Comments on stimulated scattering of electromagnetic waves by electron Bernstein modes in a plasma

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(Received 11 August 1980)

Results of a previous work on the nonlinear scattering by electrostatic waves in a laser-produced plasma are reconsidered and corrected.

The nonlinear scattering of laser radiation by electron Bernstein modes in a plasma was recently studied by Sharma, Salimullah, and Tripathi.[1] They showed that such processes can be important in laser target irradiation experiments where strong magnetic fields are generated. Similar effects may also be found in incoherent scatter radar observations of the ionosphere.[2,3] Reconsidering the theory for the above-mentioned nonlinear effect, we shall, however, in this note find results which are significantly different from those of Sharma *et al.*.[1]

Let us here study the scattering by low-frequency electrostatic modes $(\omega, \vec{k})$, supposing that the frequency ω_0 of the incident electromagnetic wave $(\omega_0, \vec{k}_0)$ is much higher than the electron plasma frequency ω_{pe}, the electron gyrofrequency ω_{ce}, as well as ω. By generalizing the theory of Drake *et al.*,[4] one then finds[5]

$$\frac{1}{\chi_e} + \frac{1}{1+\chi_i} \approx k^2 \sum_{+,-} \frac{|\vec{k}_\pm \times \vec{v}_0|^2}{k_\pm^2 (k_\pm^2 c^2 - \omega_\pm^2 + \omega_{pe}^2 + i\nu\omega_{pe}^2/\omega_0)}, \tag{1}$$

where $\omega_\pm = \omega \pm \omega_0$, $\vec{k}_\pm = \vec{k} \pm \vec{k}_0$, c is the velocity of light, ν is the electron collision frequency, and $\vec{v}_0$ is the constant amplitude of the induced electron velocity in the pump field. The dispersion relation (1) is similar in form to Eq. (14) of Drake *et al.*,[4] and it is valid for scattering by most low-frequency longitudinal modes, as χ_e and χ_i now represent the electron and ion susceptibilities of a slightly nonuniform magnetized plasma. The lengthy expressions for those susceptibilities have been derived by means of linear theory and can be found in many textbooks.[6] It must, in addition, be stressed that previous authors[5] have studied Eq. (1) for magnetized plasmas. However, the equation has not, as far as the author knows, been used for investigations of the parametric excitation of the electron Bernstein modes[7] $\omega \approx n\omega_{ce}$, where $n \geq 2$.

Considering the particularly interesting case of resonant three-wave interaction (which here requires $k \simeq 2k_0 \cos\theta$, where θ is the angle between $\vec{k}$ and $\vec{k}_0$) and neglecting ν (i.e., the pump wave amplitude is supposed to be well above threshold) one finds[4] from (1) the instability growth rate

$$\operatorname{Im}\omega = \frac{v_0}{c}\left(\frac{2\omega_0}{\partial(\chi_e+\chi_i)/\partial\omega}\right)^{1/2} |1+\chi_i|\,|\sin\phi|\cos\theta, \tag{2}$$

where ϕ is the angle between $\vec{k}_-$ and $\vec{v}_0$.

Equations (1) and (2) can be improved by means of, for example, the "generalized ponderomotive force" method[8] in order to obtain expressions which are valid for arbitrary pump wave frequencies, but as the results[9] are rather complicated and as the basic assumptions in the beginning of this paper will cover our further analysis, we shall omit any lengthy formulas here, and instead turn our attention to the growth rate which was derived in Ref. 1 [Eq. (26)]. When comparing that expression, evaluated in the limit $\omega_0 \gg \omega_{ce}$, with (2), one finds that the results do not agree, however. This discrepancy can be explained by the fact that Sharma *et al.*,[1] when calculating the nonlinear high-frequency current density [Eq. (14)], have neglected the important nonlinear term $\vec{E}_{\omega_0} \cdot (\partial f_\omega/\partial\vec{v})$ in the Vlasov equation and instead retained the term $\vec{E}_\omega \cdot \partial f_{\omega_0}/\partial\vec{v}$.

