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Induced bremsstrahlung of Langmuir waves from interaction between ion-wave turbulence and electrons

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An induced bremsstrahlung process is found to originate from electrons resonating with the ion-wave turbulence. The growth rate due to this process gives the same order of magnitude as those of decay and nonlinear scattering.

I. INTRODUCTION

According to the conventional weak turbulence theory,[1] the lowest-order mode couplings are composed of two parts. The first one is the well known three-wave decay interaction, and the other one is the nonlinear scattering. These processes are responsible for the generation of Langmuir waves in the presence of ion-density fluctuations and drifting electrons.[2] Owing to the growth of long-wavelength plasma oscillations or electromagnetic mode, the idea of a plasma laser was considered.

In this paper, we discuss a new possibility of Langmuir-wave excitation in a turbulent plasma consisting of ion-wave turbulence. This process arises from those electrons which resonate with the ion waves in a plasma. Specifically, in the presence of the ion-wave turbulence, the resonant electrons feel a strong acceleration.[3] Consequently, the free energy of the electrons is liberated in form of the induced bremsstrahlung radiation. The resonant electrons can emit and absorb the Langmuir waves. The difference of the emission and the absorption processes gives rise to the growth (or damping) of the Langmuir waves. Correspondingly, it turns out that the increment rate crucially depends on the slope of the electron distribution function computed at the resonant velocity between electrons and the ion waves. Here we shall emphasize the importance of this third process which, to the author's best knowledge, was not considered earlier.[4]

The organization of our paper is as follows. In Sec. II, the unperturbed steady-ion-wave turbulent state is determined. The effective dielectric constant of the Langmuir waves in the presence of the ion-wave turbulence is obtained in Sec. III. Induced bremsstrahlung of Langmuir waves by electrons scattered on ion waves is investigated in Sec. IV. The fully developed turbulent state is proposed in Sec. V. The three mode couplings are discussed in Sec. VI. Discussions and conclusions are contained in Sec. VII.

II. BASIC EQUATIONS

A relative drift between the electrons and the ions causes ion waves to become unstable.[5] Nonlinear interaction between the fields and particles leads to a steady turbulent state. Expanding the electron distribution function (f) in powers of the ion-wave amplitude, we have

$$f=f_{0e}+\epsilon f_{1e}+\epsilon^2 f_{2e}+\cdots , \tag{1}$$

where $f_{0e}=(m/2\pi T_e)^{1/2}\exp[-m(v-v_0)^2/2T_e]$ is the initial distribution function, ϵ is a small new parameter which can be associated with the amplitude of the ion-wave turbulence, and $v_0[v_0 \ll v_e =(2T_e/m)^{1/2}]$ is the electron drift velocity. The other notation is standard.

To order in ϵ, the Vlasov equation becomes

$$\left(\frac{\partial}{\partial t}+v\nabla\right)f_{1e}+\frac{e}{m}E_l\frac{\partial}{\partial v}f_{0e}=0 , \tag{2}$$

where E_l is the electrostatic field of the ion waves. Introducing Fourier transforms in space and time for various quantities according to

$$A(x,t)=\sum_{k,\omega}A(k,\omega)\exp(ikx-i\omega t) , \tag{3}$$

we obtain the following equation for f_{1e}:

$$f_{1e}(k,\omega)=\frac{(e/m)E_l(k,\omega)(\partial/\partial v)f_{0e}}{i(\omega-kv)} , \tag{4}$$

where k, ω are the wave number and the frequency of the ion-wave fields.

In a straightforward manner, we also obtain the higher-order contributions from the turbulent fields:

$$\begin{aligned} f_{2e}(k,\omega)&=\left(\frac{e}{m}\right)^2\sum_{k'}E_l(k')\frac{\partial}{\partial v}\frac{E_l(k-k',\omega-\omega')\frac{\partial}{\partial v}f_{0e}}{i[\omega-\omega'-(k-k')v]}\Big/ i(\omega-kv) ,\\ f_{3e}(k,\omega)&=\sum_{k'}\frac{e}{m}E_l(k')\frac{\partial}{\partial v}f_{2e}(k-k',\omega-\omega')/i(\omega-kv) ,\\ &\cdots . \end{aligned} \tag{5}$$

We note that all of the contributions $(f_{1e}, f_{2e}, \ldots)$ show the change of the electron distribution function from the initial value f_{0e}. To obtain the quasilinear plateau solution, we should sum up all the contributions $(f_{0e}+f_{1e}+f_{2e}+f_{3e}+\cdots)$. However, within a weak-turbulence model, the dominant change of the distribution comes from f_{1e}. Hence, we shall retain only f_{1e} in the following.

