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Measurement of higher-order corrections to stopping power for relativistic Ne, Ar, and Fe beams

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Measurements of range of 600 MeV/amu ^{20}Ne, ^{40}Ar, and ^{56}Fe in a variety of absorbers have been made to test the accuracy of the higher-order Mott, Bloch, and polarization correction terms to the Bethe expression of stopping power. These corrections give an acceptable ($<2\sigma$) fit to energies determined by time-of-flight methods. A dramatically improved ($<1\sigma$) fit follows if the relativistic Bloch term has half its currently calculated magnitude.

This article presents relativistic heavy ion range-energy data which for the first time explicitly demonstrate the increasing importance of certain higher-order corrections in the expression for stopping power, dE/dx, as the ion charge Z increases. We have measured ranges of ~600 MeV/amu ^{20}Ne, ^{40}Ar, and ^{56}Fe in Al, Cu, and Pb absorbers, and, by means of a range-energy program[1] developed for the purpose, have compared them to independent evaluations of the accelerator beam (extraction) energies. Much of the detail of this analysis is similar to that presented in an earlier article,[2] to which we refer the reader, where ranges of a single ion (^{56}Fe) were compared to less accurately determined beam energies resulting in weaker conclusions that we state here.

The increasing use of relativistic heavy ions in a wide variety of applications has led to an increased need for higher accuracy in range-energy relations for the regime of high Z and high energy E. The standard Bethe expression[3,4] for dE/dx, a quantal first Born approximation calculation, fails in this regime. Ahlen[5,6] has emphasized the need for three major corrections to the Bethe expression in this regime: the Mott term, originating from higher order terms in $Z\alpha$ ($\alpha=\frac{1}{137}$) in the expansion of the exact Mott collision cross-section[7]; the Bloch term, calculated by Bloch[8] to bridge the calculations in the Bethe regime ($Z\alpha/\beta \ll 1$) and in the Bohr regime[9] ($Z\alpha/\beta \gg 1$); and a low-velocity polarization term,[10,11] accounting for inadequacies in the impulse approximation used in the above treatments. The magnitudes of these correction terms as a function of energy are shown for Ne, Ar, and Fe in Pb in Fig. 1, calculated using a program[1] based on Ahlen's[5] treatment.

The measurements were made at Lawrence Berkeley Laboratory's Bevalac, where after extraction from the accelerator and passage through a series of accurately known upstream materials (beam line windows, etc.), the ions were slowed to nearly stopping in each of the three primary (Al, Cu, Pb) absorbers. The residual range was traversed in a stack consisting of a few hundred thin (~0.03 g/cm^2) sheets of Lexan polycarbonate plastic; subsequent chemical etching of these sheets[12] exposed tracks of stopping ions yielding a distribution of ranges [see Fig. 2(a)] whose peak gives the mean range and whose width is due to accelerator phase oscillations and range straggling.[9] The errors in the Lexan and absorber thicknesses were comparable to upstream matter uncertainties. By integrating dE/dx through the complete list of material (Table I) required to stop each charge, the initial (extraction) beam

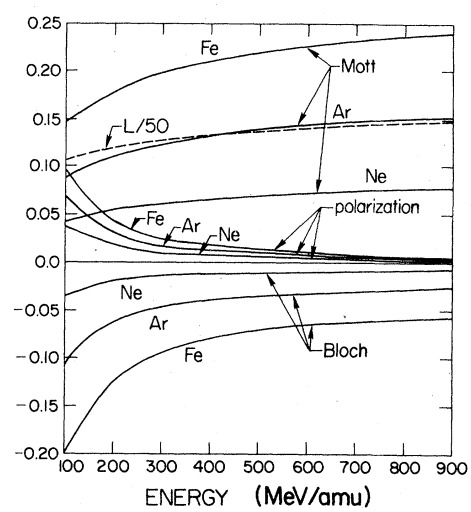

FIG. 1. Magnitudes of the Bloch, Mott, and polarization corrections for Ne, Ar, and Fe in a Pb absorber are shown along with the reduced stopping power L, which is independent of ion charge and is defined by $dE/dx=k[L+\text{Bloch term}+\text{Mott term}+\text{polarization term}]$, with $k\propto Z^2/\beta^2$.

