Behavior of classical particles immersed in the classical electromagnetic zero-point field

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This article presents a general analysis of some aspects of the interaction of classical particles with the classical electromagnetic zero-point field (cemzpf). The analysis provides a possible observational test for stochastic electrodynamics (SED). A convergence form factor derived semiclassically supports the narrow linewidth and related approximations of SED by introducing a typically sharp frequency cutoff. An extended classical charge monopole can then be shown to perform a simple jiggling motion under the influence of the cemzpf. Besides this motion (same as polarizable particles), monopolar particles also display a random walk in velocity space which leads them to ever-increasing translational kinetic energies. Hence, classical particles under the influence of the cemzpf display a conspicuous behavior because of the following well-known interrelated results: First, no velocity-dependent forces exist for classical particles moving exclusively through the cemzpf. Second, both monopolar and polarizable particles in SED are predicted to perform a random walk in velocity space due to the action of this field. Only collisions may provide a stopping mechanism. An analysis of the work of Boyer and others concerning particle collisions with walls, suggests the idea that collisions transfer energy from an unconfined gas of mutually colliding particles to the random field. Using this, a Fokker-Planck model for an unconfined gas of mutually colliding classical particles is constructed. It displays a universal equilibrium energy spectrum $E^{-\text{const}}$ for the gas particles under the cemzpf as seen from any point fixed to co-moving coordinates. Primary cosmic rays have such an energy distribution. This motivated the proposal of a zero-point field (zpf) cosmic-ray acceleration mechanism in a previous work. Such a proposal requires a careful examination. However, methodologically speaking, one should first examine the alternative possibility that the behavior predicted in SED for classical particles does not occur in nature. If that would happen to be the case, then SED and the cemzpf concept should be critically revised. That the cemzpf concept may apparently lead to difficulties, is seen by presenting a paradoxical example where a monopolar particle moving through the cemzpf is predicted to suffer an enormous frictional force due to the surrounding zpf. The prediction obviously violates the Lorentz invariance of the cemzpf energy density spectrum. But the paradox is easily resolved by realizing the improper ultrarelativistic behavior of the Lorentz-Dirac equation which is used in the example. Extreme care must then be exerted in the use of the equations of motion of classical charged particles when moving under the influence of a zpf. The search for internal contradictions in SED, not related with the well-known renormalization and other difficulties of classical electrodynamics, has so far been unsuccessful. This and several points of rigor here and elsewhere included, are enough to indicate that the conspicuous behavior of classical particles discussed here is correctly predicted from the assumptions of SED. It is therefore proposed that this predicted behavior may serve as an observational test for the validity of SED.

I. INTRODUCTION

In a recent review article Enz[1] has presented in a provocative way the problem of the existence of an electromagnetic zero-point field (zpf). This problem is discussed here from the classical point of view. A brief historical account follows. The proposal of a physically real emzpf goes back to Planck.[2] Nevertheless, it was Nernst[3] who emphatically stressed this idea. Nernst thought that at zero temperature there remained an energy per mode of $\hbar\omega$ in the cavity field instead of the $\frac{1}{2}\hbar\omega$ we consider today. Multiplying $\frac{1}{2}\hbar\omega$ by the density of modes gives the well-known divergent energy density spectrum of the zpf,

$$\rho(\omega)d\omega=\frac{\hbar\omega^3}{2\pi^2c^3}d\omega \; . \tag{1}$$

This divergent spectrum, with the implied infinite mass density and associated gravitational difficulties, lead Pauli to deny the reality of the zpf.[1] However, matters did not stay that way for long. Soon after, Welton[4] explained the Lamb shift of the $2P_{1/2}-2S_{1/2}$ energy levels of atomic hydrogen by introducing the heuristic idea of an interaction of the zpf fluctuations with the atomic electron. The Casimir[5] effect was next theoretically discovered[5] and later on experimentally[6] verified. This effect could also be presented as an attraction between two parallel plates induced by the em fluctuations of the vacuum. In a review article V. Weisskopf[7] again raised the zpf to the category of a real field. In the last few decades there have been emphatic endorsements of the idea of a real zpf. First, we recall the interesting speculations on the structure of the vacuum by Sakharov[8] and by Wheeler and co-workers.[9,10] Second, we mention the appearance of a new classical theory, called random or stochastic electrodynamics (SED), where a classical version of the zpf is presented. This theory has been extensively reviewed.[11-14] It essentially consists of introducing in the old Lorentz theory of the electron, instead of the traditional null homogeneous solution for the field potentials, a new homogeneous solution where it is assumed that there is a background of a randomly fluctuating radiation that is homogeneously distributed in space that is isotropic and that has a Lorentz-invariant energy density spectrum.[12] The associated random field has identical form in all inertial frames of reference. It can be shown[15,16] that this implies that the spectrum becomes exactly the same as the spectrum of the quantum theoretical zpf of Eq. (1). Consequently, this classical random field is called the classical electromagnetic zero-point field[12] (cemzpf). Although quantization is not introduced, Planck's constant does appear as the parameter that fixes the scale of the field.

It could be expected that advances in SED should come rather fast. However, that has not been the case. Due to mathematical and conceptual difficulties the rate of progress in this new theory has been rather slow. There are nevertheless a few interesting results. The Planck distribution without quantum assumptions has been derived.[16,17] Another important contribution was the classical derivation of the Casimir effect and of several other van der Waals forces.[18] It could furthermore be shown that various features of the quantum harmonic oscillator could be reproduced for the classical harmonic oscillator interacting with the cemzpf.[19-21] One of the outcomes of SED has been an elucidation of the unique properties of the zpf. The zpf is the unique spectrum with a Lorentz-invariant energy density.[15,16] It is the unique spectrum that remains invariant under an adiabatic compression of a cavity.[17] It is the unique spectrum that preserves as adiabatic invariants for charged systems the adiabatic invariants of some uncharged mechanical systems,[22] and it is the unique spectrum that does not give rise to frictional forces, i.e., to veloctiy-dependent forces (some aspects of this last statement are discussed at length in this work). Furthermore, the zpf produces a phase-space distribution of an ensemble of classical harmonic oscillators in the ground state that is stationary in time. The classical ensemble corresponds to the quantum mechanical oscillator in the ground state.[15]

In order to gain universal acceptance as a theory that may well complement and perhaps even compete with quantum theory, SED requires more relevant results. Among these, of course, is the solution of the problem of the hydrogen atom.[23] The theory also presents the difficulty of introducing as a very basic feature a field that has a divergent energy density spectrum.[24] This zpf has in SED the character of a real field in contradistinction with the purely formal virtual photon zpf of traditional quantum theory.