III. FORMULATION

To investigate the problem of induced bremsstrahlung, we apply an external high-frequency small perturbation field ($\mu\delta E_h$, $\epsilon \gg \mu$) to the system. Then the coupling between the perturbation field ($\mu\delta E_h$) and the quasisteady finite-amplitude turbulent field (ϵE_l) would occur through the mode-coupling processes.

Following the notion of a weak-turbulence theory, we represent the perturbed electric field and the perturbed number density of the electrons as[6]

$$\delta E = \mu\delta E_h + \mu\epsilon\delta E_{lh} \tag{6}$$

and

$$\delta f = \mu\delta f_h + \mu\epsilon\delta f_{lh}\,, \tag{7}$$

where δE_{lh} is the mixed-mode perturbation field and δf_{lh} is the mixed-mode number-density perturbation.

By means of the standard formulation, to order in $\mu\epsilon^2$, the Vlasov equation can be written as

$$\left(\frac{\partial}{\partial t}+v\nabla\right)\delta f_h = -\left(\frac{e}{m}\right)\left\langle E_l\frac{\partial}{\partial v}\delta f_{lh}\right\rangle - \left(\frac{e}{m}\right)\delta E_h\frac{\partial}{\partial v}f_{0e} - \left(\frac{e}{m}\right)\left\langle \delta E_{lh}\frac{\partial}{\partial v}f_{1e}\right\rangle, \tag{8}$$

where the angular brackets indicate the time average corresponding to the low-frequency wave. To order in $\mu\epsilon$, we obtain from the Vlasov equation

$$\left(\frac{\partial}{\partial t}+v\nabla\right)\delta f_{lh} = -\left(\frac{e}{m}\right)E_l\frac{\partial}{\partial v}\delta f_h - \left(\frac{e}{m}\right)\delta E_h\frac{\partial}{\partial v}f_{1e} - \left(\frac{e}{m}\right)\delta E_{lh}\frac{\partial}{\partial v}f_{0e}\,. \tag{9}$$

Using the representation (3) and Eq. (2), we have

$$\delta f_{lh}(k,\omega) = \frac{1}{i(\omega - kv)}\left(\frac{e}{m}\sum_K \delta E_h(K,\Omega)\frac{\partial}{\partial v}f_{1e}(k-K,\omega-\Omega) + \frac{e}{m}\delta E_{lh}(k,\omega)\frac{\partial}{\partial v}f_{0e} + \frac{e}{m}\sum_K E_l(k-K,\omega-\Omega)\frac{\partial}{\partial v}\delta f_h(K,\Omega)\right). \tag{10}$$

From Eq. (8), we also obtain

$$(-i\Omega + iKv)\delta f_h(K,\Omega) = -\frac{e}{m}\delta E_h(K,\Omega)\frac{\partial}{\partial v}f_{0e} - \frac{e}{m}\left\langle\sum_k \delta E_{lh}(K-k,\Omega-\omega)\frac{\partial}{\partial v}f_{1e}(k,\omega)\right\rangle - \frac{e}{m}\left\langle\sum_k E_l(k,\omega)\frac{\partial}{\partial v}\delta f_{lh}(K-k,\Omega-\omega)\right\rangle. \tag{11}$$

Making use of the above equations and the Poisson equation we obtain, after a lengthy but straightforward calculation, the effective dielectric constant of the Langmuir wave $[\epsilon_h(K,\Omega)]$ in the presence of the ion-wave turbulence. The result is

