Electron-impact-ionization cross sections and rates for highly ionized berylliumlike ions

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Electron-impact-ionization cross sections and rates have been calculated for several berylliumlike ions in the $1s^22s^2\ {}^1S$ ground state and the $1s^22s2p\ {}^3P$ metastable state using a distorted-wave Born exchange approximation. The ground-state cross section showed only minor deviations from classical scaling along the isoelectronic sequence. The $2s2p\ {}^3P$ cross section was more than twice that of the ground state at C III, decreasing to a factor of 1.35 greater at Ar XV. Dominant ground-state configuration interaction of the form $2s^2+2p^2$ changed the total cross section by only a few percent. An analytic fit to the distorted-wave results is given, and this allows the calculation of nonrelativistic electron-impact-ionization cross sections and rates for berylliumlike ions. A comparison with available experimental data shows good agreement between measurements and distorted-wave calculations.

I. INTRODUCTION

Electron-impact-ionization cross sections and rates are central components in the modeling and diagnostics of high-temperature plasmas. A detailed understanding of the characteristics of thermonuclear fusion machines, for example, requires accurate predictions of the rate at which high-Z impurity atoms are ionized.

Until recently, very little quantitative information concerning the electron-impact ionization of positive ions has been available.[1,2] Precise cross-section measurements using crossed electron and ion beams have been limited to the first few ionization stages.[3] Ionization-rate measurements in plasma machines have reached higher charge states, but at a reduced level of accuracy compared to the crossed-beam technique.[4-6] Theoretically, most of the studies on highly charged ions have employed variations of the Coulomb-Born[7] and the scaled hydrogenic methods.[8] Distorted-wave calculations have also been performed for a number of simple isoelectronic sequences.[9-12]

The present work concerns the electron-impact ionization of highly ionized berylliumlike ions, using a distorted-wave Born exchange approximation previously applied to the H,[9] He,[10] Li,[9] and Ne (Ref. 11) isoelectronic sequences. Berylliumlike ions are of theoretical interest in two respects. First, the $1s^22s^2\ {}^1S$ ground state is represented fairly accurately by the two-configuration approximation $2s^2+2p^2$, presenting a tractable yet nontrivial example of target wave function correlation affecting the ionization cross section. Second, the presence of a long-lived metastable state, $2s2p\ {}^3P$, allows the study of ionization from an excited state stable against rapid radiative decay. Such metastable states can be produced in a plasma either by excitation from the berylliumlike ground state ($1s^22s^2\ {}^1S \to 1s^22s2p\ {}^3P$) or by inner-shell ionization of boronlike ions ($1s^22s^22p\ {}^2P \to 1s^22s2p\ {}^3P+e^-$) and could constitute a significant fraction of the berylliumlike ion population.

Section II gives a brief summary of the theoretical and computational procedures. Section III presents the results, with comparisons to other theoretical and experimental data. Section IV is a discussion, with special emphasis on the effects of target correlation, metastable-state ionization, and isoelectronic scaling. Section V concludes the paper.

II. THEORETICAL AND COMPUTATIONAL TECHNIQUES

The application of the distorted-wave Born exchange approximation to the electron-impact ionization of positive ions has been described in detail in previous publications.[9-11] Briefly, a partial-wave decomposition of the initial-state and two final-state continuum electron wave functions was made using distorted waves generated in a local energy-dependent potential. The incident-electron distorted waves for a ground-state target were computed in the potential

$$V^N=-\frac{Z}{r}+2J_{1s}+2J_{2s}+V^N_{\rm SCE}\ , \qquad (1)$$

where

$$J_i=\frac{1}{r}\int_0^r [P(\rho)]^2d\rho+\int_r^\infty \frac{[P(\rho)]^2}{\rho}\,d\rho \qquad (2)$$

is the direct electrostatic potential associated with the ith radial orbital P_i, and where Z is the nuclear charge. $V^N_{\rm SCE}$ is a semiclassical energy-dependent exchange potential given by[13]