The typical attitude of physicists with respect to the existence of a physically real zpf is understandably cautious and ambiguous. Consider, for example, the presentation of Lautrup[25] when proving in a quantum fashion the Casimir effect. He takes two parallel plates of finite size and assumes that there is a zpf inside in order to show that a net attractive force therefore results between the plates. However, outside the inner spacing between the plates he denies the existence of a zpf.[25] Many other authors are nevertheless willing to invoke the ubiquitous existence of a zpf in order to explain certain quantum effects like spontaneous emission, etc., but naturally deny any real meaning to the divergent sum of energies over the modes $\sum \frac{1}{2}\hbar\omega_i = \infty$.

From this it follows that the notion of the structure and reality of an electromagnetic zero-point field is still unclear. Furthermore, studying the problem in all generality may require unsuspected advances in fundamental physics still not available at present. Given this state of affairs a more practical approach to the zpf problem in classical theory is to study critically some of the most conspicuous effects predicted in SED and to check if such predictions are seen experimentally. The more general study of the structure and properties of the vacuum should be left as a future task. Following this idea we select a particular phenomenon that has been theoretically predicted in SED and that does not have a known counterpart in quantum theory. Classically, at least, particles that suffer electromagnetic interactions are polarized under the action of an electromagnetic field. It has furthermore been shown that polarizable particles submitted to the action of the cemzpf steadily increase their translational kinetic energies by performing a random walk in velocity space.[16,26-28] The same phenomenon can also be shown to occur in simple monopolar particles (Appendix B). One may then surmise that all electromagnetically interacting classical particles perform a random walk in velocity space due to the action of the cemzpf. This growing energy trend is only stopped by collisions. Because of the strong acceleration, in a collision, a colliding particle returns back energy to the random background field.[12,16,17] It has also been shown that collisions may strongly inhibit the acceleration process and that only when the particle number densities are low enough does the acceleration process take off.[26] The interesting thing, however, is that the energy spectrum of a gas of low enough volume number density acted by the zpf should, if it exists, be observable under suitable conditions most likely present in astrophysical situations.

A description of the contents of this article follows. A simple classical analysis of monopoles is first performed. A purely monopolar particle submitted to the action of the cemzpf can be shown to display two kinds of motions. First, is performs a jiggling motion that is just a Brownian-motion-like perturbation of its classical trajectory. Second, and as occurs with polarizable particles, it displays a random walk in velocity space. Hence, both monopolar and dipolar particles in SED can be shown to perform a vibratory motion (dipolar vibrations for polarizable particles or jiggling motion for monopolar particles) and a superimposed translational motion with ever-increasing translational velocities. Both motions are caused by the cemzpf. A byproduct of the analysis of finite-size monopolar particles is a proof that, despite the divergent nature of the zpf energy density spectrum, there is only a finite energy available for the particle. This follows from a semiclassical derivation of a convergence form factor that is *equivalent* to imposing a frequency cutoff in the zpf energy density spectrum. This convergence form factor is very sharp and it serves to justify the narrow linewidth and other approximations of frequent use in SED.[29] *The sharpness* of the form factor is very important when considering the accelerating action of the zpf on monopolar particles.

Using a suitable model and the associated form of the Fokker-Planck equation, an energy spectrum of the form $E^{-\mathrm{const}}$ is derived for the case of an unconfined gas of mutally colliding classical particles interacting with the zpf. However, the effect of collisions with the cavity walls has been proposed by Boyer as responsible for maintaining equilibrium between the background random zpf and the polarizable particles in a gas confined to a cavity, where collisions with the cavity walls are only considered.[16,17] A revision of the collisionally induced equilibration process suggested by Boyer is presented. This idea is extended to the case of an unconfined gas of polarizable particles in equilibrium with the zpf, where collisions among the particles are considered instead.

Because of the Lorentz invariance of the spectrum no velocity-dependent drag forces can be induced by the cemzpf. The case of polarizable particles moving through the zpf has been studied by Boyer and others.[27,28] It was shown that no such em friction can occur for polarizable particles moving through the Lorentz-invariant zpf. However, for the case of monopolar particles there are some difficulties. A calculation based on the Lorentz-Dirac equation gives an enormaous em frictional effect for monopolar particles moving through the cemzpf. The predicted frictional effect may with confidence be attributed to a failure of the Lorentz-Dirac equation in ultrarelativistic regimes. This wrong result serves to warn that a correct classical treatment of the motion of charged particles in the vacuum may demand so far unspecified refinements of the theory.

It is concluded that the classical version of the zpf of Eq. (2) (see below) is not a complete version of the em aspect of the vacuum and that the energy spectrum for a gas of classical particles, predicted when the classical zpf background is taken for granted, should be examined in a critical fashion. It is true that, as previously proposed,[26] the $E^{-\mathrm{const}}$ energy spectrum may correspond to the well-known energy spectrum of cosmic-ray particles. However, for seriously considering a zpf cosmic-ray acceleration mechanism it is methodologically wise to search first for possible alternative astrophyisical explanations within conventional theory, where no resort to a field with a divergent energy density spectrum is required. The predicted conspicuous behavior of classical particles (random walk in velocity space, $E^{-\mathrm{const}}$ energy spectrum, etc.) is an exclusive result of SED that has no counterpart in quantum theory. It is proposed that such predicted behavior may serve to test the validity of the classical version of the zpf (version from which the predicted behavior rigorously follows).

Despite many interesting features of SED, one thing is certain: It is not a complete theory of nature. To start with, SED deals only with one of the interactions, namely, the electromagnetic. It does not consider gravitational phenomena. It also does not explain the high-energy phenomena of particle physics. In consequence, SED introduces a picture of the vacuum where only an emzpf is present. In other words, the purely geometrical model of the vacuum of classical theory is physically implemented in SED by imbedding it in the zpf. The picture does not include, though, other interesting properties of the vaccum. Omission is made, for example, of vaccuum polarizability which is an essential property of the vacuum for explaining several phenomena in particle physics. Hence we should limit our expectations concerning SED, although how much we have to limit them still remains to be seen.