$$\begin{aligned}\epsilon_h(K,\Omega) = {} & 1+\left(\frac{\omega_{pe}}{K}\right)^2\int L_{K,\Omega}^{-1}K\frac{\partial}{\partial v}f_{0e}dv + \left(\frac{e}{m}\right)^2\left(\frac{\omega_{pe}}{K}\right)^2\int\sum_k L_{K,\Omega}^{-1}k\frac{\partial}{\partial v}\frac{1}{[\Omega-\omega-(K-k)v]}K\frac{\partial}{\partial v}\frac{1}{(\omega-kv)}k\frac{\partial}{\partial v}f_{0e}|\Phi_l(k,\omega)|^2dv \\ & - \left(\frac{e}{m}\right)^2\left(\frac{\omega_{pe}}{K}\right)^2\int\sum_k dv\, L_{K,\Omega}^{-1}|\Phi_l(k,\omega)|^2k\frac{\partial}{\partial v}\frac{1}{[\Omega-\omega-(K-k)v]}k\frac{\partial}{\partial v}L_{K,\Omega}^{-1}K\frac{\partial}{\partial v}f_{0e} \\ & + \left[\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\int\sum_k L_{K,\Omega}^{-1}\frac{k}{(K-k)}\frac{|\Phi_l(k,\omega)|^2}{\epsilon_0(K-k,\Omega-\omega)}\frac{\partial}{\partial v}\left(\frac{1}{\omega-kv}+\frac{1}{\Omega-\omega-(K-k)v}\right)\frac{\partial}{\partial v}f_{0e}dv\right] \\ & \times\left[\int\frac{\omega_{pe}^2}{[\Omega-\omega-(K-k)v]}kK\frac{\partial}{\partial v}\left(\frac{1}{kv-\omega}+\frac{1}{\Omega-Kv}\right)\frac{\partial}{\partial v}f_{0e}dv\right],\end{aligned} \tag{12}$$

where

$$L_{K,\Omega} = \Omega - Kv \tag{13}$$

and $|\Phi_l(k,\omega)|^2$ is the turbulent potential fluctuation of the ion waves. $\epsilon_0(K-k,\Omega-\omega)$ is the linear dielectric function of the electrostatic waves.

IV. TURBULENT BREMSSTRAHLUNG OF LANGMUIR WAVES

The third and fourth terms in Eq. (12) are summarized as

$$\left(\frac{e}{m}\right)^2\left(\frac{\omega_{pe}}{K}\right)^2\int\sum_k dv\frac{|\Phi_l(k,\omega)|^2}{(\Omega-Kv)}k\frac{\partial}{\partial v}\frac{1}{[\Omega-\omega-(K-k)v]}\left(K\frac{\partial}{\partial v}\frac{1}{(\omega-kv)}k\frac{\partial}{\partial v}-k\frac{\partial}{\partial v}\frac{1}{(\Omega-Kv)}K\frac{\partial}{\partial v}\right)f_{0e}. \quad (14)$$

Bearing in mind the fact that Langmuir-wave resonances are not important for our problem, we can neglect the last term in Eq. (14), and replace $(\omega-kv)^{-1}$ by $-\pi i\delta(\omega-kv)$ in order to find the imaginary part of Eq. (14). Partial integration of Eq. (14) then leads to

$$\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\int dv\sum_k\frac{|\Phi_l|^2}{\omega_{pe}^4}kK(3K^2-kK)\frac{1}{\omega-kv}k\frac{\partial}{\partial v}f_{0e}. \quad (15)$$

In deriving Eq. (15), we have replaced $(\Omega-Kv)^{-4}$ by ω_{pe}^{-4}.

Equation (15) can be written as

$$-\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\sum_k\frac{|\Phi_l|^2}{\omega_{pe}^4}kK(3K^2-kK)\times\frac{m}{T}\left[1+\frac{\omega-kv_0}{kv_e}Z\left(\frac{\omega-kv_0}{kv_e}\right)\right], \quad (16)$$

where $Z(z)$ is the plasma dispersion function.[7]

Under the small argument limit $[(\omega-kv_0)/kv_e\ll 1]$, the imaginary part of Eq. (16) becomes

$$-\sum_k\pi 2^{-1/2}(K/k)[3-(k/K)][(\omega-kv_0)/kv_e]U_l, \quad (17)$$

where $U_l=k^2|\Phi_l(k,\omega)|^2/4\pi NT_e$. To the lowest order, the real part of Eq. (12) is

$$1=\omega_{pe}^2/(\Omega-Kv_0)^2+K^2v_e^2\omega_{pe}^2/(\Omega-Kv_0)^4. \quad (18)$$

Accordingly, the growth rate of the Langmuir wave (γ_N) is given by

$$\gamma_N=\pm\tfrac{1}{2}\omega_{pe}\times\ldots,$$
$$\gamma_N=\mp\omega_{pe}\sum_k\pi 2^{-3/2}(K/k)[3-(k/K)][(\omega-kv_0)/kv_e]U_l, \quad (19)$$

where the ellipsis stands for Eq. (17). This result is basically obtained in Ref. 8.