$$V^N_{\rm SCE}=\frac{E-V^N_D}{2}-\tfrac{1}{2}\,[(E-V^N_D)^2+\alpha^2]^{1/2}\ , \qquad (3)$$

where

$$\alpha^2 = \frac{2}{r^2} \sum_{i=1}^{N} [P_i(r)]^2 \quad (4)$$

and

$$V_D^N = V^N - V_{\rm SCE}^N \ . \quad (5)$$

E is the free-electron energy in Rydbergs, and N is the number of electrons in the ion. This simple expression has been found to reproduce accurately phase shifts resulting from a more complex Hartree-Fock exchange calculation.[11]

In the direct scattering matrix element the scattered (high-energy) final-state partial waves were computed in the same potential as the initial state, while the ejected (low-energy) final-state waves were computed in the potential of the residual ion:

$$V^{N-1} = -\frac{Z}{r} + 2J_{1s} + J_{2s} + V_{\rm SCE}^{N-1} \ , \quad (6)$$

where $V_{\rm SCE}^{N-1}$ is the semiclassical exchange potential omitting one $2s$ electron. In the exchange matrix element the final-state potentials were reversed, i.e., the ejected wave was computed in the N-electron potential V^N, and the scattered wave in the $(N-1)$-electron potential V^{N-1}. This choice insures orthogonality between overlapping orbitals in the scattering matrix element.

For ionization from the metastable state $2s2p\,^3P$ the corresponding potentials were

$$V^N = -\frac{Z}{r} + 2J_{1s} + J_{2s} + J_{2p} + V_{\rm SCE}^N(2s2p) \quad (7)$$

and

$$V_{2s}^{N-1} = -\frac{Z}{r} + 2J_{1s} + J_{2s} + V_{\rm SCE}^{N-1}(2s) \ , \quad (8)$$

$$V_{2p}^{N-1} = -\frac{Z}{r} + 2J_{1s} + J_{2p} + V_{\rm SCE}^{N-1}(2p) \ , \quad (9)$$

depending on whether the residual ion is left in the $2s\,^2S$ [Eq. (8)] or the $2p\,^2P$ [Eq. (9)] state.

The $2s2p\,^3P$ metastable stable was described by a single-configuration term-dependent Hartree-Fock wave function.[14] The ground-state wave function was approximated by a two-configuration expansion[14]

$$\Psi(1s^2 2s^2\,^1S) = c_1\phi_{1s^2 2s^2} + c_2\phi_{1s^2 2p^2} \ , \quad (10)$$

where c_i are the mixing coefficients and ϕ_i are component configurations. Note that Eqs. (1)–(6) do not consider the ground-state configuration interaction when computing the distorted-wave potentials, where its effect is expected to be small.

The electrostatic interaction operator $1/r_{ij}$, effecting the ionization process, allows at most two electrons to differ between the initial- and final-state total wave functions. Since one of these must be the scattered electron, it follows that the initial and final target wave functions can differ only in the replacement of the bound electron to be ionized by the ejected orbital. Thus initial ground-state configuration interaction involving double excitations cannot couple to the ground state of the next ion, but must proceed to an ionic excited state, a process having a different threshold than the ground state to ground-state transition. The total cross section for ionization of beryllium-like ions in the two-configuration approximation is then

$$Q^{\rm tot}(2s^2 + 2p^2) = |c_1|^2 Q(2s^2\,^1S \to 2s\,^2S) + |c_2|^2 Q(2p^2\,^1S \to 2p\,^2P) \ , \quad (11)$$

representing an incoherent addition of the scattering cross sections for each configuration component. Wave functions for the lithiumlike ions were constructed from the ground-state beryllium-like orbitals in order to avoid problems due to orbital overlap terms. Such an approximation is not expected to significantly affect the present results.