II. CLASSICAL MONOPOLAR PARTICLES IN EQUILIBRIUM WITH THE CLASSICAL ELECTROMAGNETIC ZERO-POINT FIELD

We limit ourselves here to the case of a purely monopolar particle that interacts with the cemzpf and study the particle motion due only to the interaction of the random field with the zeroth charge moment of the particle. A discussion on the motion of polarizable particles (first charge moment) subjected to the action of the cemzpf is left for the next and the other sections. Let us first study then the problem of the monopolar particle.

For ideal point charges of zero volume the problem is not rigorously solvable as the whole divergent spectrum of Eq. (1) contributes to the field-induced translational energy of the charges. The divergent expressions that result are usually forced to converge by the introduction of *ad hoc* cutoffs in the spectrum[29] or, in the case of dipoles, by invoking a narrow linewidth approximation where the divergent part at high frequencies is neglected.[30] However, it can be rigorously shown (see Appendix A) that if classical particles of nonzero volume are instead considered, a very sharp convergence form factor is obtained in the spectral integral for the particle energy that is available from the field. This form factor guarantees the convergence of the expressions. Owing to its sharp cutoff character, which produces a cut at a fairly distinct frequency, for all practical purposes this form factor can be replaced by a suitable frequency cutoff.

Assume for simplicity a rigid spherical homogeneously charged classical particle of radius $R>0$. The cemzpf is a superposition of plane waves with random phases whose overall spectrum is Lorentz invariant. Boyer writes it as[16]

$$\vec{E}(\vec{x},t)=\left(\frac{V}{(2\pi)^3}\right)^{1/2}\sum_{\lambda=1}^{2}\int d^3k\,\hat{\epsilon}(\vec{k},\lambda)\,\mu(\omega_{\vec{k}})\times\cos[\omega_{\vec{k}}t-\vec{k}\cdot\vec{x}-\Theta(\vec{k},\lambda)], \tag{2}$$

where $[V/(2\pi)^3]^{1/2}$ is a normalization constant introduced here for dimensional purposes and V is the cavity volume, which for some purposes may be made arbitrarily large. For each wave vector $\vec{k}$ there are two mutually perpendicular directions of polarization

$$\hat{\epsilon}(\vec{k},\lambda)\cdot\vec{k}=0;\ \ \hat{\epsilon}(\vec{k},\lambda)\cdot\hat{\epsilon}(\vec{k},\lambda')=\delta_{\lambda\lambda'},\ \lambda,\lambda'=1,2\ . \tag{3}$$

The factor $\mu(\omega_{\vec{k}})$ sets the scale of the field

$$\mu^2(\omega)=\frac{c^3\rho(\omega)}{\omega^2}\ . \tag{4}$$

The function $\Theta(\vec{k},\lambda)$ represents a random phase that, for each set of $\vec{k}$ and λ values, takes an arbitrary value between 0 and 2π with a uniform probability distribution in that interval. The magnetic field $\vec{B}(\vec{x},t)$ results from replacing $\hat{\epsilon}(\vec{k},\lambda)$ by $[\vec{k}\times\hat{\epsilon}(\vec{k},\lambda)]/|\vec{k}|$ in Eq. (2).

Consider a free monopolar charge in free space. Let the particle at time $t<0$, be located at the origin $\vec{x}=0$, with zero velocity $\vec{v}=\dot{\vec{x}}=0$. Assume for simplicity that the zpf is turned on at $t=0$. After a sufficiently long time interval $t=\tau$, in the nonrelativistic approximation the particle acquires a momentum given by

$$m\vec{v}_\tau=\vec{\Delta}_\tau=\int_0^\tau e\vec{E}dt=\left(\frac{V}{(2\pi)^3}\right)^{1/2}e\int_0^\tau dt\sum_{\lambda=1}^{2}\int d^3k\,\hat{\epsilon}(\vec{k},\lambda)\,\mu(\omega_{\vec{k}})\cos[\omega_{\vec{k}}t-\vec{k}\cdot\vec{x}-\Theta(\vec{k},\lambda)]\ , \tag{5}$$

where, because $|\vec{v}|\ll c$, the magnetic field contribution is neglected. Equation (5) is derived from the Abraham-Lorentz equation,

$$m\dot{\vec{v}}=\Gamma m\ddot{\vec{v}}+e\vec{E}(\vec{x},t)+(e/c)\vec{v}\times\vec{B}(\vec{x},t)\ , \tag{6}$$

where $\vec{v}=\dot{\vec{x}}$, and

$$\Gamma=\frac{2e^2}{3mc^3}\ . \tag{7}$$

In a first approximation and for nonrelativistic velocities we can neglect the effect of the magnetic field. We can also neglect the radiation reaction term. This last approximation can be performed provided the spectrum of relevant frequencies is such that $\Gamma\omega\ll1$. It can easily be shown that this condition, which incidentally is required also for the validity of the Abraham-Lorentz equation itself, is obeyed in the present case (see Appendix A). The time integration in Eq. (5) is performed first and then after squaring and averaging over the random phases we obtain the translational kinetic energy

$$\langle\epsilon\rangle\equiv\frac{\langle\vec{\Delta}^2\rangle}{2m}=\frac{e^2}{m}\int d^3k\,\gamma(\omega_{\vec{k}})\,\frac{\mu^2(\omega_{\vec{k}})}{\omega_{\vec{k}}^2}[1-\cos(\omega_{\vec{k}}t)]\ , \tag{8}$$

where $\gamma(\omega_{\vec{k}})$ represents the corresponding convergence form factor derived in Appendix A. The averaging is carried out by means of the relations[16]

$$\langle\cos[\omega_{\vec{k}}t-\vec{k}\cdot\vec{x}-\Theta(\vec{k},\lambda)]\cos[\omega_{\vec{k}'}t-\vec{k}'\cdot\vec{x}-\Theta(\vec{k}',\lambda')]\rangle=\frac{(2\pi)^3}{V}\frac{1}{2}\delta_{\lambda\lambda'}\delta(\vec{k}-\vec{k}'), \tag{9}$$

$$\langle\cos[\omega_{\vec{k}}t-\vec{k}\cdot\vec{x}-\Theta(\vec{k},\lambda)]\sin[\omega_{\vec{k}'}t-\vec{k}'\cdot\vec{x}-\Theta(\vec{k}',\lambda')]\rangle=0, \tag{10}$$

$$\langle \cos[\omega_{\vec{k}} t - \vec{k} \cdot \vec{x} - \Theta(\vec{k}, \lambda)] \rangle = 0 \,, \tag{11}$$

with corresponding expressions for the sines. Replacing the form factor by the cutoff at ω_m we then obtain

$$\langle \epsilon \rangle = \frac{2e^2 \hbar}{\pi m c^3} \left(\frac{\omega_m^2}{2} + \frac{1}{\tau^2} \{ [1 - \cos(\omega_m \tau)] - (\omega_m \tau) \sin(\omega_m \tau) \} \right). \tag{12}$$