However, the dominant imaginary part of the last term of Eq. (12) comes from the following:

$$AB, \quad (20)$$

where

$$A=\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\int\sum_k\frac{1}{\Omega-Kv}\left(\frac{1}{K-k}\right)\epsilon_0(K-k,\Omega-\omega)^{-1}\times\frac{\partial}{\partial v}\frac{1}{\omega-kv}k\frac{\partial}{\partial v}f_{0e}|\Phi_l(k,\omega)|^2dv \quad (21)$$

and

$$B=\omega_{pe}^2\int\frac{1}{\Omega-\omega-(K-k)v}K\frac{\partial}{\partial v}\frac{1}{kv-\omega}k\frac{\partial}{\partial v}f_{0e}dv. \quad (22)$$

Partial integration of Eq. (21) leads to

$$-\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\int\sum_k dv\left(\frac{K}{K-k}\right)\frac{1}{\Omega^2}\frac{1}{\epsilon_0}\frac{|\Phi_l|^2}{\omega-kv}k\frac{\partial}{\partial v}f_{0e}, \quad (23)$$

where we replaced $(\Omega-Kv)^{-2}$ by Ω^{-2}. Equation (23) may be rewritten in the form

$$-\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\sum_k\Omega^{-2}\frac{K}{K-k}\epsilon_0^{-1}\frac{m}{T_e}|\Phi_l|^2\times\left[1+\frac{\omega-kV_0}{kv_e}Z\left(\frac{\omega-kv_0}{kv_e}\right)\right]. \quad (24)$$

Under the small argument limit, the plasma dispersion function is expanded as

$$Z\left(\frac{\omega-kv_0}{kv_e}\right)\approx i\left(\frac{\pi}{2}\right)^{1/2}. \quad (25)$$

Thus, we obtain

$$A=A_r+iA_i, \quad (26)$$

where

$$A_r=-\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\sum_k\frac{|\Phi_l|^2}{\Omega^2}\frac{K}{K-k}\frac{1}{\epsilon_0}\frac{m}{T_e} \quad (27)$$

and

$$A_i=-\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\times\sum_k\frac{|\Phi_l|^2}{\Omega^2}\frac{K}{K-k}\frac{1}{\epsilon_0}\frac{m}{T_e}\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{\omega-kv_0}{kv_e}\right). \quad (28)$$

Furthermore, Eq. (22) reduces to

$$\left(\frac{\omega_{pe}}{\Omega}\right)^2 K(K-k)\frac{m}{T_e}\left[1+\left(\frac{\omega-kv_0}{kv_e}\right)Z\left(\frac{\omega-kv_0}{kv_e}\right)\right]$$
$$\simeq\left(\frac{\omega_{pe}}{\Omega}\right)^2 K(K-k)\frac{m}{T_e}\left[1+i\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{\omega-kv_0}{kv_e}\right)\right]. \quad (29)$$

Thus,

$$B=B_r+iB_i, \quad (30)$$

with

$$B_r=\left(\frac{\omega_{pe}}{\Omega}\right)^2 K(K-k)\frac{m}{T_e}, \quad (31)$$

$$B_i=\left(\frac{\omega_{pe}}{\Omega}\right)^2 K(K-k)\frac{m}{T_e}\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{\omega-kv_0}{kv_e}\right). \quad (32)$$

Hence, the imaginary part of Eq. (20) becomes

$$A_rB_i+A_iB_r=-\sum_k\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\frac{K^2|\Phi_l|^2}{\Omega^2}\left(\frac{m}{T_e}\right)^2 \times\left(\frac{\omega_{pe}}{\Omega}\right)^2(2\pi)^{1/2}\left(\frac{\omega-kv_0}{kv_e}\right)\frac{1}{\epsilon_0}. \quad (33)$$

