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Lasers with three-level absorbers

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Output characteristics of a traveling-wave monomode laser with an intracavity three-level absorber are investigated. The laser frequency is assumed to be symmetrically detuned from the two ground-state sublevels for a Λ-type transition scheme. It is shown that bistability and hysteresis in the laser output, characteristics of a first-order phase transition, may arise much before the absorber saturates. For this reason, bistability may be observed even if the gain medium saturates faster than the absorber, in contrast with the case of a two-level saturable absorber. The physical mechanism for bistable behavior is coherent population trapping which accumulates all absorbing atoms in the ground-state sublevels with a sharp decrease in absorption.

I. INTRODUCTION

It is well known that lasers with saturable absorbers can exhibit bistability and hysteresis in their output.[1-7] In terms of the phase-transition analogy,[4,8] this behavior corresponds to a first-order phase transition in contrast to a second-order phase transition, which takes place at lasing threshold in the absence of an intracavity absorber. In most of the previous work, the absorber is modeled as a homogeneously broadened two-level system and bistability arises from an interplay between the amplifying and absorbing media with different saturation characteristics. In particular, the ratio of their saturation intensities plays an important role and bistability can be observed only if the absorber saturates more easily than the gain medium.

In this paper we consider the case wherein three resonant levels are required to describe the absorber. This is the situation when either the ground state or the excited state has two closely spaced sublevels radiatively coupled to a common level (so-called Λ- or V-type three-level systems). Although intracavity three-level absorbers have been considered in high-resolution spectroscopy,[9] their effect on laser output does not appear to have been studied in the past. Recently, the case of an intracavity two-photon absorber has been considered.[10] However, intermediate levels are taken to be far from one-photon resonances so that this system is similar to a two-level system.

We consider a Λ-type three-level absorbing medium and obtain its susceptibility under certain simplifying assumptions. The steady-state solutions of the laser intensity show that both first- and second-order phase transitions are exhibited depending on the parameter values. An important difference is, however, that bistability is observed even for the case wherein the absorber saturates slowly in comparison to the gain medium, in sharp contrast with the two-level absorber case. This is due to the fact that the physical mechanism responsible for bistable behavior sets in much before the saturation effects come into play; it is called coherent population trapping[11] and arises from transverse optical pumping which accumulates all the atoms in a coherent superposition of the ground-state sublevels. This is the phenomenon responsible for nonabsorption resonance[12,13] observed in spectroscopy. Recently, intensity dependence of this resonance was explored in the context of optical bistability[14] and phase conjugation.[15]

II. INTENSITY AND PHASE EQUATIONS

We consider a laser cavity containing the amplifying and absorbing media as shown in Fig. 1. Using Maxwell's wave equation for the electric field

$$\mathcal{E}(z,t)=\tfrac{1}{2}E(t)\exp[-i(\omega t+\phi-kz)]+\text{c.c.}, \qquad (1)$$

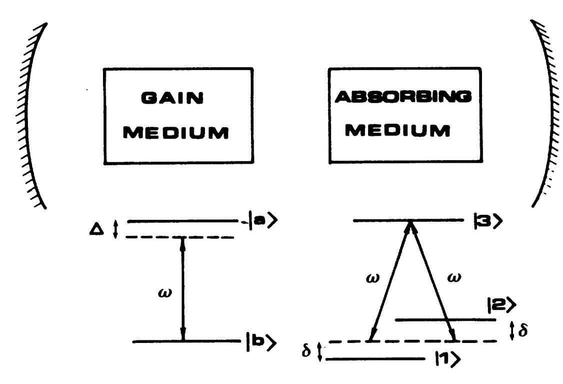

FIG. 1. Schematic view of a laser with an intracavity absorber. The transition schemes associated with the gain and absorbing media are also shown. The laser frequency ω is assumed to lie midway between the ground-state sublevels $|1\rangle$ and $|2\rangle$ and δ is the corresponding detuning parameter.

the slowly varying amplitude E and the phase ϕ are found to satisfy[16]

$$\frac{dE}{dt} = -\frac{1}{2}\frac{\omega}{Q}E - \tfrac{1}{2}\omega\chi'' E, \quad (2a)$$

$$\frac{d\phi}{dt} = (\omega_c - \omega) - \tfrac{1}{2}\omega\chi', \quad (2b)$$

where ω is the laser frequency, ω_c is the cavity-mode frequency, Q accounts for cavity losses, and the complex susceptibility

$$\chi = (\chi' + i\chi'') = \chi_{amp} + \chi_{abs} \quad (3)$$

consists of contributions arising from the amplifier and the absorber. We assume that the laser oscillates in a single linearly polarized plane-wave mode. Furthermore, standing-wave effects will be ignored by restricting attention to a unidirectional ring cavity.