Observe that $\Gamma\omega \leq \Gamma\omega_m \ll 1$. The time ω_m^{-1} is of the order of the time required for a light ray to travel a distance equal to the size of the particle. It is then reasonable to consider times τ much larger than ω_m^{-1}. If $\omega_m \tau \gg 1$, Eq. (11) yields[31]

$$\langle \epsilon \rangle = \frac{4\pi e^2 \hbar}{m c \lambda_m^2} \,, \tag{13}$$

where $\lambda_m = 2\pi c/\omega_m$ (Appendix A). The rigid monopolar particle performs a jiggling motion with a finite well-defined average energy. The motion is slightly reminiscent of the Zitterbewegung of quantum theory that is present between positive and negative energy states in the Dirac equation for the electron. However, this similarity is most likely not meaningful. The classical equation (13) has, however, an exact counterpart in quantum electrodynamics. It corresponds to the transverse self-energy of the electron under the electromagnetic fluctuations of the vacuum.[32]

Satisfying as it may seem, because of its correspondance with a phenomenon derived in quantum theory, the result of Eq. (13) is, however, incomplete. On the one hand Eq. (13) implies that the particle has an infinite memory of its initial velocity state, despite the fact that the particle is being submitted to the perennial action of a random force. This fact is not entirely satisfactory. On the other hand, polarizable particles have been predicted to perform, according to SED, a random walk in velocity space with ever-growing translational kinetic energies. This growing translational kinetic energy trend is an additional motion superimposed on the internal vibratory motion of the dipole caused by the cemzpf. The vibratory motion of the dipole for a polarizable particle in a sense corresponds to the jiggling motion of the monopole. However, there should be an additional correspondance that should be physically expected also for monopoles; namely as in the case of polarizable particles, monopolar particles should also perform a random walk in velocity space to ever-increasing translational kinetic energies. However, this last effect is missing from our analysis above.

The clue to the missing prediction of the above derivation was very recently given to us by Boyer.[33] Observe that in the above derivation and under the assumption that $v/c \ll 1$ we neglected the last term of Eq. (6). However, in the case of polarizable particles, it is precisely this term that is responsible for the random walk in velocity space. This consequently means that the random walk in velocity space is a distinct relativistic effect that until the present has not been uncovered, due to the usual neglect of the magnetic field in the treatment of classical equations for the motion of charge monopoles under random em radiation.

Consider a monopolar particle of charge e, mass m, and velocity $\vec{v}_t$ at time t. Let it be submitted to the action of thermal plus zpf radiation ($T > 0$):

$$\rho(\omega, T) = \frac{\omega^2}{\pi^2 c^3} \left(\frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} + \frac{1}{2}\hbar\omega \right) \tag{14}$$

[which for $T = 0$ reduces to Eq. (1)]. In three dimensions, and if no walls confine the system of particle and radiation, we may write

$$m\vec{v}_{t+\tau} = m\vec{v}_t + \vec{I}_\tau \,, \tag{15}$$

where τ represents a short enough time interval, during which the particle state of motion does not substantially change, but which is much longer than typical oscillation periods of the fluctuating field. The net impulse acquired during this time interval we denote by $\vec{I}_\tau$. Traditionally, this impulse has been broken up into two separate parts:

$$\vec{I}_\tau = \vec{\Delta}_\tau + \vec{\phi} \,, \tag{16}$$

where $\vec{\Delta}_\tau$ represents a fluctuating part and $\vec{\phi}$ a velocity-dependent drag force. In Appendix B we present a detailed proof that to first order in v/c it follows that

$$\vec{\phi} \equiv \langle \vec{I}_\tau \rangle = -P\tau\vec{v}_t \,. \tag{17}$$

It also follows that for the case of the cemzpf alone [$T = 0$ in Eq. (14)], to first order in v/c we have $P = 0$. Owing to the Lorentz invariance of the field energy density spectrum, it should be expected that $\vec{\phi} = 0$ for the cemzpf case to all orders in v/c. This proof is also included in Appendix B.

Although the average value of $\vec{\Delta}_\tau$ is equal to zero, $\langle \vec{\Delta}_\tau \rangle = 0$, it is shown, however, that as in the case of polarizable particles the average of the square of $\vec{\Delta}_\tau$ grows with the time τ in such a manner that the translational kinetic energy of the monopolar particle grows at a rate (Appendix B)

$$\Omega \equiv \frac{dE}{dt} = \frac{\langle \vec{\Delta}_\tau^2 \rangle}{2m\tau} \cong \frac{3}{5\pi} (\Gamma\omega_m)^2 \left(\frac{\hbar\omega_m}{mc^2} \right) (\hbar\omega_m)\omega_m \,. \tag{18}$$

For a more precise expression in the case of homogeneously charged spherical particles we can use the form factor $\gamma(\omega)$ of Appendix A instead of the simple cutoff at ω_m [see Eq. (B.15)]. However, Eq. (18) is enough to show the very strong dependence of Ω on the highest frequency ranges. As an example for the case of electrons, Eq. (18) reads

$$\Omega = 5\times 10^{-84}\omega_m^5 \ , \tag{19}$$

which for an ω_m of about 10^{18} s^{-1} would give a reasonable value of $\Omega \cong 10^6$–10^7 eV s^{-1}. However, if we use $\omega_m = \omega_c = 2\pi c/\lambda_c = \pi c/R$; $R = \Gamma c$ (Appendix A), then $\omega_m \simeq 5\times 10^{23}\,s^{-1}$, which gives for Ω the unreasonable value of $\Omega \cong 10^{35}$ eV s^{-1}. Furthermore, experiments in particle physics have been unable to determine a radius for the electron. This indicates that due to the very strong dependence on the highest frequency ranges available for the transmission of purely translational motion to the particle, the rate of translational energy growth Ω is very strongly dependent on the geometrical characteristics of the particle model. However, for the most typical charge monopole known, which is the electron, no definitive model has until the present been found, and hence a determination of Ω, even within a much more detailed formulation than the one here, is premature.

