}\times\vec{\nabla}U+\tfrac{1}{4}\alpha_1\vec{w}\times\vec{\nabla}U -\tfrac{1}{4}(7\Delta_1+\Delta_2)\vec{\nabla}\times\vec{V}] . \tag{19}$$

In Eq. (19), we have substituted for $\vec{g}^{\rm PPN}$ from Eq. (16). We have also neglected the term $\vec{v}\times\vec{A}$ since its magnitude is much smaller than other terms in Eq. (19).

For the present discussion, it is sufficiently accurate to treat the earth as a uniform, rigidly rotating sphere. The Newtonian potential U is then given by $r_s/2r_{\oplus}$ where $r_s=2GM/c^2$ is the Schwarzschild radius (G is the universal gravitational constant and M is the total mass of the earth). The interferometer is assumed to be spinning at a rate Ω_0 about its axis $\hat{a}=(\cos\theta_x, \cos\theta_y, \cos\theta_z)$ and the velocity of the earth relative to fixed stars is assumed to be $\vec{w}=|\vec{w}|\;(A(t_s),-B(t_s),C)$ (t_s is the sidereal time). It then follows from Eqs. (11) and (22) that the frequency difference $\Delta\omega$ is given by[13]

$$\Delta\omega=\frac{4A}{\lambda P}[\Omega_0+\Omega_{\oplus}\cos\theta_z+\alpha_1 T_{\alpha_1}+\tfrac{1}{8}(7\Delta_1+\Delta_2)T_{\Delta} +(\gamma+1)T_{\gamma}] , \tag{20}$$

where the terms T_{α_1}, T_{Δ}, and T_{γ} associated with the preferred-frame effect, Lense-Thirring effect, and space curvature effect, respectively, are as follows:

$$T_{\alpha_1}=\frac{1}{8}\frac{r_s}{r_{\oplus}^2}\,|\vec{w}|\,\{\,[A(t_s)\sin l - C\cos l]\cos\theta_y + B(t_s) \times(\sin l\,\cos\theta_x-\cos l\,\cos\theta_z)\,\} , \tag{21a}$$

$$T_{\Delta}=\frac{1}{10}\Omega_{\oplus}\frac{r_s}{r_{\oplus}}[-3\sin 2l\,\cos\theta_x+(1-3\sin^2 l)\cos\theta_z] , \tag{21b}$$

$$T_{\gamma}=\frac{1}{2}\Omega_{\oplus}\,\frac{r_s}{r_{\oplus}}(\tfrac{1}{2}\sin 2l\,\cos\theta_x-\cos^2 l\,\cos\theta_z) . \tag{21c}$$

According to the measurements of Smoot *et al.*[14] of the dipolar anisotropy of the microwave background,

$$\begin{aligned} |\vec{w}| &= (390\pm 60)\ \text{km/sec} ,\\ A(t_s) &- \cos\delta\,\cos\alpha ,\\ B(t_s) &= \cos\delta\,\sin\alpha ,\\ C &= \sin\delta , \end{aligned} \tag{22}$$

with

$$\delta=6^\circ\pm 10^\circ,\ \ \alpha=\Omega_{\oplus}\,[t_s-(11\pm 0.6)\ \text{h}] . \tag{23}$$

The contributions to $\Delta\omega$ due to bias Ω_0 to the earth rotation $\Omega_{\oplus}$ are apparent in Eq. (23). The sidereal day periodicity of the α_1-dependent preferred-frame term and its θ_y-dependent component both facilitate the separation of this effect from others and noise. With optimized latitude and tilt angles, the amplitude of this periodic effect is at least $1.2\times 10^{-7}\alpha_1\Omega_{\oplus}$. The γ-dependent geodetic rotation rate is $1.4\times 10^{-9}[(\gamma+1)/2]\Omega_{\oplus}$, while that of Lense-Thirring is 5.6×10^{-10} $[(7\Delta_1+\Delta_2)/8]\Omega_{\oplus}$.

III. DISCUSSION

The Sagnac frequency difference $\Delta\omega$ can be measured in two different types of experiments depending on whether the laser is placed inside or outside the ring cavity. When the laser is placed inside the ring cavity, the two modes oscillate at a frequency difference which is proportional to the rate of rotation of the device.[9] However, at low rotation rates, backscattering couples the counterpropagating waves, and the beat note between the two waves disappears. This "locking" problem associated with internally driven ring lasers has been the subject of recent investigations,[15] and these devices can now operate down to a rate of 10^{-5} to 10^{-6} of earth rate using a ring of 1-m diameter. The ring radii larger than this are difficult to work with as the laser tends to go multimode. There are several methods of biasing a ring laser in order to avoid the locking problem, however, an alternative approach using a passive device with a laser located outside a ring interferometer looks even more promising. This method avoids the locking problem and very encouraging estimates of the sensitivity of these devices have been offered.