The gain medium is modeled as a homogeneously broadened two-level system described by the susceptibility[16]

$$\chi_{amp} = -\frac{\mu^2 N T_2}{\epsilon_0 \hbar}\left(\frac{\Delta + i}{1 + \Delta^2 + E^2/I_s}\right), \quad (4)$$

where $\Delta = (\omega_{ab} - \omega)T_2$ is the atomic detuning, and $I_s = (\hbar^2/\mu^2 T_1 T_2)$ is the saturation intensity. The various atomic parameters N, μ, ω_{ab}, T_1, and T_2 represent, respectively, the population-inversion density, the dipole moment, the transition frequency, and the longitudinal and transverse relaxation times.

The absorbing medium is modeled as a homogeneously broadened Λ-type three-level system as shown in Fig. 1. Although the susceptibility χ_{abs} for arbitrary level detunings can be obtained,[17] the resulting expression is too involved to be useful. Substantial simplification occurs if we assume that the laser frequency lies midway between sublevels $|1\rangle$ and $|2\rangle$, and the dipole moments between levels $|1\rangle$ and $|3\rangle$ and $|2\rangle$ and $|3\rangle$ have equal values.[14] For further simplicity we neglect nonradiative transverse decay between ground-state sublevels; this restriction can easily be removed, if necessary, and does not introduce any qualitative changes as long as the nonradiative decay rate is much smaller than the radiative decay rate $T_{1,2}^{-1}$. Under these conditions, the following expression for χ_{abs} is obtained[14,15]:

$$\chi_{abs} = (i\bar{\mu}^2 \bar{N} \bar{T}_2/\epsilon_0\hbar)(1 + \tilde{\delta}^2 + \tfrac{3}{2}E^2/\bar{I}_s)^{-1}, \quad (5)$$

$$\tilde{\delta} = [\delta - \tfrac{1}{2}(r/\delta)E^2/\bar{I}_s], \quad \delta = \tfrac{1}{2}\Omega_{21}T_2, \quad (6)$$

where $r = (\bar{T}_2/2\bar{T}_1)$ and Ω_{21} is the frequency splitting of levels $|1\rangle$ and $|2\rangle$. The absorber parameters $\bar{\mu}$, $\bar{N}$, $\bar{I}_s$, $\bar{T}_1$, and $\bar{T}_2$ have definitions analogous to the gain medium parameters.[18]

Several features of χ_{abs} given by Eq. (5) are noteworthy. It is purely imaginary indicating that, for the case considered here, dispersive effects due to laser detuning from levels $|1\rangle$ and $|2\rangle$ mutually cancel. Further, the form of χ_{abs} is similar to the two-level case [Eq. (4)] except that the effective detuning parameter $\tilde{\delta}$ is itself intensity dependent. This gives rise to nonabsorption resonance[13] mentioned in the Introduction. The decreased absorption near $\delta = 0$ is a consequence of the coupled nature of transitions $1 \to 3$ and $2 \to 3$ and arises from coherent population trapping.[11] In particular, $\chi_{abs} = 0$ when $\delta = 0$. When nonradiative damping between levels $|1\rangle$ and $|2\rangle$ is allowed, the sharp dip in the absorption spectrum of three-level system no longer goes to zero.[13]

Using Eqs. (2)–(5) we obtain the following equations for the dimensionless intensity $I = E^2/I_s$ and the phase ϕ:

$$\frac{dI}{dt} = \frac{\omega}{Q}\left(\frac{A}{1 + \Delta^2 + I} - \frac{\bar{A}}{1 + \tilde{\delta}^2 + \frac{3}{2}aI} - 1\right)I, \quad (7)$$

$$\frac{d\phi}{dt} = (\omega_c - \omega) + \frac{\omega}{2Q}\left(\frac{A\Delta}{1 + \Delta^2 + I}\right), \quad (8)$$

where

$$A = (\mu^2 N T_2 Q/\epsilon_0 \hbar), \quad (9)$$

with a similar definition for $\bar{A}$. The parameter

$$a = I_s/\bar{I}_s = (\bar{\mu}^2 \bar{T}_1 \bar{T}_2/\mu^2 T_1 T_2) \quad (10)$$

is a relative measure of the saturation characteristics of the amplifying and absorbing media.