III. ENERGY SPECTRUM OF AN INFINITE GAS OF MUTUALLY COLLIDING CLASSICAL PARTICLES IN EQUILIBRIUM WITH THE CEMZPF

Here we search for the energy spectrum of an unconfined ultrarelativistic gas with infinitely many mutually colliding classical particles in equilibrium with the cemzpf. For simplicity of presentation, we deal only with the case of polarizable particles. The treatment of monopolar particles can be performed in a very analogous manner with correspondingly analogous results. The energy spectrum of a gas of polarizable particles in equilibrium with the cemzpf that will be obtained here was previously derived[26] in a way that did not connect with the detailed calculations of Boyer for the collisionless one-dimensional case.[27,28] The present revision attempts such a connection. The complexity of the problem where particle collisions are considered demands several simplifying assumptions. A Fokker-Planck model is therefore used. Both the nonrelativistic and the relativistic one-dimensional gases of collisionless particles in equilibrium with the cemzpf have been studied by Boyer in considerable detail.[16,17,27,28]

Boyer gives the net fluctuating impulse transmitted to the polarizable particle in a short time interval τ. He restricts the motion of the particle to the x axis and the direction of vibration of the dipole along an axis parallel to the z axis. He then obtains

$$\langle\Delta_\tau^2(\beta)\rangle = \gamma\langle\Delta^2(0)\rangle \ , \tag{20}$$

and

$$\langle\Delta_\tau^2(0)\rangle = \tau\,\frac{4}{5}\,\Gamma_q\,\frac{\pi^4 c^4}{\omega_0^2}\,\rho^2(\omega_0) \ , \tag{21}$$

where $\beta = v/c$, $\gamma = (1 - v^2/c^2)^{1/2}$, and ω_0 is the characteristic frequency of the dipole. Γ_q is the radiation damping constant for polarizable particles.[26] Equations (20) and (21) can be extended to the full three-dimensional case, i.e., for both the particle translational motion and the internal dipole vibration. For the relativistic and nonrelativistic cases one obtains, respectively,

$$\langle\vec{\Delta}_\tau^2(\beta)\rangle = \gamma\langle\vec{\Delta}_\tau^{\,2}(0)\rangle \tag{22}$$

and[26]

$$\langle\vec{\Delta}_\tau^2(0)\rangle = 6\pi^4 c^4\Gamma_q\,\frac{\rho^2(\omega_0)}{\omega_0^2}\,\tau \ , \tag{23}$$

where instead of the numerical factor 27/5 we have the number 6, which results from an additive correction of 3/5 to the value previously reported.[34] This correction comes from a missing term in the integration over the angles. We study next the stationary distribution of the infinite unconfined gas of polarizable particles in equilibrium with the cemzpf. As the distribution should be homogeneous, there cannot be any space dependence. The probability distribution $W = W(\vec{p})$ is independent of $\vec{r}$. The particle momentum $\vec{p}$ is measured with respect to an inertial reference frame. More on that inertial frame is discussed below. The Fokker-Planck equation is[35]

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial\vec{p}}\left[\left(\frac{1}{\tau}\langle\Delta\vec{p}\rangle W\right) - \frac{1}{2}\frac{\partial}{\partial\vec{p}}\left(\frac{1}{\tau}\langle(\Delta\vec{p})^2\rangle W\right)\right] \ . \tag{24}$$

Because of the stationarity $\partial W/\partial t = 0$ and because of the isotropy of the problem, W should be independent of the direction of the $\vec{p}$ vector, i.e., of $\theta_{\vec{p}}$ and $\phi_{\vec{p}}$. Hence, $\partial/\partial\vec{p} = \hat{i}_p(\partial/\partial p)$. The problem becomes one-dimensional since only one variable, namely $p = |\vec{p}|$, appears. For this case it is known that the assumption of detailed balance[35] can safely by made.[36] There are hence no probability currents $\vec{J}_{\vec{p}}$ in momentum space and, therefore,[36]

$$\vec{J}_{\vec{p}} = \frac{1}{\tau}\,\langle\Delta\vec{p}\rangle\,W - \frac{1}{2}\,\frac{\partial}{\partial\vec{p}}\left(\frac{1}{\tau}\,\langle(\Delta\vec{p})^2\rangle\,W\right) = 0 \ . \tag{25}$$

The average change of momentum during a time interval of duration τ, $\langle \Delta\vec{p}\rangle$ has two parts: one due to the zpf, which has been rigorously shown to be strictly equal to zero,[37] and the other which is due to collisions with the surrounding particles of the gas. In order to be able to incorporate these very strong collisions into the Fokker-Planck formalism we have to introduce a model. A collision of two particles at high energies is a complex process where the two original particles are annihilated and several other unstable particles and radiation cascade out from the initial event. Viewed in momentum space, and if equilibrium and stationarity are to be maintained, the collision in our model should look as follows. Two particles that are far away from the origin disappear. They are replaced by two other particles that spring forth from the neighborhood of the origin. The kinetic energy loss of the collision eventually transforms into random electromagnetic radiation. A "black box approach" is hence used. A more detailed description of the collision process is beyond the phenomenology required by the thermodynamic level of our model. If we assume that a collision completely stops the colliding particle it is easy to see that

$$\langle \Delta\vec{p}\rangle = -\hat{i}_{\vec{p}}\, p\sigma(\vec{p})\beta c\tau n \ , \tag{26}$$

where $\sigma(\vec{p})$ is the collision cross section at momentum $\vec{p}$ and n is the volume number density of the gas particles. The average momentum change $\langle \Delta\vec{p}\rangle$, between t and $t+\tau$ has two contributions, one due to the zpf whose average value is zero,[37] and the other due to collisions, which obviously depends on the value of $\vec{p}$ at time t. The Fokker-Planck formalism requires that the momentum change occurring during the time interval τ be very small in comparison with the value of $|\vec{p}| = p$. It can be assumed then that instead of occasional sharp collisions the effect of the other gas particles is replaced by a fictitious surrounding thin viscous fluid whose average drag force is as given by Eq. (26). For the other term in Eq. (25) we know that

$$\langle (\Delta\vec{p})^2\rangle = \langle \vec{\Delta}_\tau^2(\beta)\rangle \ . \tag{27}$$

