PHYSICAL REVIEW A VOLUME 23, NUMBER 1 JANUARY 1981

Excitation of convective cells by drift waves

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(Received 21 July 1980)

Parametric excitation of convective cells by drift waves is considered. Noting that the convective-cell fluctuation of the electrostatic potential is much larger than that of the density, we derive a set of coupled differential equations describing the interaction of the drift waves and the convective cells. For a weak pump, a general dispersion relation which describes three-wave decay and modulational instabilities is given. The growth rate of the latter is found to be generally smaller than that of the decay instability. Our analytical result agrees with computer simulation in which enhanced cells appear in the presence of drift-wave turbulence. The problem of anomalous cross-field diffusion is also discussed.

I. INTRODUCTION

Numerical simulations[1] have shown that vortex-like convective-cell motion[1,2] plays an important role in the diffusion of plasma across an external magnetic field. The cells appear as a result of particle convection in crossed electric and magnetic fields at the drift velocity $c\vec{E}\times\vec{B}/B^2$. Being homogeneous along the magnetic field, these modes are characterized by their two-dimensional nature ($k_\parallel = 0$) as well as aperiodicity ($\mathrm{Re}\,\omega = 0$). Convective-cell motion is therefore similar to that of two-dimensional vortices in a incompressible fluid, both motions being divergence free. In the presence of collisions, the cells are damped because of ion viscosity μ_i leading to a normal mode with purely imaginary frequency. An important property[3] of these cells is the absence of significant density variation $\tilde{n}_i^c$ accompanying the electrostatic potential fluctuation ($\tilde{n}_i^c/n_0 \ll e\psi/T_e$). The slow motion of the convective cells can lead to anomalously rapid plasma transport across the magnetic field even in thermal equilibrium.[1,2] The process becomes especially pronounced in the presence of mechanisms which can cause instabilities leading to convective-cell excitation. In the simulations of Cheng and Okuda[4,5] it was found that rapid generation of convective cells is associated with drift-wave turbulence in an inhomogeneous magnetized plasma. They explained this result in terms of a simple mode-coupling process[5] in which initially given linearly unstable drift waves beat to excite the convective cells. Sagdeev *et al.*[6] reconsidered the problem self-consistently in terms of a three-wave decay process. Their results are, however, of limited application. The reason is that they assumed that the density perturbations in the two-dimensional low-frequency motion are much larger than that of the electrostatic potential ($n_i^c/n_0 \gg e\psi/T_e$). This, except for very large wavelengths, is in contradiction to the basic properties of the convective cells. Using a wave packet formulation, Mima and Lee[7] recently considered convective-cell generation from a modulationally unstable drift wave. They assumed three-dimensional low-frequency motion, and $n_i^c/n_0 \simeq e\psi/T_e$, which is actually within the drift wave scaling as discussed by Hasegawa and Mima.[8] Thus, the convective-cell motion considered by them is not the two-dimensional one which Cheng and Okuda[4] found to be most important. Recalling that the latter are characterized[3] by potential fluctuations much larger than the density fluctuations, we consider here in detail the nonlinear coupling between finite-amplitude drift waves and convective cells of this kind.

In the next section, we present the basic set of equations which govern the dynamics of drift waves and convective cells. We then obtain a coupled set of equations describing the drift and convective-cell modes together with their parametric interaction. A general dispersion relation for the weak pump case is given in Sec. III. In Sec. IV, we analyze the three-wave decay interaction leading to a purely growing instability. Explicit expression for the growth rate is obtained and its dependence on the various plasma parameters is discussed. A four-wave process which gives rise to a new kind of modulational instability is considered in Sec. V. It is found that in general the decay process has a higher growth rate than the modulational instability. The corresponding plasma diffusion is discussed in the last section.