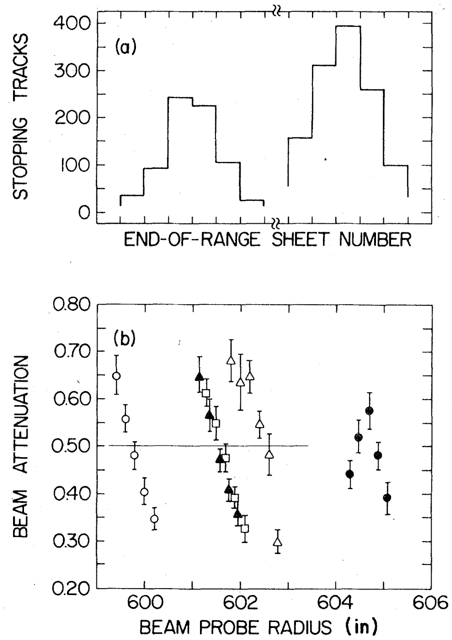

FIG. 2. (a) Two representative histograms show the number of stopping ion tracks, measured by microscopic examination of chemically etched Lexan polycarbonate, versus the individual sheet number in the plastic stack. The distribution provides us with a mean range measurement. The peak, whose full-width-half-maximum of ~0.06 g/cm^2 corresponds to less than 0.3% of total range, lies in the middle of the plastic stack. (b) Beam radius measurements: In each of four accelerator quadrants, North (○), West (▲), East (□), and South (△), relative beam position measurements were made by recording the ratio of extraction intensities before and after insertion of a probe finger which swept out a fraction of the beam. Choosing an arbitrary beam attenuation factor, 0.5, gave a probe radius for each quadrant that corresponds to a particular common point in the radial distribution of the beam. The absolute radial positions were then fixed by using a double-finger harp in the East quad which determined the location of the beam distribution maximum (●).

energy was computed. These computed energies varied, depending upon which correction terms described above were or were not included in the expression for dE/dx.

To be compared to these variously calculated (integrated) energies are the actual beam energies; since comparison of stopping power corrections hinges on the accuracy of the beam-energy measurements, an effort was made to achieve unparalleled accuracy by using the Bevalac itself as a time-of-flight device, with a flight path per revolution of about 120 m. The details of this calculation are quite involved and are discussed elsewhere.[13] The closed orbit flight path of each ion species was determined by radial measurements

TABLE I. List of material from beam extraction to end-of-range for Ar with Pb as the primary absorber. The values of I_{adj} used for integration of dE/dx through this list are also shown. Where given, uncertainties listed contributed to error calculations.

Material	Thickness (g/cm^2)	I_{adj} (eV)
Al	0.4033 ± 0.018	166
air	0.6924 ± 0.029	85
Kapton	0.3060 ± 0.011	71.8
CO_2	0.0116 ± 0.001	84.9
Ar	0.0420 ± 0.002	187
polyvinyltoluene	0.1468 ± 0.003	59.7
Al	0.3026 ± 0.006	166
Pb	28.525 ± 0.010	811.4 ± 50
air	0.2235 ± 0.009	85
Lexan	2.3323 ± 0.033	68 ± 5

made during separate Bevalac runs. Figure 2(b) shows how relative and absolute radial measurements were made in each accelerator quadrant. Harmonic analysis yielded the flight path (4th harmonic components were determined to be insignificant); coupled with accurate (1 part in 10^6) machine frequency readings, beam energies were determined for each of the ion species ^{20}Ne, ^{40}Ar, and ^{56}Fe, being, respectively, 601.5 ± 1.5, 601.0 ± 1.5, and 600.7 ± 1.5 MeV/amu. The equivalent fractional range errors are roughly 1.6 times the fractional energy errors.