Observe that Eqs. (26) and (27) adjust well to the first and second transition moments given by Eq. (225) of Chandrasekhar.[35] From Eqs. (23), (25), (26), and (27) we obtain a reduced version of the Fokker-Planck equation for the energy

$$\frac{nc}{\Omega}\,\sigma(E)WE + \frac{d}{dE}(WE) = 0 \ , \tag{28}$$

where, because the particles are ultrarelativistic, $E = pc$. We introduce again the constant[26]

$$\Omega = \frac{\langle \vec{\Delta}^2(0)\rangle}{2mc\tau} \ , \tag{29}$$

where m is the rest mass of the gas particles. After integration the model then gives

$$W(E) = \frac{\text{const}}{E}\exp\left(-\frac{nc}{\Omega}\int_{E_0}^{E}\sigma(E')dE'\right) \ . \tag{30}$$

This distribution, despite the special assumptions that the model implies, is similar to the previous result,[26] differing only in an additional E^{-1} factor. For the case of collisionless particles, $\sigma = 0$, we recover the result of Boyer[27,28] for an infinite unbounded relativistic gas, $W \sim E^{-1}$. It follows that in asymptotia, which is the spectral region where there are some observational possibilities (Sec. IV. C), we obtain a particle current density at energy E and per unit energy of the form[38]

$$J(E) \sim E^{-\text{const}} \ . \tag{31}$$

The exponent is

$$\text{const} = 1 + \frac{\eta\alpha^2 n}{2\Omega mc^2} = 1 + \frac{\eta(3ec)^2 Mn}{(\hbar\omega_0)^4} \ , \tag{32}$$

with const $\to 1$ if $n \to 0$, where η is a constant coming from scale invariance in asymptia,[26] α is the fine-structure constant, and M is a parameter with the dimensions of mass previously discussed[26] in more detail. For large number densities Eq. (32) reduces to the previous result. As before,[26] it follows that if n is not small enough the whole spectrum is quenched because of strong exponential attenuation.

It is interesting to note that we obtained a one-dimensional Fokker-Planck equation which does not allow for probability accumulations anywhere, not even at infinity. We used the assumption of zero probability currents, Eq. (25). This assumption is formally justified by the *potential condition*[36,39] trivially obeyed in stationary one-dimensional cases, and in particular in this case as

$$W(E) \sim E^{-\text{const}} \ .$$

Last but not least, the inertial frame of reference with respect to which the particle momenta $\vec{p}$ are measured has to be specified in more detail. For the introduction of a well-defined unique frictional force in Eq. (26), such an inertial frame has to be given by the average local motion of the fictitions vicous fluid introduced above. Such a viscous fluid can then be made to coincide with a cosmological fluid of an expanding universe.[40,9] The fluid then defines locally a preferred frame of reference which is the frame with respect to which the average momenta of the surrounding matter is zero. The expanding cosmological fluid defines everywhere the so-called co-moving coordinates, i.e., a special coordinate system that expands with the universe and with respect to which the average local motion of the cosmological fluid is zero.[40] For any two separate points (galaxies) in the cosmological fluid, the corresponding coordinate interval, as given by the co-moving coordinates, remains constant and the expansion of the universe results, not from any change in the coordinates position of the points (galaxies), but rather from a change in the metric of the space.

The co-moving coordinates define at every point, then, a local preferred inertial frame of reference with respect to which the distribution W of polarizable particles should be homogeneous and isotropic. The distribution is not necessarily, though, homogeneous and isotropic, when observed from a different inertial frame. Locality is understood here in the cosmological sense: The size of the relevant region should be small in comparison with the Hubble length; but it should, of course, be large with respect to the mean free path of the colliding particles of the polarizable gas. It is an easy task to check that, for an intergalactic gas with $n \simeq 10^{-5}$–10^{-6} cm^{-3}, these conditions are met within wide margins.[26]

Something similar to the thermal 2.7-K background radiation occurs with the preferred local frame. The distribution appears homogeneous and isotropic as seen from the preferred local frame that co-moves with the cosmological fluid. The particle distribution, as well as the 2.7 K background radiation, are not, however, in general Lorentz invariant, and they should not be expected to be so. Only, until when the particle density becomes very small [see Eq. (28)] and no collisions take place, does the distribution recover the Lorentz-invariant character, $W \sim E^{-1}$, recently uncovered by Boyer.[27,28] Thus the $n \to 0$ limit for particles [see Eq. (28)] is analogous to the $T \to 0$ limit for thermal radiation. In each one of those limits the respective distribution becomes Lorentz invariant.

IV. DISCUSSION

Here we comment on some general features of the above results with the related thermodynamic aspects and ensuing observational consequences.

A. General aspects—the thermodynamic problem

In the previous section we found the energy spectrum of an infinite gas of mutually colliding classical particles that are in equilibrium with the cemzpf. Attention was focused on the high-energy tail of the distribution which is presumably the only part of the spectrum able to reach observers in galactic interiors. Were it not for collisions the E^{-1}, spectrum of Boyer[27,28] for an infinite gas of collisionless particles in equilibrium with the zpf would have been found.

In a collisionless gas of unconfined free particles the zpf, as viewed in SED, should force the particles to diffuse out to infinity. Simultaneously, the particles are predicted to perform a random walk in velocity space to ever-increasing average translational kinetic energies.[16,17,26] Energy is thus transferred one way only, from the field to the particles, and no equilibrium spectrum may thus be found.[27,28] It is in the case of an infinite gas, when the particles uniformly fill all space, that an equilibrium spectrum can be obtained.[27,28] For a finite gas, though, equilibrium may occur only if the gas is confined to a cavity. When, in addition to the zpf, temperature radiation is also present, the effect of the walls is to impose a radiation distribution with a preferred frame of reference, namely the Planck distribution with the zpf term[16,17] [Eq. (14)]. We study here a slightly different case. The gas has an infinite number of particles and is unconfined, but the particles are allowed to collide. Collisions are manifested furthermore in producing an additional attenuation in the energy spectrum which is of the form $E^{-\text{const}}$, const ≥ 1 (const $= 1$ only for the collisionless case). For the case of an infinite unconfined collisionless gas, energy moves one way only from the field to the particles and there is no return energy path. In our Fokker-Planck model for an infinite gas, particles collide among themselves. They are assumed to be stopped by the collisions.