Distorted waves were numerically calculated on a 350-point block linear radial grid. For incident-electron energies less than or equal to 1.5 ionization threshold units the maximum initial, ejected, and scattered partial-wave angular momenta were 10, 6, and 10, respectively. For incident energies greater than 1.5 ionization threshold units the maxima were 15, 10, and 15, respectively. At the highest relative energies an extrapolation over higher incident-state partial-wave angular momenta was included in the partial-cross-section summation; such extrapolations amounted to only a few percent of the total cross section.

Radial matrix elements were evaluated by means of a double application of Simpson's rule. Integration over the final-state continuum electron energy distribution was accomplished by three- and five-point Gauss-Legendre formulas in the low and high incident- electron-energy ranges. The scattering matrix element was constructed in the "maximum-interference" (MIA) approximation of Peterkop,[15] designed to maximize the effect of exchange and hence minimize the total cross section in the Born exchange approximation. Rudge[1] has compared the MIA to other scattering approximations for the electron ionization of neutral hydrogen and helium, finding it to produce cross sections in better agreement with experiment than other simple approximations for exchange. A definitive study of scattering exchange in electron ionization has not yet been performed, however, so that the case for employing the MIA over other approximations is based mainly on the agreement

TABLE I. Ionization energies of berylliumlike ions (eV).

	C III	N IV	O V	F VI	Ar XV
$1s^22s^2\,{}^1S \rightarrow 1s^22s\,{}^2S$	47.887	77.472	113.90	157.16	854.75
$1s^22s^2\,{}^1S \rightarrow 1s^22p\,{}^2P$	55.887	87.464	125.89	171.15	888.71
$1s^22s2p\,{}^3P \rightarrow 1s^22s\,{}^2S$	41.385	69.129	103.68	145.11	825.52
$1s^22s2p\,{}^3P \rightarrow 1s^22p\,{}^2P$	49.387	79.126	115.68	159.11	859.50

of its predictions with rather sparce available experimental data.

The $Z=\infty$ approximation is essentially a scaled hydrogenic model in which all continuum functions as well as the bound target orbital are $Z=1$ Coulomb functions. The present $Z=\infty$ cross sections differ slightly from the data of Ref. 8 due to a different phase approximation in the exchange matrix element.

Experimental ionization energies were employed.[16] For inner-shell ionization of the $2s2p\,{}^3P$ state the ionization energy was calculated from the formula

$$I(2s2p\,{}^3P \rightarrow 2p^2\,P) = I(2s2p\,{}^3P \rightarrow 2s\,{}^2S) + \Delta E(2s\,{}^2S \rightarrow 2p\,{}^2P)\,, \qquad (12)$$

where the right most term is the excitation energy of the associated lithiumlike ion.[17,18] Ionization energies used in the present work are given in Table I.

III. RESULTS

Electron impact ionization cross sections for the berylliumlike ions C III, N IV, O V, F VI, and Ar XV, computed in the distorted-wave Born exchange approximation described above are given in Table II. In order to suppress the gross Z dependence of the cross section we tabulate the scaled quantity uI^2Q, where u is the incident energy in threshold units, I is the *ground-state* ionization energy in electron volts, and Q is the cross section in cm^2. Separate cross sections are listed for ionization of the $2s2p\,{}^3P$ state into the $2s\,{}^2S$ and $2p\,{}^2P$ lithiumlike states. The ground-state cross sections include the effect of $2s^2 + 2p^2$ configuration mixing. For the purpose of comparison single-configuration ground-state cross sections are given in parentheses.

Figure 1 compares the present distorted-wave Born exchange scaled cross sections C III with the Coulomb-Born no-exchange results of Moores for the ground state only, the crossed-beam measurements of Woodruff *et al.*,[20] and the scaled hydrogenic ($Z=\infty$) approximation. Since the experimental data consist of a mixture of ground and metastable states, we are unable to make an exact comparison between theory and measurement. In order to provide theoretical brackets within the distorted-wave approximation, we show separate theoretical curves for the ground and metastable states. There is a nonzero cross section at $u<1$ due to metastable-state ionization $1s^22s2p\,{}^3P \rightarrow 1s^22s\,{}^2S$ which has a lower threshold than the ground state.