Next, we estimate $\epsilon_0(K-k,\Omega-\omega)$. By definition, we have

$$\epsilon_0(K-k,\ \Omega-\omega)=1+\left(\frac{k_e}{k-K}\right)^2\left[1+\frac{\omega-\Omega-(k-K)v_0}{(k-K)v_e}Z\left(\frac{\omega-\Omega-(k-K)v_0}{(k-K)v_e}\right)\right] +\left(\frac{k_i}{k-K}\right)^2\left[1+\frac{\omega-\Omega}{(k-K)v_i}Z\left(\frac{\omega-\Omega}{(k-K)v_i}\right)\right], \quad (34)$$

where k_e and k_i are the Debye wave numbers of the electrons and protons, respectively. For the large argument limit, Eq. (34) is simplified to yield

$$1-\frac{\omega_{pe}^2}{[\Omega-(K-k)v_0]^2}-\frac{(k-K)^2k_e^2v_e^4}{[\Omega-(K-k)v_0]^4}-\frac{\omega_{pi}^2}{\Omega^2}. \quad (35)$$

Assuming $K\sim k\ll k_e$, Eq. (35) is reduced to

$$1-\left(\frac{\omega_{pe}}{\Omega}\right)^2-\left(\frac{\omega_{pi}}{\Omega}\right)^2\cong\frac{\omega_{pe}^2}{(\Omega-Kv_0)^2}+\frac{K^2v_e^2\omega_{pe}^2}{(\Omega-Kv_0)^4}-\frac{\omega_{pe}^2}{\Omega^2}$$
$$\cong\frac{\omega_{pe}^2}{(\Omega-Kv_0)^2}. \quad (36)$$

On substituting Eq. (36) into Eq. (33), we obtain

$$A_rB_i+A_iB_r=-2^{3/2}\pi^{1/2}\left(\frac{\omega_{pe}}{\Omega}\right)^4 \times\sum_k\left(\frac{\Omega-Kv_0}{kv_e}\right)^2\left(\frac{\omega-kv_0}{kv_e}\right)U_l. \quad (37)$$

Thus, the nonlinear growth rate of the Langmuir wave is found to be

$$\gamma_N=\pm\tfrac{1}{2}\omega_{pe}\times(A_rB_i+A_iB_r)$$
$$=\mp(2\pi)^{1/2}(\omega_{pe}/\Omega)^4\sum_k[(\Omega-Kv_0)/kv_e]^2 \times[(\omega-kv_0)/kv_e]\omega_{pe}U_l$$
$$\cong\pm(2\pi)^{1/2}\sum_k\left(\frac{k_e}{k}\right)^2\left(\frac{v_0}{v_e}\right)\omega_{pe}U_l. \quad (38)$$

In deriving Eq. (38), we have replaced $\Omega-Kv_0$ by $\pm\omega_{pe}$. The contribution of Eq. (38) is larger than that of Eq. (19).

As we have mentioned earlier, in the presence of ionwave turbulence, the resonant electrons[3] can emit and absorb the Langmuir waves through induced bremsstrahlung interaction. The difference of the emission and the absorption processes gives the growth of the Langmuir waves. This essence is visualized if we follow the structure of the growth rate [Eq. (38)] of the Langmuir waves which is simply $-\mathrm{I}_m\epsilon_h(K,\Omega)/[(\partial/\partial\Omega)\mathrm{R}_e\epsilon_h(K,\Omega)]$. The sign of $\mathrm{I}_m\epsilon_h(K,\Omega)$ depends on the slope of the electron distribution function computed at the resonance velocity between the electrons and the ion waves. $\mathrm{R}_e\epsilon_h(K,\Omega)$ is determined from the dispersion relation of the Langmuir waves. Therefore, the test Langmuir wave with a particular propagation direction is amplified. The other wave with different propagation is damed. Accordingly, the induced bremsstrahlung interaction between electrons and ion waves gives rise to the enhanced Langmuir waves for any type of electron distribution except the plateau one.

As we are considering only the lowest-order contribution (f_{1e}), the growth rate depends only on the first derivative of the electron distribution. However, if we take into account the higher-order contributions ($f_{2e}, f_{3e}, \ldots$), the growth rate also depends on the higher-order derivatives of the electron distribution function.