The picture of the equilibration mechanism just described is based on the idea, first introduced by Boyer,[16] that for a finite gas of confined classical particles that do not collide among themselves, but that collide with the surrounding walls, equilibrium is restored by the action of the cavity walls which accelerate particles during the collisions and force them to radiate. However, the equilibrium mechanism of the cavity walls has recently been criticized by Jiménez, de la Peña, and Brody.[41] The main point of their criticism may nevertheless be avoided by a rather minor change in the set of assumptions proposed by Boyer in his treatment of the cavity walls.[16,17]

B. The cavity walls

From the above it follows that, for our equilibration mechanism by particle collisions of Sec.

III, the importance of the problem of collisions with the cavity walls need not be over emphasized here. The thermodynamic analysis of the particle collisions with the cavity walls is also crucial in the proof[16] that the Planck distribution for thermal radiation can be obtained without quantum assumptions. We proceed then to a re-analysis of the problem.

In free space a polarizable particle suffers momentum fluctuations that satisfy,[16,17] as in the case of monopolar particles, Eqs. (15)–(17). The corresponding expression for P can be found in the literature[16,26]; it is, of course, not the same as the one for monopolar particles of Eq. (B.25) and (B.26). A one-dimensional analysis suffices. Squaring and averaging in Eq. (15) and deleting the vector notation, one obtains

$$\langle (mv_{t+\tau})^2\rangle = \langle (mv_t)^2\rangle + \langle I_\tau^2\rangle , \tag{33}$$

and the energy runs one way only, from the field to the particle. Equilibrium does not occur. In the case of confinement within a cavity, the cavity walls serve to restore equilibrium.[16,17] Another term is then added to account for the effect of the wall collisions:

$$mv_{t+\tau} = mv_t + I_\tau + J , \tag{34}$$

where J is the impulse transmitted in a collision, if there happens to be one during the time interval τ. At $T=0$, $I_\tau = \Delta_\tau$. After squaring and averaging and assuming a steady state $\langle (mv_{t+\tau})^2\rangle = \langle (mv_t)^2\rangle$, one obtains

$$0 = 2\langle mv_t\Delta_\tau\rangle_0 + 2\langle mv_t J\rangle_0 + 2\langle \Delta_\tau J\rangle_0 + \langle J^2\rangle_0 + \langle \Delta_\tau^2\rangle_0 . \tag{35}$$

At $T>0$, $I_\tau = \Delta_\tau - Pv_t\tau$, and Eq. (35) becomes

$$0 = 2\langle mv_t\Delta_\tau\rangle_T + 2\langle mv_t J\rangle_T + 2\langle J\Delta_\tau\rangle_T + \langle J^2\rangle_T + \langle \Delta_\tau^2\rangle_T - [\langle mv_t^2\rangle_T + \langle v_t\Delta_\tau\rangle_T + \langle J v_t\rangle_T]2P\tau + \langle v_t^2\rangle_T P^2\tau^2 . \tag{36}$$

Boyer proposes then the equations

$$\langle mv_t\Delta_\tau\rangle_T = \langle mv_t\Delta_\tau\rangle_0 = 0 \tag{37}$$

and

$$\langle J\Delta_\tau\rangle_T = \langle J\Delta_\tau\rangle_0 = 0 \tag{38}$$

that follow from the random character of the incoming radiation that is uncorrelated with the actual dynamical state of the particle or of the walls. Furthermore, he correctly argues that

$$\langle mv_t J\rangle_0 < 0 . \tag{39}$$

τ can be made small enough so that

$$P\tau/m \ll 1 . \tag{40}$$

Boyer then assumes that if the cavity is large enough, then

$$\langle J^2\rangle_0 \ll \langle \Delta_\tau^2\rangle_0 . \tag{41}$$

He finally proposes that

$$\langle mv_t J\rangle_0 = \langle mv_t J\rangle_T , \tag{42}$$

arguing that, in the case when only purely thermal energy is involved, there should not be a net transfer of energy between the field and the gas particles at the walls. After collisions with the walls, particles should emerge with a spectrum of kinetic energies the same as the entering particles, except for the crucial contribution of the zero-point energy which can be removed only at the walls. (It is precisely the last two equations, namely, Eqs. (41) and (42), that Jiménez and coworkers[41] claim are invalid.) Using Eqs. (41) and (42) we then obtain

$$2\langle mv_t J\rangle_0 = -\langle \Delta_\tau^2\rangle_0 , \tag{43}$$

and introducing Eqs. (37) to (43) in Eq. (36) we finally get

$$\langle \Delta_\tau^2\rangle_T - \langle \Delta_\tau^2\rangle_0 = 2Pm\tau\langle v_t^2\rangle_T , \tag{44}$$

as on the average J can be neglected as compared with mv_t.

If the values for $\langle \Delta_\tau^2\rangle$ and P of the model of Einstein and Hopf[42] are replaced in Eq. (44), the Planck spectrum with the zero-point term is obtained.[16] Turning to the objection of Jiménez and co-workers, their argument runs as follows: Equation (41) is not compatible with Eq. (43). Let K be the average momentum exchange for a particle per collision; then, $K = -2[\langle (mv_t)^2\rangle]^{1/2}$, provided collisions with the walls do not cause radiation and kinetic energy loss. However, if Boyer's supposition holds, then K will be less in absolute value, but it cannot be smaller than $-[\langle (mv_t)^2\rangle]^{1/2}$; at this value the particles stick to the walls. Therefore, $\langle J^2\rangle = fK^2$, where f is the fraction of intervals of the same duration τ

during which a collision occurs. However, an analogous argument also gives $\langle mv_t J\rangle \approx -fK^2$. Now, f decreases with the cube root of the volume of the cavity. Hence one can choose a cavity of such a size that Eq. (41) is satisfied or, alternatively, Eq. (43), but not both. Moreover, f depends on the temperature, as follows from elementary considerations. Hence it is unlikely that Eq. (42) is satisfied also. The difficulty is, however, not grave. A rather minor change in the proposed expressions, Eqs. (41) and (42) and the ensuing Eq. (43), safely avoids the difficulty.

Consider the net momentum loss l caused by a wall collision. It is easy to see that $l = 2mv_t + J$. From simple kinetic considerations it follows that K and f, and hence $\langle J^2\rangle$, are increasing functions of the temperature. We then write $(\langle J^2\rangle)^{1/2} \sim g(T)$ where $g(T)$ is an increasing function of its argument. From the discussion above one knows that l is instead a decreasing function of the temperature.