II. DERIVATION OF EQUATIONS

We shall use the fluid equations in the drift approximation to describe the coupling between the drift waves and the convective cells because all the frequencies involved are much smaller than the ion cyclotron frequency. In the absence of magnetic shear, the governing equations are

$$\partial_t \tilde{n}_j + \nabla \cdot n_j \vec{v}_j = 0, \tag{1}$$

$$\vec{v}_{\perp j} = D_j \left(\hat{z} \times \nabla - \frac{1}{\Omega_j} (\partial_t + \vec{v}_{\perp j} \cdot \nabla - \mu_j \nabla^2) \nabla \right) \times \left(\frac{e_j \Phi}{T_j} - \frac{\tilde{n}_j}{n_0} \right), \tag{2a}$$

$$\partial_t v_{zj} + \vec{v}_j \cdot \nabla v_{zj} = - \frac{e_j}{m_j} \partial_z \Phi - \frac{T_j}{m_j n_j} \partial_z \tilde{n}_j, \tag{2b}$$

$$\nabla^2 \Phi = -4\pi \sum_{j=i,e} e_j \tilde{n}_j, \tag{3}$$

where $n_j = n_0(x) + \tilde{n}_j(\vec{x}, t)$, $\Omega_j = e_j B / m_j c$, $D_j = cT_j / e_j B$. The external magnetic field $\vec{B}$ is taken to be along the z axis. The other notations are standard.

For a low-β plasma, the parallel phase velocity of the drift waves lies between the ion and electron thermal velocities. The electrons maintain equilibrium by streaming along the magnetic field, so that $\tilde{n}_e^d = n_0(x) e\phi / T_e$. The ions move in a plane perpendicular to $\vec{B}$. On the other hand, for convective cells both ions and electrons move perpendicular to the magnetic field, and the potential fluctuation ψ is much greater than the density fluctuation $\tilde{n}_j^c / n_0 \ll e\psi^c / T_e$. Decomposing the field quantities into their high- and low-frequency components, that is $\Phi = \phi + \psi$, $n_j = n_0(x) + \tilde{n}_j^d + \tilde{n}_j^c$, and $\vec{v}_{\perp j} = \vec{v}_{\perp j}^d + \vec{v}_{\perp j}^c$, we obtain

$$\vec{v}_{\perp i}^d = D\eta \hat{z} \times \nabla\phi - \frac{D\eta}{\Omega_i} \frac{\partial \nabla \phi}{\partial t} - \frac{D^2 \eta}{\Omega_i} \left[(\hat{z} \times \nabla\psi \cdot \nabla) \nabla\phi + (\hat{z} \times \nabla\phi \cdot \nabla) \nabla\psi \right], \tag{4}$$

$$\vec{v}_{\perp i}^c = D\hat{z} \times \nabla\psi - \frac{D}{\Omega_i} \left(\frac{\partial}{\partial t} + \mu_i \nabla^2 \right) \nabla\psi - \frac{D^2 \eta^2}{\Omega_i} (\hat{z} \times \nabla\phi \cdot \nabla) \nabla\phi, \tag{5}$$

$$\vec{v}_{\perp e}^c = D\hat{z} \times \nabla\psi, \tag{6}$$

where $D = |D_e|$, $\eta = 1 + T_i / T_e$, $\mu_i = 3\nu_i \rho_i^2 / 10\Omega_i$, $\rho_i = (T_i / m_i \Omega_i^2)^{1/2}$, and ν_i is the ion-ion collision frequency. The potentials ϕ and ψ are normalized by T_e / e. Note that the electron $\vec{E} \times \vec{B}$ drift is exactly canceled by its diamagnetic drift. Hence, there is no net perpendicular electron drift.

Matching terms of the same frequency in the ion continuity equation, and using (4) to (6) as well as the quasineutrality condition, we find an evolution equation for the drift waves including the effect of the slowly varying potential ψ of the convective cells,

$$(1 - \eta \rho_s^2 \nabla^2) \partial_t \phi - \eta \rho_s^2 \Omega_i \nabla\phi \times \hat{z} \cdot \nabla \ln n_0 = D \nabla\psi \times \hat{z} \cdot \nabla\phi + D\rho_s^2 \eta (\nabla\psi \times \hat{z} \cdot \nabla\nabla^2\phi + \vec{\nabla}\phi \times \hat{z} \cdot \nabla\nabla^2\psi). \tag{7}$$

Here, as well as in the dynamics of convective cells discussed below, we neglect nonlinear self-interaction terms.

Next, matching terms of low frequency (that is, for the convective-cell mode) in both the electron and ion continutiy equations, using (3) to (6), one readily finds an equation for the two-dimensional convective cells including the effect of beating of the high-frequency waves,[5]

$$\left(\partial_t - \frac{\mu_i}{1+a} \nabla^2 \right) \nabla^2 \psi = \frac{D\eta^2}{1+a} \langle (\nabla\phi \times \hat{z} \cdot \nabla) \nabla^2 \phi \rangle, \tag{8}$$

where the angular brackets denote time average over a drift wave period. We have defined $a = (\Omega_i / \omega_{pi})^2$.