In this approach, due to Ezekiel and coworkers,[8] a passive ring Fabry-Perot interferometer is employed as the rotation sensing element and an external laser is used to measure any difference between the clockwise and counterclockwise lengths of the cavity cause by inertial rotation. The external laser is split into two beams whose frequencies are shifted by electro-optic means. The cavity path-length difference can be determined by locking the clockwise and counterclockwise resonance frequencies of the cavity to the shifted frequencies of the laser beams. The precision with which the frequency difference $\Delta\omega$ of the counterpropagating beams can then be measured depends on the signal-to-noise ratio in the measurement. It is argued that, by invoking the photon shot noise limit of heterodyne detection, it should be possible to reach a sensitivity to 10^{-10} of earth rate in an integration time of 1000 sec. The major problems associated with achieving this high level of sensitivity have been related to the long-term stability of the laser frequency, to the misalignment effects due to thermal and mechanical stresses, and to the backscattering from the mirrors.

There are of course, many other interesting problems associated with such high-precision ring laser interferometry. Some of these problems are discussed by Ezekiel and Balsamo.[8] It is perhaps well to mention here one other subtle source of noise arising due to the effect of finite beam widths of the counterpropagating beams on the frequency difference $\Delta\omega$. For a Gaussian beam of rms width σ, the relationship between the inertial rotation rate Ω and frequency difference $\Delta\omega$ can be easily determined in the absence of gravitation effects.[16] To a good approximation it is given by

$$\Delta\omega=\frac{4A}{\lambda P}\left[\Omega-\frac{\lambda^2}{\sigma^2}\Omega\right]. \tag{24}$$

For a beam of rms width $\sigma\approx 0.1$ cm, we obtain $\lambda^2/\sigma^2\approx 10^{-8}$. It is apparent from our earlier discussion that the term corresponding to the preferred-frame effect in Eq. (20) is of the order $10^{-7}\alpha_1\Omega_\oplus$. The effect of beam width is of order $10^{-8}\Omega_\oplus$. Thus we see from Eq. (24) that variation in beam width could lead to a source of error in such precision measurements.[17] We now turn our attention to the presicion with which the various quantities that occur in the expression of the frequency shift [cf. Eq. (20)] can be experimentally determined.

A. Scale factor $4A/\lambda P$

We cannot hope to make sufficiently accurate, independent measurements of the area A and perimeter P of the interferometer's ray path or of the wavelength λ. However, when we have "fixed" the modes of the interferometer these quantities are not independent. Indeed, the scale factor itself will be essentially unaffected by any isotropic distortion of the interferometer, for example, caused by changes of temperature or of the biasing rotation rate. We can determine the scale factor by measuring $\Delta\omega$ for different biasing rates Ω_0. An alternative strategy for measuring rotation rate would be to adjust the bias frequency so that $\Delta\omega$ was zero and then to measure Ω_0. In this situation, independent of the scale factor, the magnitude of Ω_0 would be $\Omega_0\cos\theta_z$ plus the gravitational contributions. We discuss precision measurements of Ω_0 below.

B. Bias frequency Ω_0

We now consider the precision to which we can measure the bias frequency Ω_0 as produced by, for example, rotating the entire interferometer. One can measure Ω_0 by means of the time for the interferometer table to turn through 2π. Because this time can be observed to more than satisfactory precision, the error in this measurement is associated with the angular uncertainty that the table has indeed turned through 2π. This uncertainty may be minimized by, for example, using a mirror mounted on the turntable with a large autocollimator mirror located in the laboratory adjacent to the table. In this way it should be possible to determine that the mirror has returned to its original position to within 10^{-4} sec of arc.[18] This corresponds to an error of around $10^{-10}\,\Omega_0$.[19]

C. Earth rotation rate term $\Omega_\oplus\cos\theta_z$

The magnitude and direction of the earth's angular velocity $\vec{\Omega}_\oplus$ are currently being determined by classical astrometric measurements and new space techniques. Within a couple of years it is expected that two of the new techniques will achieve accuracies of about 10^{-9} for measuring the quantity over a one day interval. One approach is to use very long baseline radio interferometry, as discussed in a review article by Counselman.[20] Table I of that paper lists the expected limitation on measuring the earth's angular position to be 0.001 sec of arc, which corresponds to about 10^{-9} in rotation rate for one day. The other approach involves laser-range measurements to the Lageos satellite and to the moon.[21] The Lageos measurements would give the highest accuracy for short periods, while the lunar measurements would determine the intermediate and longer period variations. The overall accuracy again is ecpected to be about 10^{-9} for a one-day interval.[22,23]

The angle θ_z is the angle between the interferometer axis and $\vec{\Omega}_\oplus$. Notice that in the special interferometer configuration with these vectors aligned $\theta_z \approx 0$ the fractional error in the earth rate term induced by uncertainty in θ_z is proportional to the square of this uncertainty. Limiting the factional error to 10^{-10} requires alignment to within 10^{-5} rad, or about 1 in. of arc, which should be achievable. An added benefit of operating in the aligned configuration is that variable "noise" contributions to θ_z as large as 10^{-5} rad will not change the fractional error of the earth rate term. Of course, this assumes that the initial alignment has been carried out to within 10^{-5} rad. Tilting of the interferometer due to seismic activity, to differential heating of the support structure, and to earth tides are among the possible sources of "noise" in this context.