Figure 2 is a comparison between the present results for N IV, the crossed-beam measurements of Crandall *et al.*,[21] the Coulomb-Born no-exchange data of Moores,[19] and the scaled hydrogenic approximation. Figure 3 illustrates the same comparison for O V.

Figures 4 and 5 are isoelectronic plots of the scaled ionization cross sections for berylliumlike ions in the ground and metastable states, respectively. These plots are useful in that they clearly demonstrate the isoelectronic scaling of

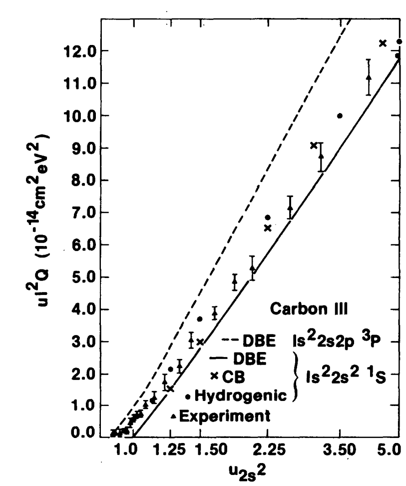

FIG. 1. Scaled electron-impact-ionization cross section uI^2Q for C III in units of 10^{-14} cm^{2}eV^2. - - -: $2s2p\,{}^3P$, distorted-wave Born exchange; —: $2s^2\,{}^1S$ distorted-wave Born exchange; ×: $2s^2\,{}^1S$, Coulomb-Born, no exchange, Ref. 19; ●: $2s^2\,{}^1S$, scaled hydrogenic; ▲: crossed-beam experiment, Ref. 20. Here $u=E/I$, where E is the incident electron energy and I is the ground-state ionization energy.

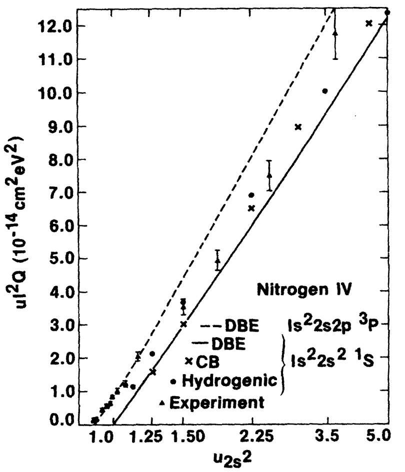

FIG. 2. Scaled electron-impact-ionization cross section uI^2Q for N IV in units of 10^{-14} cm^{2}eV^2. ---: $2s2p\,^3P$, distorted-wave Born exchange; —: $2s^2\,^1S$, distorted-wave Born exchange; ×: $2s^2\,^1S$, Coulomb-Born, no exchange, Ref. 19; •: $2s^2\,^1S$, scaled hydrogenic; ▲: crossed-beam experiment, Ref. 21. Here $u=E/I$, where E is the incident electron energy and I is the ground-state ionization energy.

the cross section and provide a means for rapid interpolation of nonrelativistic electron impact ionization cross sections for any ion in the beryllium isoelectronic sequence. Parametrizations of these curves in terms of simple analytic expressions for the cross sections and rates are given below.

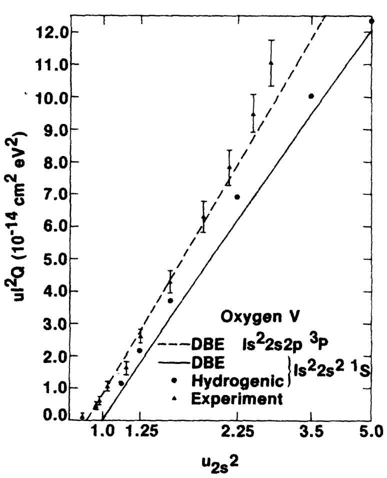

FIG. 3. Scaled electron-impact-ionization cross section uI^2Q for O v in units of 10^{-14} cm^{2}eV^2. ---: $2s2p\,^3P$, distorted-wave Born exchange; —: $2s^2\,^1S$, distorted-wave Born exchange; •: $2s^2\,^1S$, scaled hydrogenic; ▲: crossed-beam experiment, Ref. 21. Here $u=E/I$, where E is the incident electron energy and I is the ground-state ionization energy.