Since the convective cells involve rather weak density fluctuations ($\tilde{n}_i^c / n_0 = -\lambda_e^2 \nabla^2 e\psi / T_e$, where λ_e is the electron Debye length), the usual density-ponderomotive force relation does not hold. Here, the ponderomotive force term originates from the nonlinear ion flux due to the nonlinear ion polarization drift, since the electron perpendicular drift vanishes, and the contribution due to $n_i^d v_i^d$ is smaller by a factor $\omega_0^* / \Omega_i k_0^2 \rho_s^2$, where $\omega_0^* = -cT_e \vec{k}_0 \times \hat{z} \cdot \nabla \ln n_0 / eB$ is the drift frequency and $\vec{k}_0$ is the wave vector of drift waves.

Before proceeding to analyze (7) and (8), we should point out that for the case simulated by Cheng and Okuda,[4,5] in which $T_e / T_i = 4$, the assumption $k_0 \rho_i \ll 1$ required for the fluid approach is only marginally satisfied. Since for the decay and modulational instabilities we are considering, the waves in the $k_0 \rho_s \sim 1$ range of the drift-wave spectrum are of dominant interest. This is because only in this part of the spectrum waves with similar values of frequency and parallel wave vector, but different perpendicular wave vectors, exist. Only such waves can couple parametrically to convective-cell modes having zero frequency and no parallel wave vector. However, due to the particular functional dependence of the dispersion relation on $k_0 \rho_i$ when kinetic theory is used, it turns out that the fluid approach is adequate.[6]

III. THE DISPERSION RELATION

We now consider decay and modulational instabilities of drift waves in the presence of convective cell motion. For this purpose, we split the high-frequency potential in (7) and (8) into three components, namely the pump and the upper and lower sidebands. Thus,

$$\phi = \phi_0 \exp(-i\omega_0 t + i\vec{k}_0\cdot\vec{x}) + \phi_+\exp(-i\omega_+ t + i\vec{k}_+\cdot\vec{x}) + \phi_-\exp(-i\omega_- t + i\vec{k}_-\cdot\vec{x}), \tag{9}$$

where $\omega_\pm = \omega \pm \omega_0$ and $\vec{k}_\pm = \vec{k} \pm \vec{k}_0$. The low-frequency potential ψ is assumed to have a time-space dependence of $\exp(-i\omega t + i\vec{k}\cdot\vec{x})$. Equations (7) and (8) then become

$$(\omega_\pm - D\hat{z}\times\vec{k}_\pm\cdot\nabla\ln n_0 + \rho_s^2\eta\omega_\pm k_\pm^2)\phi_\pm = \pm iD\hat{z}\times\vec{k}\cdot\vec{k}_0\psi\begin{pmatrix}\phi_0\\ \phi_0^*\end{pmatrix}, \tag{10}$$

$$(\omega + i\Gamma)\psi = iD\frac{\eta^2}{k^2(1+a)}\hat{z}\times\vec{k}_0\cdot\langle\vec{k}_-(k_-^2-k_0^2)\phi_0\phi_- - \vec{k}_+(k_+^2-k_0^2)\phi_0^*\phi_+\rangle, \tag{11}$$

where $\Gamma = \mu_i k^2/(1+a)$ and $b \equiv 1 + \eta(k^2 - k_0^2)\rho_s^2$. Combining these equations, we obtain the following general dispersion relation for decay and modulational instabilities,

$$\omega + i\Gamma = -D^2\frac{\eta^2 b}{k^2(1+a)}|\hat{z}\times\vec{k}_0\cdot\vec{k}|^2 \times\left(\frac{k_+^2-k_0^2}{\alpha_+} + \frac{k_-^2-k_0^2}{\alpha_-}\right)|\phi_0|^2, \tag{12}$$

where

$$\alpha_\pm = (1+\eta k_\pm^2\rho_s^2)(\omega_\pm - \omega_{k_\pm})$$

and

$$\omega_{k_\pm} = \omega_\pm^*/(1+\eta k_\pm^2\rho_s^2).$$

Equation (12) describes the parametric coupling between drift waves and convective cells.