D. Tilt angles θ_x, θ_y, θ_z

Once the interferometer axis has been aligned with $\vec{\Omega}_\oplus$, variations of the orientation of the interferometer relative to the local vertical can be monitored with commercially available "tilt meters" to $\delta\theta \approx 10^{-9}$ rad.[24] Such variations translate simply into variations in θ_x, θ_y, and θ_z.

We conclude this section with a few remarks on the gravitation physics measurements we envision. In the search for a possible preferred-frame effect, one would exploit the sidereal periodicity of the α_1-dependent rotation. With the interferometer fixed in the laboratory and aligned with $\vec{\Omega}_\oplus$, $\Delta\omega$ would be scanned for a tiny component with this period and the appropriate phase. Noise with diurnal periodicity can be expected. However, the relative insensitivity of $\Delta\omega$ to temperature variation and thermally induced θ_z variations, as described above, should reduce such noise. An experiment running for a few weeks will allow one to distinguish between the nearly equal sidereal and diurnal periodicities. The attempt to isolate the geodetic and Lense-Thirring rotations requires the subtraction of the earth's rotation rate $\Omega_\oplus$, the earth rate term in Eq. (20) for $\theta_z \approx 0$, from the rotation rate measured by the ring interferometer. Clearly, one requires measurements of the highest quality for both these quantitites if this procedure is to reveal the tiny gravitational effects. Accuracies of a part in 10^{10} for both $\Omega_\oplus$ and the ring-measured rate of rotation are required. Long-term averaging of the data may help in achieving the necessary accuracy.

IV. CONCLUSION

In this paper, we have suggested and analyzed an earthbound optical test of metric gravitation theories based on high-precision ring laser interferometry ($10^{-10}\Omega_\oplus$ sensitivity) with good time resolution (10^3 sec to make a measurement). The proposed experiment is a challenging one but should be able to place tight new constraints on the preferred-frame parameter α_1, and is sensitive to principle to the as yet unobserved geodetic and Lense-Thirring rotations of the local inertial frames relative to the fixed stars. In this connection, we should mention that, in 1960, Schiff[25] suggested an experiment to test Einstein's general theory of relativity by measuring the precession of the spin of gyroscopes in orbit around the earth. This experiment, which is being set up at Stanford,[26,27] would be sensitive mainly to the geodetic and Lense-Thirring rotations. It would, however, not be able to improve our knowledge of α_1.

Finally, it should be remarked that motivation to undertake the development of a ring interferometer system of the type described in this paper does not come only from gravitation physics. Indeed, the consideration of such systems has previosly been motivated by potential geophysical applications (e.g., measurements of earth wobble, of rotational seismic motions, etc.) and by the need for high-precision test equipment for turntables and gyroscopes. The opportunity to perform a significant gravitational experiment does, however, make an interesting and important addition to such applications.

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APPENDIX

In Eq. (15), Z axis coincides with the earth's axis. Without loss of generality we center the laboratory at $X_0 \equiv (r_\oplus \cos l,\, 0,\, r_\oplus \sin l)$, where $r_\oplus$ is the radius of earth and l is the latitude. We now transform to a rotating set to coordinates $(t',\vec{x}')$, defined by

$$T=t' ,$$
$$X=(r_\oplus \cos l + x')\cos(\bar{\Omega}t') - y'\sin(\Omega t') ,$$
$$Y=(r_\oplus \cos l + x')\sin(\bar{\Omega}t') + y'\cos(\bar{\Omega}t') ,$$
$$Z=r_\oplus \sin l + z' . \quad \text{(A1)}$$

The rotation rate $\bar{\Omega}$ is chosen such that the laboratory is fixed in the new coordinate system. Its value will be related to the laboratory-measured earth-rotation rate later. In terms of the new coordinates $(t',\vec{x}')$, the line element becomes

$$ds^2=\left[1-2U-2\vec{x}'\cdot\vec{\nabla}U-\frac{\bar{\Omega}^2}{c^2}r_\oplus\cos l(r_\oplus\cos l+2x')\right]c^2dt'^2-2r_\oplus\cos l\,\bar{\Omega}(1+2\gamma U+2\gamma\vec{x}'\cdot\vec{\nabla}U)dy'c\,dt'$$
$$+2\bar{\Omega}(1+2\gamma U)(y'dx'-x'dy')c\,dt'+2(g_l^{\rm PPN}+\vec{x}'\cdot\vec{\nabla}g_l^{\rm PPN})dx'^l c\,dt'-(1+2\gamma U+2\vec{x}'\cdot\vec{\nabla}U)(dx'^l)^2 . \quad \text{(A2)}$$