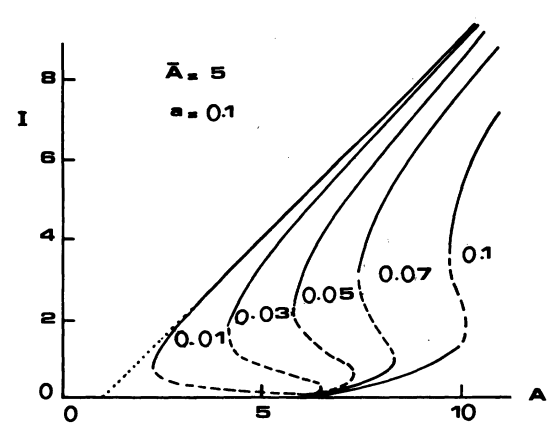

FIG. 2. The laser intensity I versus the pump parameter A for several values of the detuning parameter δ. The fixed parameters are $\bar{A} = 5$, $a = 0.1$ and $r = 1$ $(\bar{T}_2 = 2\bar{T}_1)$. The unstable portion of the intensity curve is shown by a dashed line and gives rise to bistability and hysteresis in the laser output. The dotted line corresponds to the case wherein the absorber is absent.

III. BISTABILITY IN THE LASER INTENSITY

Under steady-state conditions, the laser intensity I and the operating frequency ω is obtained through simultaneous solution of Eqs. (7) and (8) after setting $dI/dt = d\phi/dt = 0$. Although mode-pulling and mode-pushing effects may be of interest under certain conditions, here we are primarily interested in the laser intensity. We therefore consider the specific case $\omega_c = \omega_{ab}$ so that the laser operates at the gain line center $\omega = \omega_{ab}$ and we may set $\Delta = 0$ in Eq. (7).[19] The steady-state solutions are then given by

$$\frac{dI}{dt} = \left(\frac{\omega}{Q}\right) I f(I) = 0 , \tag{11}$$

where $f(I) = \sum_{j=0}^{3} C_j I^j$ is a third-degree polynomial with coefficients

$$\begin{aligned} C_0 &= \delta^2[(1+\delta^2)(1-A)+\bar{A}] , \\ C_1 &= \delta^2[a(\tfrac{3}{2}-r)(1-A)+(1+\delta^2+\bar{A})] , \\ C_2 &= \tfrac{1}{4}a^2r^2(1-A)+a\delta^2(\tfrac{3}{2}-r) , \\ C_3 &= \tfrac{1}{4}a^2r^2 . \end{aligned} \tag{12}$$

A straightforward root analysis shows that for some parameter values three solutions for I are possible. However, each physically admissable root is to be checked for its stability under small perturbations.

A linear-stability analysis[20] in its most general form requires diagonalization of a matrix whose dimension is equal to the number of time-dependent variables involved in the problem. In the present case, eight absorber variables, three gain-medium variables, and two electromagnetic variables (intensity and phase) render such an analysis extremely involved. Simplification occurs in the good cavity limit, when the atomic decay times $T_{1,2}$ and $\bar{T}_{1,2}$ associated with the gain and absorbing media are much less than the cavity decay time $\tau_c \sim Q/\omega$. For further simplicity, we neglect phase fluctuations. A linear-stability analysis of Eq. (11) shows that a stationary solution I is stable if

$$\left[f(I) + I\left(\frac{df}{dI}\right)\right] < 0 . \tag{13}$$

In Figs. 2 and 3 we have shown variation of the intensity I with the pump parameter A for $a = 0.1$ and 1, respectively. In each case, different curves are obtained by varying the absorber detuning parameter $\delta = \frac{1}{2}\Omega_{21}\bar{T}_2$. The dahsed portion of each curve corresponds to the unstable branch and is physically inaccessible. The stability condition (13) is equivalent to saying that the negative-slope regions are unstable. The dotted line

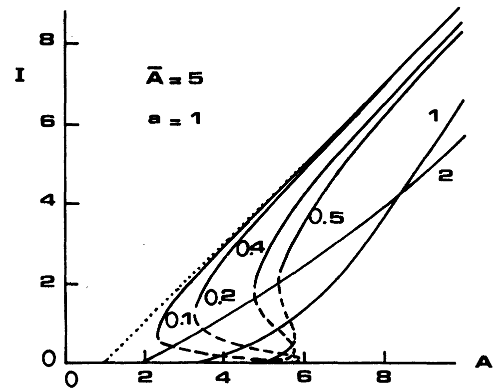

FIG. 3. Same as in Fig. 2 except that the parameter $a = 1$ which corresponds to the case when the gain and absorbing media saturate at the same rate.

in Figs. 2 and 3 corresponds to the situation wherein the absorber is absent and indicates the usual second-order transition taking place at the lasing threshold $A = 1$.