V. FULLY DEVELOPED TURBULENCE

The linear damping rate of the Langmuir waves arises from the Landau resonance. The linear part of Eq. (12) yields

$$1+\left(\frac{k_e}{K}\right)^2\left[1+\frac{\Omega-Kv_0}{Kv_e}\,Z\left(\frac{\Omega-Kv_0}{Kv_e}\right)\right]=0\ . \qquad (39)$$

For the large argument limit of the Z function, Eq. (39) yields

$$1-\frac{\omega_{pe}^2}{(\Omega-Kv_0)^2}+i\left(\frac{\pi}{2}\right)^{1/2}\left(\frac{\Omega-Kv_0}{Kv_e}\right)\left(\frac{k_e}{K}\right)^2 \times \exp\left(-\frac{(\Omega-Kv_0)^2}{2K^2v_e^2}\right)=0\ . \qquad (40)$$

Thus, the linear Landau damping (γ_0) is given by

$$\gamma_0=-\left(\frac{\pi}{8}\right)^{1/2}\omega_{pe}\left(\frac{k_e}{K}\right)^3\exp\left[-\frac{1}{2}\left(\frac{k_e}{K}\right)^2\right]\ . \qquad (41)$$

It is instructive to compute the ratio (R) of Eqs. (38) and (41) which can be written as

$$R=|\gamma_N/\gamma_0|$$
$$=|\text{Eq. (38)/Eq. (41)}|$$
$$=\sum_k 4\left(\frac{k_e}{k}\right)^2\left(\frac{K}{k_e}\right)^3 U_l\left[\frac{v_0}{v_e}-\left(\frac{m}{M}\right)^{1/2}\right]\exp\left[\frac{1}{2}\left(\frac{k_e}{K}\right)^2\right]\ . \qquad (42)$$

It emerges that one can have the turbulent plasma with enhanced both the Lagmuir and ion-wave frequency fluctuation under the condition $R>1$:

$$U_l>\frac{1}{4}\frac{v_e}{v_0}\left(\frac{k_0}{k_e}\right)^2\left(\frac{k_e}{K}\right)^3\exp\left[-\frac{1}{2}\left(\frac{k_e}{K}\right)^2\right]\ . \qquad (43)$$

In deriving at this result, we have assumed that the ion-waves energy is mainly contained in mode k_0, and $v_0 \gg (m/M)^{1/2}v_e$. Figure 1 is a plot of the critical turbulent energy (U_l) vs (K/k_e). Note that the region of a fully developed turbulent state lies above the solid line, and such a state is observed in Ref. 9.

VI. THREE MODE COUPLINGS

Here we discuss the relation between our results and existing weak-turbulence theory. In the following, we estimate the growth rate in the limit of zero drift ($v_0=0$). In deriving the growth rate from Eq. (12), we have three possibilities.[10]

The first one is the induced radiation discussed in this paper. From Eq. (33), we obtain the imaginary contribution in the limit of zero drift as

$$A_rB_i+A_iB_r=-\sum_k\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\frac{K^2|\Phi_l(k,\omega)|^2}{\Omega^2}\left(\frac{m}{T}\right)^2\left(\frac{\omega_{pe}}{\Omega}\right)^2(2\pi)^{1/2}\left(\frac{\omega}{kv_e}\right)\mathrm{Re}\,\epsilon_0(K-k,\Omega-\omega)^{-1}\ , \qquad (44)$$

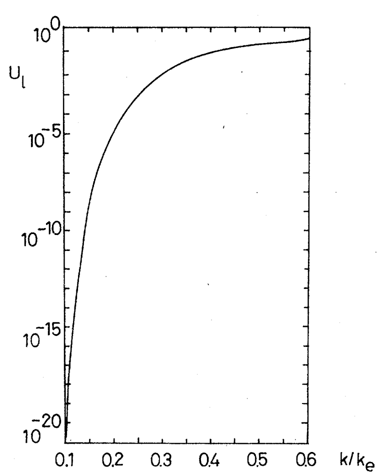

FIG. 1. The critical ion-wave turbulent energy [Eq. (43)] versus k_0/k_e for $v_0=(\frac{1}{4})v_e$, and $K=k_0$; here k_0, K, and v_0 show the characteristic wave numbers of the ion sound and Langmuir waves, and the drift velocity of the electrons, respectively.

where Re stands for the real part of the relevant term. From Eq. (35),

$$\epsilon_0(K-k,\Omega-\omega)=1-\frac{\omega_{pe}^2}{(\Omega-\omega)^2}-\frac{(k-K)^2k_e^2v_e^4}{(\Omega-\omega)^4}-\frac{\omega_{pi}^2}{(\Omega-\omega)^2} \qquad (45)$$

for $\omega \ll \Omega$, we expand $\epsilon_0(K-k,\Omega-\omega)$ and find

$$\epsilon_0(K-k,\Omega-\omega)\approx\frac{2\omega}{\Omega}-\frac{v_e^2(k^2-2kK)}{\Omega^2}\ . \qquad (46)$$