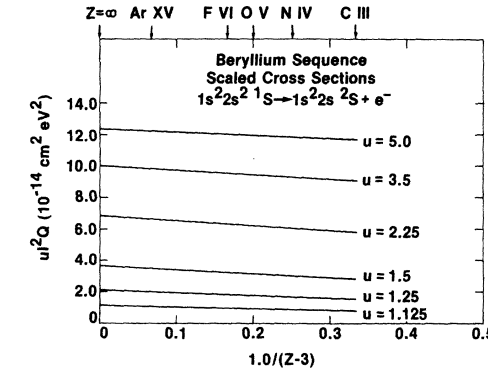

FIG. 4. Isoelectronic plot of the scaled electron-impact-ionization cross section for the ground state of berylliumlike ions computed in the distorted-wave Born exchange approximation. Each curve corresponds to a fixed incident electron energy measured in ground-state ionization-threshold units $u=E/I$.

IV. DISCUSSION

A. Ground-state configuration interaction

As is evident from Table II, ground-state configuration interaction of the form $2s^2+2p^2$ has a relatively minor effect on the total ionization cross section of berylliumlike ions. This result is in contrast to other ground-state properties such as the correlation energy, optical transition probabilities, excitation cross sections, etc., which are significantly affected by such angular correlations. In the ionization cross section, however, such a mixing of correlation orbitals results in an incoherent addition of the cross sec-

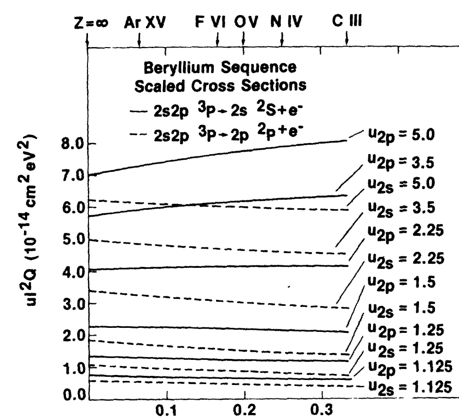

FIG. 5. Isoelectronic plot of the scaled electron-impact-ionization cross section for the $2s2p\,^3P$ metastable state of berylliumlike ions computed in the distorted-wave Born exchange approximation. Each curve corresponds to a fixed incident electron energy measured in metastable-state ionization-threshold units.

TABLE II. Scaled electron-impact-ionization cross sections uI^2Q for berylliumlike ions[a] (10^{-14} cm^{2}eV^2).

