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Brief Reports

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Aharonov-Bohm effect in bound states: Theoretical and experimental status

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The Aharonov-Bohm effect has recently been questioned on theoretical and experimental grounds. Such discussions have suffered from ambiguities which resulted from their focusing on scattering states. It is noted here that the bound-state problem is much simpler. To avoid the Aharonov-Bohm effect in theory requires us to abandon the most fundamental ideas of quantum mechanics. The quantization of flux in superconducting rings and Josephson junctions is a powerful experimental confirmation.

According to conventional quantum mechanics, the behavior of charged particles can be influenced by external magnetic fields which are confined to a region from which the charged particles are excluded. That comes about in the theory because the Hamiltonian for an electron with coordinate $\vec{x}$ depends upon the external vector potential $\vec{A}(\vec{x})$, and $\vec{A}(\vec{x})$ in turn depends upon the magnetic field $\vec{B}(\vec{x}')$ at all positions $\vec{x}'$, even those from which the electron is excluded. Aharonov and Bohm[1] pointed out in 1959 that when electrons are confined to a multiply connected region, such as the exterior of a cylinder, this has observable consequences; the scattering of electrons from the cylinder depends upon the magnetic flux Φ through the cylinder and the scattering cross section is in fact periodic in Φ with period hc/e, about 10^{-7} G cm^2. That observation stimulated several scattering experiments,[2] which confirmed it in the opinions of the experimenters. There was also much theoretical discussion at that time as to how we should interpret the influence of remote magnetic fields on charged particles.[3]

Lately, it has been claimed[4,5] that the Aharonov-Bohm effect does not exist in theory or does not imply an effect of inaccessible fields, and that the experiments really do not confirm it. Such claims invariably involve scattering situations, where the opportunities for ambiguous interpretation of the theory are almost unlimited, and where experiments can always be challenged on the grounds that return fields, however small or remote, are in principle accessible to the electron and may somehow be responsible for the observed phenomena. The arguments are difficult to evaluate objectively because the flux Φ through the cylinder is equal to the integrated return flux in the accessible region so that the value of Φ can be said to be available in some sense to the electron.

In this paper, I will bypass the discussions of scattering states and consider bound states, where irrelevant ambiguities do not appear. I will show in a direct way that nonexistence of the Aharonov-Bohm effect denies the very basis of quantum mechanics, and that the observed quantization of the fluxoid in superconducting rings with quanta equal to $hc/2e$ is a powerful experimental confirmation of the Aharonov-Bohm effect.

Consider a single spinless electron of charge e, confined to a torus of major radius r and negligible minor radius, lying in the xy plane and centered on the z axis. The Hamiltonian in the presence of an external magnetic field is given by

$$H=(2m)^{-1}(\vec{p}-e\vec{A}/c)^2 \ .$$

This is a gauge-invariant quantity. If the external magnetic field does not touch the torus, then the gauge can be chosen so that $\vec{A}_r=\vec{A}_z=0$ on the domain of H, where the electron moves. In that case,

$$H=(2mr^2)^{-1}(\vec{L}_z-er\vec{A}_\theta/c)^2$$
$$=(2mr^2)^{-1}(\vec{L}_z-e\Phi/2\pi c)^2 \ ,$$

where Φ is the flux threading the torus. The eigenvalues of the energy are then given by

$$E_l=(\hbar^2/2mr^2)(l-e\Phi/hc)^2$$

where the values of l are the integers. Thus the eigenvalue spectrum of H depends upon the flux Φ. Anybody who says that the Aharonov-Bohm effect does not exist in theory either has to say that the Schrödinger equation (and the Dirac equation) is wrong or that energy differences are not observable in principle. Giving the torus a finite minor radius, thereby allowing the electron to move in three dimensions, changes nothing essential.

It has been observed in experiments with widely differing geometries[6] that the equilibrium state of a superconducting ring depends upon the external magnetic flux which threads the ring, even when that flux is confined not to touch the ring. If the flux is a multiple of hc/e (in fact, of $hc/2e$), there is an equilibrium state in which no net current flows around the ring. For other external flux values, a net current generally does flow and its magnitude depends upon the flux. This is an ideal test of the Aharonov-Bohm effect in principle because there is no need for any external field to touch the superconductor where the electrons are, and no need for unphysical currents at infinity.[7] (In the experiments which involve Josephson junctions, external flux does touch the superconductor. However the phenomena are observed to depend upon the flux in the nonsuperconducting region with the same periodicity in $hc/2e$.) The periodicity in Φ with period hc/e follows from the general form of the Schrödinger equation with electromagnetic current interactions, and requires no assumptions about the dynamics of superconductors. That Φ also has period $hc/2e$ follows from nothing more than the assumption that superconductivity is based on time-reversal pairs,[8] or that the superconducting state has off-diagonal long-range order,[9] assumptions which are central to everything we know about superconductivity.

In practice, there will always be some small leakage fields which do touch the superconductor. However, the quantum of necessary external flux threading the ring to achieve zero net current at equilibrium is equal to $hc/2e$, independently of the flux that actually strikes the ring, and the flux threading the ring is not determined even in principle by the external field which is accessible to the electrons.

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