PHYSICAL REVIEW A VOLUME 24, NUMBER 2 AUGUST 1981

Radiation from a relativistic electron beam in a molecular medium due to parametric pumping by a strong electromagnetic wave

L. Stenflo
Department of Plasma Physics, Umeå University, S-90187 Umeå, Sweden

H. Wilhelmsson
Institute for Electromagnetic Field Theory and Euratom-Fusion Research (EUR-NE), Chalmers University of Technology, S-41296 Göteborg, Sweden
and Laboratory for Plasma and Fusion Energy Studies, University of Maryland, College Park, Maryland 20742
(Received 2 March 1981)

Parametric interaction of longitudinal and transverse waves is studied for a system consisting of a relativistic electron beam interpenetrating a molecular medium. The analysis suggests new possibilities for free-electron generation of coherent radiation.

Plasmas containing molecules or ions, for which the internal level structure offers possibilities of radiative transitions between discrete levels, have attracted some recent attention.[1] Energy couplings between plasma fluctuations and molecular excitations are of particular interest in this connection, as inverted populations of the molecular levels may contribute free energy to the system.

An interesting way of obtaining resonance between the plasma waves and the molecular transitions is to vary the velocity of a relativistic electron beam and to make use of the coupling of longitudinal and transverse waves by a wiggler[2] or a strong wave field. Such systems may offer advantages of enhanced radiation and possibly improvements in the coherence properties also.

It is the purpose of the present short paper to formulate and interpret the dispersion relation that governs the wave interactions for a system consisting of a relativistic electron beam (density N_B, velocity v_{z0}) which penetrates a molecular medium (level population differences $N_{mj}=N_{lj}-N_{hj}$, with N_{lj} and N_{hj} denoting the populations of the lower and higher levels).

A constant external magnetic field $B_{z0}\hat{z}$ is directed along the electron beam, and a helical magnetic field (constant amplitude $B_{\perp 0}$, frequency ω_0, wave number $k_0\hat{z}$) is also assumed to be present in the medium.

Considering density perturbations of the form $\exp(-i\omega t+ikz)$ and electromagnetic field perturbations $\exp[-i(\omega\mp\omega_0)t+i(k\mp k_0)z]$, it is convenient to introduce the wave dispersion constants,

$$\epsilon_z=\epsilon_z(\omega,k)=1-\frac{\omega_B^2}{(\omega-kv_{z0})^2\gamma_0\gamma_{z0}^2}-\sum_j\frac{\omega_{mj}^2}{\omega^2-\omega_{rj}^2+i\omega\nu_j}$$

and

$$\epsilon_\pm=\epsilon_\pm(\omega\mp\omega_0,k\mp k_0)=(k\mp k_0)^2c^2-(\omega\mp\omega_0)^2+\frac{\omega_B^2}{\gamma_0}\frac{\omega\mp\omega_0-(k\mp k_0)v_{z0}}{\omega\mp\omega_0-(k\mp k_0)v_{z0}\mp\omega_{ce}/\gamma_0}+\sum_j\frac{\omega_{mj}^2(\omega\mp\omega_0)^2}{(\omega\mp\omega_0)^2-\omega_{rj}^2\mp(\omega\mp\omega_0)\omega_{cmj}+i(\omega\mp\omega_0)\nu_j},$$

where the $\pm$ signs refer to right- and left-hand polarization of the transverse waves, $\omega_B=(N_Be^2/\epsilon_0 m)^{1/2}$ is the beam plasma frequency, e is the electron charge, m is the electron rest mass, $\omega_{mj}=(g_{mj}N_{mj}e^2/\epsilon_0 m_j)^{1/2}$, m_j is the effective mass of the jth bound electron, $\omega_{ce}=eB_{z0}/m$, $\omega_{cmj}=eB_{z0}/m_j$, ω_{rj} is a molecular resonance frequency (Ref. 2), g_{mj} is the oscillator strength, and ν_j is an effective damping constant. The relativistic factors are $\gamma_{z0}=(1-v_{z0}^2/c^2)^{-1/2}$ and $\gamma_0=(1-v_{\perp 0}^2/c^2-v_{z0}^2/c^2)^{-1/2}$, where $v_{\perp 0}$ is the oscillatory transverse velocity in the pump field.[3]