$u=E/I$	C III	N IV	O V	F VI	Ar XV	$Z=\infty$
			$1s^22s^2\,{}^1S \rightarrow 1s^22s\,{}^2S+e^-$			
1.125	0.770	0.849	0.841	0.926	1.04	1.17
	(0.770)[b]	(0.849)	(0.895)	(0.929)	(1.05)	(1.15)
1.25	1.41	1.57	1.67	1.73	1.96	2.14
	(1.46)	(1.59)	(1.68)	(1.73)	(1.94)	(2.11)
1.50	2.70	2.90	3.04	3.14	3.48	3.71
	(2.68)	(2.86)	(2.99)	(3.09)	(3.42)	(3.67)
2.25	5.71	5.94	6.09	6.22	6.61	3.90
	(5.54)	(5.77)	(5.94)	(6.07)	(6.51)	(6.83)
3.50	9.02	9.27	9.41	9.50	9.83	10.0
	(8.68)	(8.98)	(9.15)	(9.27)	(9.69)	(9.95)
5.00	11.6	11.8	11.9	12.0	12.2	12.4
	(11.2)	(11.4)	(11.6)	(11.7)	(12.1)	(12.3)
			$1s^22s2p\,{}^3P \rightarrow 1s^22p\,{}^2P+e^-$			
$u=E/I$	C III	N IV	O V	F VI	Ar XV	$Z=\infty$
1.125	0.309	0.346	0.390	0.416	0.502	0.575
1.25	0.625	0.718	0.779	0.821	0.951	1.06
1.50	1.23	1.36	1.44	1.50	1.69	1.83
2.25	2.67	2.82	2.91	3.00	3.24	3.41
3.50	4.28	4.44	4.55	4.63	4.84	4.98
5.00	5.54	5.70	5.80	5.87	6.04	6.15
			$1s^22s2p\,{}^3P \rightarrow 1s^22s\,{}^2S+e^-$			
$u=E/I$	C III	N IV	O V	F VI	Ar XV	$Z=\infty$
1.125	1.60	1.42	1.30	1.22	0.953	0.738
1.25	2.22	2.03	1.91	1.82	1.55	1.34
1.50	3.30	3.06	2.92	2.82	2.52	2.28
2.25	5.68	5.24	5.04	4.89	4.42	4.07
3.50	8.20	7.60	7.17	6.99	6.28	5.69
5.00	10.2	9.28	8.81	8.50	7.23	6.82

[a] For consistency, ground-state values of u and I were used throughout this table.
[b] Cross sections for the single-configuration ground state are given in parentheses.

tions associated with each configuration component, weighted by the *squares* of the appropriate mixing coefficients. Since typical wave-function-mixing coefficients for berylliumlike ions are of the order $c_{2s}\approx 0.97$ and $c_{2p}\approx 0.25$, the cross sections are mixed by factors of $|c_{2s}|^2\approx 0.94$ and $|c_{2p}|^2=0.06$. Even if the $2p$ cross section is substantially different from the $2s$, the small mixing coefficient results in only a minor modification of the total cross section. Such an incoherent addition of component cross sections will occur in other atoms whose correlation structure is dominated by double-excitation configuration interaction. In systems where single-substitution configuration interaction is important, this relation may break down and interference between excited- and ground-configuration matrix elements may be important.

B. Ionization from the $2s2p\,{}^3P$ metastable state

Metastable states could be of some importance in the understanding of high-temperature plasmas. In berylliumlike ions the presence of a substantial metastable population gives access to the $2snl\,{}^3L$ levels which otherwise would occur only through relatively weak intermediate coupling or by electron excitation from the ground state. The presence of strong triplet-triplet transitions in plasmas might be an indication of substantial metastable populations.

While metastable berylliumlike ions can be produced by electron-impact excitation from the ground state, a more important mechanism would appear to be direct inner-subshell ionization of boronlike ions: $2s^22p\,{}^2P \rightarrow 2s2p\,{}^3P+e^-$. Such a process could result in substantial quantities of metastable berylliumlike ions, since the inner-subshell cross section for $2s$ ejection is approximately the same as that for $2p$ ionization, which results in ground-state berylliumlike ions.[22] Based on the statistical weights of the $2s2p$ multiplets one would expect 37.5% of the boronlike ions to go to the metastable berylliumlike multiplet, 13.5% to go to the $2s2p\,^1P$ multiplet (which rapidly decays to the ground state), and 50% to go to the $2s^2\,^1S$ berylliumlike ground state directly.

Table II indicates that for low incident energies the electron-impact-ionization cross sections for the $2s2p\,^3P$ metastable state of few times ionized berylliumlike ions are much larger than the ground-state values. Figure 6 shows the ratio $Q(2s2p\,^3P)/Q(2s^2\,^1S)$ as a function of Z for various incident energies. Most of the difference between the ground- and metastable-state cross sections is due to the process $2s2p\,^3P \rightarrow 2s\,^2S + e^-$, which for $u = 1.125$ varies by almost a factor of 2 from C III to the $Z = \infty$ limit.