IV. DECAY INSTABILITY

Here, we assume that only the lower sideband is resonant. For $|k| \ll |k_0|$, we found a purely growing instability with growth rate

$$\gamma_D = D\eta\left(\frac{2|\vec{k}\cdot\vec{k}_0|b}{(1+a)(1+\eta k_0^2\rho_s^2)}\right)^{1/2}|\hat{z}\times k_0\cdot\hat{k}\phi_0|, \tag{13}$$

where $\gamma_D \gg \Gamma$ has been assumed.

Energy is pumped into the convective cells as well as long-wavelength drift waves by this process. Thus, this decay reinforces the process of large-scale eddy formation discussed by Hasegawa and Mima,[8,9] who considered decay within the convective motion of the drift waves. We emphasize that although (13) is similar in form (but different in the coupling coefficients) to the result obtained by Sagdeev *et al.*,[6] the physical processes involved are quite different as discussed earlier.

V. MODULATIONAL INSTABILITY

Here, we assume both sidebands are active and that the wavelength of the modulation is much smaller than that of the pump. The dispersion relation (12) becomes

$$(\omega^2 - \delta\omega^2)(\omega + i\Gamma) = \frac{4D^2 b\eta^2|\hat{z}\times\vec{k}\cdot\vec{k}_0|^2\vec{k}_0\cdot\vec{k}}{k^2(1+a)(1+\eta k_0^2\rho_s^2)}\delta\omega|\phi_0|^2, \tag{14}$$

where $\delta\omega = \omega_0 - \omega_{k_0} \ll \omega_0$ is the frequency shift. Instability occurs if $(\vec{k}_0\cdot\vec{k}\delta\omega) < 0$. For $|\omega| \gg \delta\omega$ and Γ, the growth rate is

$$\gamma_M = \frac{3^{1/2}}{2}\left(\frac{4D^2 b\eta^2|\vec{k}_0\cdot\vec{k}\delta\omega|}{(1+a)(1+\eta k_0^2\rho_s^2)}|\hat{z}\times\vec{k}_0\cdot\hat{k}\phi_0|^2\right)^{1/3}. \tag{15}$$

Thus, in general the decay instability is more important than the modulational instability. For interaction among drift waves of nearly the same wavelengths, we have

$$\frac{\gamma_M}{|\delta\omega|} \simeq 1.1\left(\frac{\gamma_D}{|\delta\omega|}\right)^{2/3}. \tag{16}$$

We note that the modulational instability discussed here differs from that of Mima and Lee.[7] They considered modulational instability of drift waves within the drift-wave spectrum, in that the drift-wave scaling $\tilde{n}_i^c/n_0 \sim e\psi/T_e$ is also assumed for the low-frequency motion.

VI. DISCUSSION

We have considered modulational and decay instabilities caused by interaction between drift waves and convective-cell motion. Inherent nonlinearities which produce mode coupling within the drift or convective-cell spectra are of higher order in ∂_t/Ω_i and are therefore not included in our stability calculation. We have shown that for the convective cells ($\tilde{n}_i^c/n_0 \ll e\psi/T_e$) observed in the simulations, the coupling coefficient between drift and convective-cell branches is smaller than that proposed previously and scales with completely different parameters. Using the parameters in Cheng and Okuda's simulation,[4,5] we obtain from (13) a convective-cell generation time of $\tau = 600\omega_{pe}^{-1}$, which is in much better agreement to the simulation result than those of Refs. 6 and 7.

The maintenance of an overall stationary spectrum can be visualized as follows. The energy pumped into the drift waves, for example, by the universal instability, is first transferred to the convective cells by the instabilities considered here. It is then lost by plasma diffusion due to viscosity or destruction of the organized convective-cell motion. To consider this problem one must include the mode-coupling terms in the convective cell equation. Such a calculation is rather complex and shall not be attempted here. Instead, we mention that rough estimate such as that of Sagdeev *et al.*[6] shows that enhancement factors (convective-cell energy density versus equilibrium thermal-energy density) on the order of 400 can occur and can thus cause considerable diffusion.

Inclusion of finite-β effects,[10] kinetic effects, magnetic shear, as well as the toroidal geometry[11] in the present investigation is in progress and the results shall be presented elsewhere.

ACKNOWLEDGMENTS

This work has been performed under the auspices of the Sonderforschungsbereich 162 "Plasmaphysik Bochum/Jülich". One of us (HUR) is grateful to the Deutscher Akademischer Austauschdienst for the award of a fellowship.

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