The presence of the electromagnetic pump wave introduces a coupling between the longitudinal ($\epsilon_z=0$) and transverse (ϵ_+ or $\epsilon_-=0$) modes. Straightforward calculations, using a hydromagnetic description for the relativistic electron beam and a Lorentz model for the molecular medium, lead to an algebraically very complicated dispersion relation, cf. Ref. 3, which could, however, be given a comparatively simple form for $B_{z0}=0$, $\nu_j=0$ and $\omega_0\ll\omega_r$, as some nonlinear terms in the Lorentz model are negligible in this frequency range. We then obtain the following dispersion relation (cf. Refs. 2–4)

$$\epsilon_s\epsilon_+\epsilon_- = -\tfrac{1}{2}B_{\perp 0}^2(A_+\epsilon_- + A_-\epsilon_+)F,$$

where the coupling coefficients are

$$A_\pm = \left(\frac{\omega_B^2}{(\omega - kv_{z0})^2\gamma_0^2}\frac{e}{m}\frac{(k-\omega v_{z0}/c^2)}{k_0} + \sum_j \frac{\omega_{mj}^2}{(\omega^2-\omega_{rj}^2)[1-\omega_{rj}^2/(\omega\mp\omega_0)^2]}\frac{e}{m_j}\frac{k}{k_0}\right)^2 .$$

Higher-order terms in $B_{\perp 0}^2$ are contained in the factor F, where

$$(F)^{-1} = 1 + \sum_{+,-}\frac{B_{\perp 0}^2}{2\epsilon_\pm}\left(\frac{\omega_B^2}{(\omega-kv_{z0})^2\gamma_0^3}\frac{e^2}{m^2} + \sum_j \frac{\omega_{mj}^2}{(\omega^2-\omega_{rj}^2)[1-\omega_{rj}^2/(\omega\mp\omega_0)^2]}\frac{e^2}{m_j^2}\right)\frac{(k^2-\omega^2/c^2)}{k_0^2} .$$

The dispersion relation above describes parametric wave coupling between the various modes. It is caused by an electromagnetic pump wave, which we allow to be strong, as expressed by keeping terms of higher order than $B_{\perp 0}^2$. For the weak coupling case, we may take either ϵ_+ or $\epsilon_- \cong 0$ in which cases the dispersion relation reduces considerably. The factor F may then (as $B_{\perp 0}$ is small) be approximated by unity.

The limit of our dispersion relation in the absence of molecules has been discussed by previous authors, e.g., Ref. 3, for free-electron lasers ($\omega_0 = 0$), as well as for finite frequency pump waves. The presence of molecules has been considered recently[2,4] for the case where $\omega_0 = 0$, i.e., for a static pump field, and assuming weak coupling. The present work thus extends the possibilities of application to strong wave interactions in relativistic plasmas, considering, within a Lorentz-model description, plasma field couplings to the internal structure of the molecules. The molecular medium may have an inverted population of the energy levels and acts thus as a maser or laser, which in our study is dynamically coupled to the electron beam. The pump field may be a static or time-varying laboratory wiggler field[5,6] or simply a strong wave, such as those which may exist in astrophysical situations, e.g., in pulsar atmospheres, or in laboratory experiments concerning stimulated microwave scattering by electron beams.[7,8] The presence of such a pump field may cause coupling of wave energy between the longitudinal collective waves on the relativistic electron beam and the transverse radiation modes.[7-9] The tunability of the frequency of a transverse beam mode by the velocity of the electron beam may enable us to make its frequency coincide with molecular resonances. The beam, operating as a free-electron laser, may alternatively be used to pump the molecular medium at particular frequencies, or it may interact with the medium to cause an enhanced stimulated radiation by contributing to the radiation process its own free energy.[10]

We believe that our analysis, although not at this stage specified to particular choices of parameters, offers new possibilities and is a necessary prerequisite for further developments in certain areas of astrophysics as well as in laboratory experiments concerning generation of coherent radiation.

[1] H. Wilhelmsson and I. P. Yakimenko, Phys. Scr. 17, 523 (1978).
[2] H. Wilhelmsson, Phys. Scr. 22, 503 (1980).
[3] L. Stenflo, Phys. Scr. 21, 831 (1980).
[4] R. A. Smith and H. Wilhelmsson, Phys. Scr. 23, 797 (1981).
[5] D. A. G. Deacon, L. R. Elias, J. M. J. Madey, G. J. Ramian, H. A. Schwettman, and T. I. Smith, Phys. Rev. Lett. 38, 892 (1977).
[6] D. B. McDermott, T. C. Marshall, S. P. Schlesinger, R. K. Parker, and V. L. Granatstein, Phys. Rev. Lett. 41, 1368 (1978).
[7] P. Sprangle, V. L. Granatstein, and L. Baker, Phys. Rev. A 12, 1697 (1975).
[8] V. L. Granatstein, S. P. Schlesinger, M. Herndon, R. K. Parker, and J. A. Pasour, Appl. Phys. Lett. 30, 384 (1977).
[9] F. A. Hopf, P. Meystre, M. O. Scully, and W. H. Louisell, Phys. Rev. Lett. 37, 1342 (1976).
[10] See also several contributions in, *Free-Electron Generators of Coherent Radiation*, edited by S. F. Jacobs, H. S. Pilloff, M. Sargent III, M. O. Scully, and R. Spitzer, *Physics of Quantum Electronics* (Addison-Wesley, Reading, Mass., 1980), Vol. 7.