Let us now look at the excitation of electron Bernstein modes, $n \geq 2$, assuming that the magnetic field is in the z direction, that a density gradient $(\partial n_0/\partial x = -\kappa n_0)$ exists in the x direction, and that the electron velocity distribution function is Maxwellian with thermal velocity v_T. Using Eq. (11.42) of Miyamoto[6] (to minimize the length of the formulas below, we also neglect gravitational fields, fluid velocities, and temperature gradients), we then write

$$\chi = \frac{\omega_p^2}{k^2 v_T^2}\left\{1 - \sum_{n=-\infty}^{\infty} I_n(b)e^{-b}\left[\omega - \omega^*\left(1 - \frac{n\omega}{b\omega_c}\right)\right] \int_{-\infty}^{\infty} \frac{F_z\, dv_z}{\omega - k_z v_z - n\omega_c}\right\}, \tag{3}$$

where I_n is the modified Bessel function of order n, $b = k_\perp^2 v_T^2/\omega_c^2$, $\omega^* = -k_y \kappa v_T^2/\omega_c$ is the drift frequency, and

$$F_z = (2\pi v_T^2)^{-1/2} \exp(-v_z^2/2v_T^2).$$

Following Sharma *et al.*[1] we can then in (2) approximate $[\partial(\chi_e+\chi_i)/\partial\omega]^{-1/2}$ by

$$[1+(k\lambda_D)^{-2}]^{-1/2}(\omega - n\omega_{ce})^{1/2},$$

where $\lambda_D = v_{Te}/\omega_{pe}$ is the Debye length.

Disregarding the angular factor in (2) one thus finds

$$\mathrm{Im}\,\omega \approx \left(\frac{2}{\pi}\right)^{1/4} \frac{v_0}{c} \frac{\omega_{ce}(k\lambda_D)^{-3/2}}{[1+(k\lambda_D)^{-2}]} \left(\frac{\omega_0}{\omega_{pe}}\right)^{1/2} \times \left[n + \frac{\kappa b}{k_y}\left(1 - \frac{n^2}{b}\right)\right]^{1/2}. \tag{4}$$

Using typical values for plasmas produced by Nd: glass lasers, e.g., $\omega_0 \simeq 1.8\times 10^{15}$ rad sec^{-1}, $\omega_{ce} \simeq 3\times 10^{13}$ rad sec^{-1}, $\omega_{pe} \simeq 3\times 10^{14}$ rad sec^{-1}, $v_0/c \simeq 10^{-3}$, $k\lambda_D \simeq 0.5$, $n = 5$, and $\kappa\lambda_D b \ll 1$, one then deduces $\mathrm{Im}\,\omega \simeq 10^{11}$ rad sec^{-1}, which is of the same order of magnitude as the growth rates obtained by Sharma *et al.*.[1]

Finally, it should be stressed that it is often very difficult to identify the electrostatic wave which is responsible for the scattering. Equation (2), which is valid for scattering by any longitudinal mode, tells us, however, that $\mathrm{Im}\,\omega$ is large when $(\omega\partial/\partial\omega)(\chi_e+\chi_i)$ is small. The sign of the last mentioned term determines the sign of the wave energy and waves with small energies can thus be efficient scatterers. The scattering by electron Bernstein waves, where $(\omega - n\omega_{ce})/\omega$ is small and the energy is large, will, however, be difficult to observe. In addition, I thus think that scattering by drift cyclotron modes could be of interest as the energies of those waves can be very small (when ω_e^* is of the same order of magnitude as $n\omega_{ci}$). Expressions (2) and (3) may then be used to explain the scattering of laser radiation[1] as well as other observations,[3] where density gradient effects on cyclotron waves previously have not been calculated.

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[9]J. Larsson and L. Stenflo, Beitr. Plasmaphys. 16, 79 (1976). The result, which generalizes Eq. (2) here [or Eq. (26) in Ref. 1] is $\mathrm{Im}\,\omega = [\omega(\omega_0-\omega)/W_\omega W_{\omega_0-\omega}]^{1/2}|V|$, where W is the wave energy and where the coupling coefficient V is defined by (2*e*), p. 80.