C. Isoelectronc scaling of the cross section

Figures 4 and 5 demonstrate the smooth variation of the scaled electron-impact-ionization cross section uI^2Q with respect to the nuclear charge. In order to aid in the interpolation of our cross-section data as well as to allow calculation of the associated ionization rate coefficients we have fit the uI^2Q of Table II by the function[23]

$$uI^2Q = A\left(1-\frac{1}{u}\right) + B\left(1-\frac{1}{u}\right)^2 + C\ln u + \frac{D}{u}\ln u\,, \quad (13)$$

where A, B, C, and D are Z-dependent parameters. Parameters A, B, and D were determined by a least-squares fit to the distorted-wave Born exchange data of Table II. Parameter C is a Bethe coefficient, related to the integral of the photoionization cross section σ, as

$$C = \frac{I}{\pi\alpha}\int_I^\infty \frac{\sigma}{\epsilon}\,d\epsilon\,, \quad (14)$$

where α is the fine-structure constant.

For the ground state of C III, C was derived from the pseudopotential calculations of McGinn.[24] For the remaining ground states the more approximate distorted-wave data of Manson[25] were employed. For the $2s2p\,^3P$ metastable-state we generated photoionization cross sections using the computer code of Manson.[25]

In order to allow isoelectronic interpolation of the berylliumlike electron-impact-ionization cross section the parameters A, B, C, and D were fitted by the inverse power series

$$A = \sum_{i=0}^{3} \frac{a_i}{(Z-N+1)^i}\,, \qquad B = \sum_{i=0}^{3} \frac{b_i}{(Z-N+1)^i}\,, \quad (15)$$

etc., where Z is the nuclear charge and N is the number of electrons in the initial ion (four for a berylliumlike ion). Separate fits were performed for each subshell, with the resulting coefficients a_i, b_i, c_i, and d_i given in Table III. Note that when using Eqs. (13)–(15) one must insert the ioni-

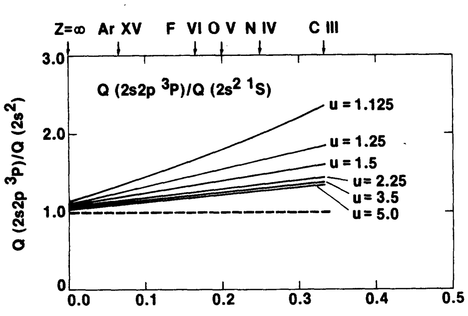

FIG. 6. Isoelectronic plot of the ratio of the total scaled cross section for electron-impact ionization from the $2s2p\,^3P$ metastable state to the cross section for ground-state ionization.

TABLE III. Isoelectronic fit parameters for ground- and metastable-state ionization cross sections and rates for highly ionized berylliumlike ions (10^{-14} cm^2 eV^2).

	$i=0$	1	2	3
		$1s^22s^2 \rightarrow 1s^22s+e^-$		
a_i	20.8	−36.4	−18.3	448
b_i	−6.78	35.9	−43.2	−210
c_i	2.68	−1.29	24.1	−65.3
d_i	−13.1	17.9	9.45	−359
		$1s^22s2p \rightarrow 1s^22s+e^-$		
a_i	13.2	18.7	−116	231
b_i	−4.14	−10.6	70.3	−139
c_i	0.862	3.61	−3.06	1.60
d_i	−7.43	−24.9	113	−223
		$1s^22s2p \rightarrow 1s^22p+e^-$		
a_i	9.07	24.1	−80.0	75.9
b_i	−2.47	−15.8	44.4	−21.6
c_i	1.34	0.661	0.366	−5.96
d_i	−5.37	−31.1	83.3	−70.8

zation energy and reduced incident energy u, corresponding to the particular subshell under consideration, which is not necessarily the ground-state value.

Cross sections for ejection of a $1s$ electron from a berylliumlike ion were not explicitly calculated in the present work, but are expected to be approximately the same as those for ionization of the corresponding heliumlike ion, given in Ref. 23. Note, however, that ejection of a $1s$ electron in a berylliumlike ion leaves the ion in an autoionizing lithiumlike state, resulting in effective double ionization of the initial target by a single electron collision. Such double ionization will occur for all isoelectronic sequences heavier than beryllium as well. For very highly charged ions the branching ratio for autoionization compared to radiative decay will decrease, producing a smaller double-ionization contribution.