Figure 3 succinctly summarizes the results. Shown are three sets of data, one for each absorber (Al, Cu, Pb). For a given absorber and ion species three data points are shown corresponding

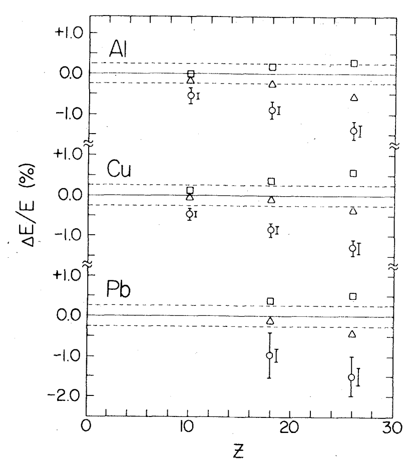

FIG. 3. Deviations of calculated (integrated) energies from energies determined by time-of-flight methods versus ion charge, for three absorbers (Al, Cu, and Pb). See text for explanation.

to the integrated energies calculated using a dE/dx expression with no correction term added (circles), with all but the Bloch term added (squares), and with all the expected correction terms added (triangles). The ordinate gives the percentage deviation of these integrated energies from the actual measured beam energies which determine the ordinate zero; the measured beam-energy errors are represented by horizontal dashed lines. Errors in calculated (integrated) energies [shown only on circle (○) points for clarity but applying to all three data points] are in pairs: the smaller errors correspond to thickness uncertainties (both systematic and random) in the matter lists (Table I), while the larger include quoted uncertainties (systematic) in the stopping power parameter $I\widehat{\text{adj}}$ (Ref. 4) (adjusted mean logarithmic ionization potential of the absorber) as well. Thus, one can see the effect of imprecise knowledge of $I\widehat{\text{adj}}$ on our calculated energies. The $I\widehat{\text{adj}}$ values used for Al, Cu, and Pb are, respectively, 166.0 ± 4, 326.6 ± 5, and 811.4 ± 50 eV, determined from recommended I (mean logarithmic ionization potential) values[14] and adjusted for the effects of finite shell corrections at $\beta = 1$.[15,6] The effect of the larger uncertainty for $I\widehat{\text{adj}}$ in Pb is evident in the large error bars for that absorber.

The results unequivocally demonstrate the failure of the standard Bethe expression (circles) for dE/dx; by the time $Z = 26$ is reached in Al and Cu the calculated energies are up to 5 standard deviations (σ) away from the measured beam energy, and over 2σ away in Pb. The results are less conclusive but nevertheless highly suggestive with respect to the Mott and Bloch corrections. The use of dE/dx values with Mott, Bloch, and polarization terms included ($M+B+P$) gives the best fits (triangles) to the measured energies. At points for $Z = 26$, however, this result also deviated by 1 to 2σ and is rivaled in accuracy by the dE/dx expression lacking the Bloch correction ($M+P$), indicated by squares. As $Z \to 0$, one demands that all higher-order corrections in Z vanish with respect to the Bethe expression, i.e., *all* the integrated energy points should converge to $\Delta E/E = 0.0$ (within errors). Extrapolation shows that this is indeed the case here, reinforcing confidence in the independent calculation of the actual beam energies. With that confidence, one may suspect that the apparent divergences of the $M+B+P$ *and* $M+P$ energies will continue with higher Z. The errors associated with the Mott correction calculation are given in Ref. 5 and are far too small to account for the apparent divergences. Also, the polarization correction,[11] whose accuracy has been established in the low-energy regime,[16,17] is too small for its errors to have a significant effect on the calculated relativistic energies. This suggests the need for a closer examination of the magnitude of the Bloch correction (originally calculated nonrelativistically) in the relativistic regime. Low-energy (3 MeV/amu) stopping power measurements[16] have confirmed the existence of a Bloch correction, with a measurement roughly one standard deviation larger than its theoretical value; on the other hand, in the relativistic regime our measurements suggest that the present (nonrelativistically calculated) theoretical value is too large. A theoretical treatment of the Bloch correction in the relativistic regime, therefore, could well yield a smaller magnitude. Should the actual relativistic Bloch correction have only one-half is currently calculated magnitude, a nearly perfect correspondence between our integrated energy and measured beam-energy data would ensue.

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