The PPN potentials and their derivates in Eq. (A2) are understood to be evaluated at X_0. We can remove the U dependence from g^{oo} and g^{ii} by rescaling $(t',\vec{x}')$ to $(t'',\vec{x}'')$ through the following transformation:

$$t'=(1+U)t''-g_l^{\rm PPN}x''^l$$
$$+[(1+\gamma)U+\bar{\Omega}^2r_\oplus^2\cos^2l/2c^2](\bar{\Omega}r_\oplus\cos l/c)y'' ,$$
$$x'=(1-\gamma U)x'' , \quad \text{(A3)}$$
$$y'=(1-\gamma U)y'' ,$$
$$z'=(1-\gamma U)z'' .$$

Next we make a transformation to a moving system of coordinates $t'''=t''$, $\vec{x}'''=\vec{x}''+\vec{v}t''$, where $\vec{v}\equiv(0,\bar{\Omega}r_\oplus\cos l,0)$ is chosen to make $g^{\mu\nu}=\eta^{\mu\nu}$ at the center of the interferometer. We then apply a Lorentz boost of $\vec{v}$ back to a system $(t,\vec{x})$ at rest in the laboratory. The resulting transformation is

$$t''=\frac{t+\bar{\Omega}r_\oplus y\cos l/c^2}{(1-\bar{\Omega}^2r_\oplus^2\cos^2l/c^2)^{1/2}}$$
$$\approx\left[1+\frac{\bar{\Omega}^2r_\oplus^2\cos^2l}{2c^2}\right]t+\left[\frac{\bar{\Omega}r_\oplus\cos l}{c}\right]y ,$$
$$x''=x ,$$
$$y''=\left[1-\bar{\Omega}^2r_\oplus^2\,\frac{\cos^2l}{c^2}\right]^{1/2}y$$
$$\approx\left[1-\frac{\bar{\Omega}^2r_\oplus^2\cos^2l}{2c^2}\right]y , \quad \text{(A4)}$$
$$z''=z .$$

It then follows, on substituting from (A4) in Eq. (A3) that

$$ds^2=\left[1-2\vec{x}\cdot\vec{\nabla}U-2r_\oplus\frac{\bar{\Omega}^2x}{c^2}\cos l\right]c^2dt^2+2p_ldx^lc\,dt$$
$$-2r_\oplus\,\frac{\bar{\Omega}^2\cos l}{c^2}(y\,dx-x\,dy)dy$$
$$-(1+2\gamma\vec{x}\cdot\vec{\nabla}U)(dx^l)^2 , \quad \text{(A5)}$$

where

$$p_ldx^l=\vec{x}\cdot\vec{\nabla}g_l^{\rm PPN}dx^l+\bar{\Omega}(1+U)(ydx-xdy)$$
$$+2r_\oplus\bar{\Omega}\cos l[r_\oplus\bar{\Omega}x\cos l-(1+\gamma)\vec{x}\cdot\vec{\nabla}U]dy . \quad \text{(A6)}$$

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[1]See, for example, C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973); V. Braginsky, C. M. Caves, and K. S. Thorne,

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[2]S. O. Campbell, Ph. D. thesis, Oklahoma State Univer-isty, 1976 (unpublished). We became aware of this work after completing our studies; our conclusions differ substantially. We are grateful to K. Stowell for bringing this reference to our attention.

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[5]Aspects of this work are discussed in M. O. Scully, in *Laser Spectroscopy IV,* edited by H. Walther and K. Rothe (Springer, Berlin, 1979), p. 21; M. P. Haugan, M. O. Scully, and K. Just, Phys. Lett. 77A, 88 (1980).

[6]The proposed experiment is significant in light of the general belief that an earthbound experiment to detect these gravitational effects would be impossible [see for example, C. W. F. Everitt, in *Experimental Gravitation,* edited by B. Bertotti (Academic, New York, 1974)].

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[12]Readers familiar with relativistic gravitation will note that sophisticated techniques may be used to drive our results. See, for example, C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), Sec. 40.7 [note that their Eq. (40.34) should read $(\gamma+1)\vec{v}\times\vec{\nabla}U$]. The presentation here seems more suitable for a general audience.

[13]It should be mentioned that the rotation rates of the earth around the sun, the solar system around the galaxy, etc., do not enter in this expression because these free-fall motions do not produce a rotation of the earth relative to the fixed stars.

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