It is interesting that for the same Z the berylliumlike ground-state-ionization cross sections are approximately twice the lithiumlike values.[9]

The rate coefficient for electron-impact ionization assuming a Maxwellian electron velocity distribution can be expressed as a correction to the often-used Seaton formula[23]

$$S = F(\chi) S^{\text{Seaton}} , \tag{16}$$

where S is the ionization rate in cm^3/s and $\chi = kT/I$ is the electron thermal energy in ionization threshold units. $F(\chi)$ is a distorted-wave factor, expressed in terms of the same four parameters as the cross section:

$$F(\chi) = \frac{3.0\times 10^{13}}{\chi}\left(A + B\left(1+\frac{1}{\chi}\right) + \left\{C - \frac{1}{\chi}\left[A + B\left(2+\frac{1}{\chi}\right)\right]\right\}\alpha(\chi) + \frac{D}{\chi}\beta(\chi)\right), \tag{17}$$

where

$$\alpha(\chi) \equiv \frac{0.001\,93 + 0.9764\chi + 0.6604\chi^2 + 0.025\,90\chi^3}{1.0 + 1.488\chi + 0.2972\chi^2 + 0.004\,925\chi^3}, \tag{18}$$

$$\beta(\chi) \equiv \frac{-0.000\,5725 + 0.01345\chi + 0.8691\chi^2 + 0.034\,04\chi^3}{1.0 + 2.197\chi + 0.2475\chi^2 + 0.002\,053\chi^3}. \tag{19}$$

The Seaton formula is[26]

$$S^{\text{Seaton}} = 2.2\times 10^{-6}\sqrt{\chi}\, I_{\text{eV}}^{-3/2} e^{-1/\chi}, \tag{20}$$

where I_{eV} is the ionization energy in eV.

D. Other effects

The present calculation has not included ionization via autoionization of core-excited states such as $1s2s^2nl$. Owing to the small electron-impact-excitation cross sections associated with core-excited states this mechanism is expected to increase the direct ionization cross sections by only 10–15%. The experimental data for C III (Ref. 20) and N IV (Ref. 21) are of insufficient accuracy in the appropriate energy ranges to indicate excitation autoionization. In O V, however, a distinct 10–15% rise in the measured cross section occurs near 600-eV incident electron energy, the expected location of the core-excited autoionizing states.

Also omitted from our work is consideration of electron correlation involving the final-state continuum electrons. Such correlation takes two forms: interaction between the slow ejected electron and the residual ion, similar to that occurring in photoionization, and correlation between the two continuum electrons themselves. Altick[27] has explored the former problem in Be I photoionization, finding strong coupling between the continuum electron and nearby autoionizing states. For berylliumlike ions, however, most of these autoionizing configurations have become bound, no longer affecting the ejected wave. The problem of continuum-continuum interaction has not been systematically addressed as yet, involving consideration of the long-range three-body problem. Previous calculations on simple ionic targets using a noncorrelated distorted-wave approximation have yielded good agreement with available experimental data, indicating that continuum-continuum interaction may not be serious at least in some cases.

V. CONCLUSION

Electron-impact-ionization cross sections for several berylliumlike ions in the ground and first metastable states have been computed in a distorted-wave Born exchange approximation. Dominant ground-state configuration interaction of the type $2s^2 + 2p^2$ had a minor effect on the total gound-state-ionization cross section due to the incoherent addition of the scattering matrix elements from the component configurations. Cross sections for electron-impact ionization of the $2s2p\,^3P$ metastable state were significantly larger than the associated ground-state values. Considering the uncertainties in metastable-state populations in crossed-beam experiments, the theoretical data agrees well with available measurements. Analytic fits to the ionization cross sections and rates were provided.

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