For $k_e \gg k > k_e(m/M)^{1/2}$, Eq. (46) reduces to

$$\epsilon_0(K-k,\Omega-\omega)=-\frac{v_e^2(k^2-2kK)}{\Omega^2}\ . \qquad (47)$$

Thus, Eq. (44) reduces to

$$-\sum_k\frac{k^2|\Phi_l|^2}{4\pi NT}\left(\frac{k_e}{k}\right)^4\left(\frac{m}{M}\right)^{1/2}\frac{1}{(2K/k)-1}\ . \qquad (48)$$

We obtain the growth rate (γ) of the Langmuir wave,

$$\frac{\gamma}{\Omega}=\sum_k \frac{k^2|\Phi_l|^2}{8\pi NT}\left(\frac{k_e}{k}\right)^4\left(\frac{m}{M}\right)^{1/2}\frac{1}{(2K/k)-1}. \qquad (49)$$

In order to obtain a qualitatively correct result, we further reduce the wave spectrum to a single unstable wave number $k=k_0 \ll k_e$. Thus, Langmuir wave with wave number $K>k_0/2$ grows even in the limit of zero drift ($v_0=0$). This does not mean that thermal equilibrium Maxwell distribution drives the unstable Langmuir wave. As is shown by Eq. (1), the unperturbed-state electron distribution function is far from Maxwell distribution because of the presence of enhanced ion-wave fluctuations. Electrons feel a strong nonlinear ponderomotive force through resonant electrons. Thus, the unp-conversion of ion-wave energy to Langmuir waves occurs.

The second one is the nonlinear scattering of waves. In a drift-free plasma, it is known that an enhanced level of ion acoustic fluctuations increases the damping of Langmuir waves. This is the so-called anomalous absorption effect of Dawson and Oberman.[11] Here, we show that Eq. (12) reduces to their results under the conditions $K=0,\ k\sim k_e,\ v_0=0$. From Eq. (12), we get

$$\lim_{K\to 0,\,\omega\to 0}\epsilon(K,\Omega)=-\omega_{pe}^4\left(\frac{e}{m}\right)^2\frac{1}{\Omega^2}\left(\frac{m}{T}\right)^2 \times\sum_k\frac{|\Phi_l(-k,0)|^2}{\epsilon_0(k,\Omega)}\left(\int\frac{f_{0e}dv}{\Omega-kv}\right)^2, \qquad (50)$$

where

$$f_{0e}=\left(\frac{m}{2\pi T}\right)^{1/2}\exp\left(-\frac{mv^2}{2T}\right).$$

For $(\Omega/k)(m/2T)^{1/2}\geq 1$, the imaginary part of Eq. (50) reduces to

$$\mathrm{Im}\,\epsilon(0,\Omega)=\pi^{-1/2}\sum_k\left(\frac{k_e}{k}\right)^4 k^2|\Phi_l|^2\frac{1}{\epsilon_0(0,\Omega)}, \qquad (51)$$

where Im shows the imaginary part of the relevant term. In obtaining Eq. (51), we have used the following realtion for $\Omega/kv_e \geq 1$:

$$\mathrm{Im}\left(\int\frac{f_{0e}dv}{\Omega-kv}\right)^2=-\frac{m}{kT\Omega}\left(\frac{2T}{m\pi}\right)^{1/2}. \qquad (52)$$

Thus, we obtain the damping rate $\gamma(0,\Omega)$ of the Langmuir waves[12]

$$\frac{\gamma(0,\Omega)}{\Omega}=-\frac{\pi^{-1/2}}{2}\frac{1}{\epsilon_0(0,\Omega)}\sum_k\left(\frac{k_e}{k}\right)^4\frac{k^2|\Phi_l|^2}{4\pi NT}. \qquad (53)$$