It is evident from Figs. 2 and 3 that, depending on the values of δ, the system may display either the second-order phase transition or the first-order phase transition (S-shaped curves with bistability and hysteresis) or both of them as the pump parameter is increased in a continuous manner. For instance, in Fig. 3, when $\delta = 0.1$ we have only first-order phase transition, when $\delta > 1$ we have only second-order phase transition, while for an intermediate value $\delta = 0.5$, a first-order phase transition follows a second-order one. For parameters corresponding to Figs. 2 and 3, $a = 0.1$ and $a = 1$, bistability does not occur for a two-level saturable absorber in one-photon resonance. In fact, it can be shown that in that case bistability arises only when $a > 1 + \bar{A}^{-1}$.[5] In our case there is no lower bound[21] on the value of a because the physical mechanism for bistability is not the optical saturation but the intensity dependence of the effective detuning parameter $\tilde{\delta}$ given by Eq. (6).

It is clear from the above discussion that bistable behavior depends critically on the ground-state sublevel spacing Ω_{21} through $\delta = \frac{1}{2}\Omega_{21}\bar{T}_2$. The critical intensities I_{max} and I_{min} at which the laser intensity jumps discontinuously can be determined by noting that at these points the slope $dA/dI = 0$. In Fig. 4 bistability loops were obtained by plotting I_{max} and I_{min} as a function of δ for several values of the parameter a. We note that as a increases, the range of δ for which bistability occurs also increases. In any case observation of bistability requires that the sublevel splitting Ω_{21} should be of the order of or less than a homogeneous linewidth associated with the absorber.

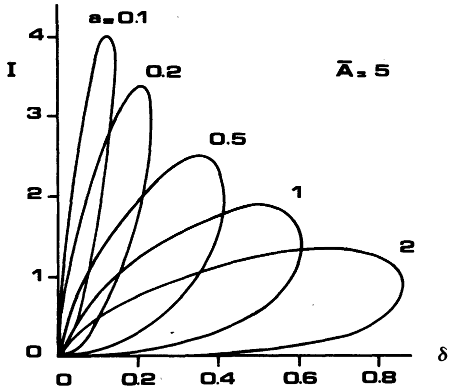

FIG. 4. Variation of bistable domains when the parameter a is varied and $\bar{A}=5$ and $r=1$. The critical intensities I_{max} and I_{min} are plotted as a function of the detuning parameter δ.

IV. DISCUSSION AND CONCLUSIONS

We have shown that a laser with an intracavity three-level absorber exhibits bistability and hysteresis, characteristics of a nonequilibrium first-order phase transition. When compared to the case of two-level saturable absorber, the main difference from a practical point of view is that the bistable features can be obtained even for the case when the absorber saturates slowly in comparison to the gain medium ($a<1$).[21] From a theoretical point of view, it should be stressed that the origin of bistability in two cases is completely different. For a three-level absorber it is not the saturable absorption but the intensity dependence of the sharp absorption dip near $\delta=0$ that plays the major role. The physical mechanism for the absorption dip is coherent population trapping which accummulates all absorbing atoms (or molecules) in a coherent superposition of the two ground-state sublevels; it arises from quadrupole-induced atomic coherence.

The present analysis can readily be extended[13] for a V-type three-level absorber where a common ground level is optically connected to two excited-state sublevels. An expression similar to Eq. (5) for χ_{abs} shows that the absorption dip near $\delta=0$ is still present. It is, however, less pronounced due to the radiative decay (spontaneous emission) which always takes place from the excited states. Here, the physical mechanism for dip formation is constructive interference between two channels of saturated induced transitions.

A possible way to observe bistable behavior in a laser with a three-level absorber is to perform a level-crossing-type experiment.[22] In this configuration, the ground state is initially doubly degenerate and the laser frequency is taken to be in exact resonance with the absorber. The degeneracy is removed by applying a static electric or magentic field and finite values of the detuning parameter $\delta=\frac{1}{2}\Omega_{21}\overline{T}_2$ are introduced. The required sublevel splitting ($\Omega_{21}\sim 1$ MHz), a fraction of the homogeneous linewidth $\overline{T}_2^{-1}$, can be obtained at moderate field strengths.

In conclusion, it may be worthwhile to point out several possible generalizations of the present analysis. If an atomic or molecular system is used an an intracavity three-level absorber, the Doppler-broadening effects should be accounted for. It may, however, be noted that for a traveling-wave monomode laser considered here, the Doppler broadening is expected to play a minor role because of the Raman-type nature of the coupled transitions.[13] For a Zeeman laser[16] described by a three-level gain medium, mode-coupling effects in the amplifying and absorbing media should be of interest. For a two-mode laser with a three-level absorber, the mode-crossing resonance, well known in spectroscopy,[23] may substantially influence laser-output characteristics. Finally, it should be mentioned that in the present semiclassical anaylsis fluctuations are ignored. Their inclusion requires derivation of a Fokker-Planck equation similar to the case of a two-level saturable absorber.[5]

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