The third process is the decay interaction. For zero drift ($v_0=0$), Eqs. (27) and (31) reduces to, respectively,

$$A_r(v_0=0)=-\left(\frac{\omega_{pe}}{K}\right)^2\left(\frac{e}{m}\right)^2\sum_k\frac{|\Phi_l|^2}{\Omega^2}\frac{K}{K-k}\frac{m}{T}\times\frac{1}{\epsilon_0(K-k,\Omega-\omega)} \qquad (54)$$

and

$$B_r(v_0=0)=\left(\frac{\omega_{pe}}{\Omega}\right)^2K(K-k)\left(\frac{m}{T}\right). \qquad (55)$$

Accordingly, the imaginary part of Eq. (12) arises from possible resonances of $\epsilon_0(K-k,\Omega-\omega)^{-1}$ and the result is

$$\mathrm{Im}\,\epsilon(K,\Omega)=-\left(\frac{\omega_{pe}}{\Omega}\right)^2\omega_{pe}^2\left(\frac{e}{m}\right)^2\sum_k\frac{|\Phi_l|^2}{\Omega^2}\left(\frac{m}{T}\right)^2\times\mathrm{Im}\frac{1}{\epsilon_0(K-k,\Omega-\omega)}. \qquad (56)$$

From Eq. (47),

$$\mathrm{Im}\frac{1}{\epsilon_0(K-k,\Omega-\omega)}=\frac{\pi\Omega^2}{kv_e^2}\delta(2K-k). \qquad (57)$$

Substituting Eqs. (56) and (57) into Eq. (19), we get

$$\frac{\gamma}{\Omega}=\sum_k\left(\frac{k_e}{k}\right)^4\frac{k^2|\Phi_l|^2}{4\pi NT}k\delta(k-2K). \qquad (58)$$

In contrast to the induced radiation [Eq. (49)], the decay interaction is effective between the particular modes due to the δ function. Thus, the growth rate is usually smaller than that of the induced radiation.[13]

VII. DISCUSSION

The effective dielectric constant of the Langmuir wave in the presence of the ion-wave turbulence is obtained within the framework of the linear response theory of a turbulent plasma. The analysis predicts that the turbulent energy is shared between the Langmuir and ion wave even when the electron drift velocity is less than the electron thermal velocity. A critical condition for the fully developed turbulence is Eq. (43). The physical mechanism of our theory is the turbulent bremsstrahlung of Langmuir waves by electrons scattered on ion waves.

As is shown in Eq. (49), the growth of the Langmuir wave occurs even in the limit of zero drift ($v_0=0$). This result is markedly different from the so-called plasma laser.[14] For an enhanced level of ion fluctuations, the plasma laser theory is effective for a drifted Maxwellian electron distribution function. Moreover, for a drifted Maxwellian electron distribution function and an equilibrium level of ion fluctuations, the amplification of Langmuir waves occurs for a sufficiently strong drift.[15] The free energy of these theories comes from the electron drift velocity ($v_0 \neq 0$). On the other hand, the free energy of induced radiation exists in the enhanced ion-wave fluctuations in addition to the drift velocity. Thus, the finite drift velocity ($v_0 \neq 0$) is not necessary for the growth of the Langmuir waves through the induced radiation process.[10]

We note that our mode coupling is effective for a general dispersion relation, because our effect does not require the matching conditions for resonant decay interaction. We may conclude that mode coupling in a turbulent plasma is much enhanced. This result significantly differs from what is generally believed to be true within the framework of the conventional weak-turbulence theory.[1] We emphasize again that in addition to the conventional three-wave decay[16] [(a) in Fig. 2: the matching conditions are $K - K' = \pm k$, $\Omega - \Omega' = \pm\omega$] and nonlinear scattering[17] [(b) in Fig. 2: the condition is $\Omega \pm \omega = (K \pm k)v$], the third mechanism as discussed here originates from the induced bremsstrahlung of Langmuir waves and is caused by electrons which resonate with the ion waves [(c) in Fig. 2: the condition is $\omega = kv$].

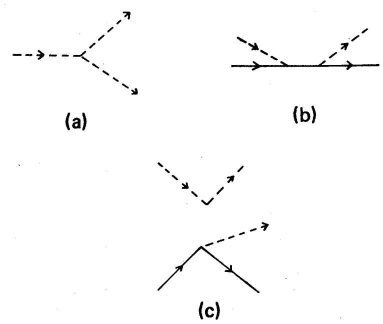

FIG. 2. Mode coupling in a turbulent plasma. (a), (b), and (c) show decay, nonlinear scattering, and induced bremsstrahlung, respectively.

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