Assume that $l \sim 1/g(T)$. With this matching it occurs then that $\langle lJ\rangle$ is indeed a constant independent of temperature. We then have

$$\langle (2mv_t + J)J\rangle_T = \langle (2mv_t + J)J\rangle_0 \,. \tag{45}$$

If we assume this relation, instead of Eqs. (41) and (42), we can in a more direct manner than before obtain Eq. (44) without ever needing Eq. (41). And from Eq. (43) the Planck distribution with the zpf term follows in the usual way.[16,11]

Observe furthermore that if l indeed is a decreasing function of temperature, $l \sim 1/g(T)$, the detection of any wall effect is much easier at low temperatures than at high temperatures. It thus may come as no surprise that such a wall effect is not easily measurable in a direct manner in cavities at ordinary temperatures. Despite all claims to the contrary, it is clear that an energy loss by negative acceleration in a collision process has to take place at least within the context of a classical theory for particles with distributed charges. Boyer's proposal is that the radiation coming from such an energy loss is so random that it can be identified with the radiation with the maximum degree of disorder, namely, with the cemzpf.[12,17]

If collisions with the cavity walls do indeed serve to maintain an energy balance between the translational energy of the particles and the zpf, it is natural to think that the same role is played by collisions in the classical case of an unconfined gas of mutually colliding particles in equilibrium with the cemzpf.[26] It may be argued,[41] however, that at least part of the energy radiated in collisions is thermalized at a higher temperature, and hence that equilibrium is broken because of the ensuing net transfer of field energy to the gas.[41] This net transfer would then imply an imbalance in energy, causing a continuous warming up of the gas. However, this unbalance cannot occur. Because of the second law of thermodynamics, there should exist some energy-balancing mechanism so that there is no net transfer of energy from a cold reservoir (field at $T=0$) to a hot reservoir (gas at $T>0$). Collisions are the only possible balancing mechanism.[26] Observe furthermore that this balancing mechanism seems crucial for the establishment of the equilibrium Fokker-Planck model of the previous section.

C. Astrophysical speculations—high-energy cosmic rays and hot intergalactic gas

If our naive version of the zpf does indeed take place in physical reality, and if the field can be isolated from other properties of the vacuum[9,10] so that it may be represented as in Eq. (2), it then necessarily follows that there is a classical accelerating mechanism for increasing the translational kinetic energy of charged or polarizable classical particles. For the second case, this mechanism has been extensively studied[16,17,11,26-28] and gives for an infinite unconfined gas an energy spectrum with a particle flux of the form $J(E) \sim E^{-\text{const}}$, const ≥ 1, the equal sign holding for a gas of collisionless particles.

If, as argued above, the accelerating effect is also present in monopolar particles it may occur in particles other than protons, i.e., electrons. For the case of protons the polarizability has been measured, and it is good enough for the satisfactory application of the mechanism.[26] For this case, the constant in the energy exponent depends on a parameter η originated in some high-energy theorems of asymptotia and on another parameter whose value remains unknown.[26] Our constant in the energy exponent cannot thus be explicitly evaluated. This is a handicap since observationally one does not know what value of the exponent to search for. There is, however, the well-known spectrum of primary cosmic rays whose shape, in rather wide energy ranges, goes as $E^{-\text{const}}$. (For the largest range 10^{11}–10^{16} eV, const $\simeq 2.8$, with not widely different values for other ranges and always with const > 1.) Apparently, this spectrum is in support of our cemzpf. There are, however, several serious attempts to explain the origin of the cosmic-ray spectrum from more traditional model sources, i.e., supernova explosions, pulsars, etc. (see Ref. 26 for further references). One has to wait for more conclusive answers from those attempts to materialize.[43] If it turns out that one or several of them are enough to explain satisfactorily the primary cosmic-ray spectrum, it would then follow that the cemzpf gives a deceptive $E^{-\mathrm{const}}$ prediction, and hence that the cemzpf does not give a good enough picture of the whole physical aspect of a more complex vacuum. In that case, some still unidentified changes would have to be made in the classical formulation of the vacuum, and presumably a certain generalization of SED (beyond purely electromagnetic theory) as a consequence would result. SED includes also only a very simple *ad hoc* phenomenological particle description whose naive character may preclude a correct prediction for the behavior of particles in several physical regimes. It is then, of course, venturesome at present to point out in which direction modifications of the cemzpf concept or of the model of charged particles will have to be carried out. Let us recall again the interesting considerations of Wheeler and others[9,10] on geometrodynamics that might give a clue for the construction of a more complete classical model of the vacuum. If, however, the cosmic-ray energy spectrum cannot be satisfactorily explained in terms of traditional model sources, we should then conclude that the cemzpf of Eq. (2), despite the naïve aspect of its formulation, which includes a divergent spectrum, may for some purposes at least adequately represent the physical structure of the vacuum. Observe[26] also that, in favor of a real zpf effect on particles in the intergalactic medium, there is the well-known x- and γ-ray background radiation, which implies a hot intergalactic medium of about 4×10^8 K. Despite some previous reviews where the reality of a very hot intergalactic gas is not taken as established,[44] there are very recent experimental measurements[45] that indicate that a hot intergalactic gas would much more easily explain the available data than other alternative interpretations. However, for definitively ruling out or accepting a hot intergalactic gas, we have to wait for more conclusive results, most likely to become available from the recently launched Einstein Observatory.

V. CLOSING REMARKS

Although interesting, the idea of a physically real zpf (as opposed to virtual), that is, Lorentz invariant and classically representable [as in Eq. (2)], is not free from difficulties. These cemzpf difficulties appear in conjunction with the use of equations of classical charged-particle motion. To illustrate the issue we present the following example. Because of the Lorentz invariance of the zpf, there should be no zpf induced frictional forces for a monopolar particle that moves through the zpf (Appendix B). However, if instead of the equation of Abraham Lorentz, as in Appendix B, the Lorentz-Dirac (LD) equation is used, it can be shown that, formally at least, a large zpf induced frictional force is obtained on a moving monopole (Appendix C). The reason for this failure can easily be attributed to the improper ultrarelativistic behavior of the Lorentz-Dirac equation. The improper behavior of the LD equation at large particle speeds has been amply discussed by Burke.[46] In this context we proceed to review very briefly the approximate nature of the LD equation. Consider the classical nonrelativistic generalized equation of Abraham Lorentz,

$$m\frac{d\vec{v}}{dt}=\vec{K}+\vec{G}\ , \tag{46}$$

where m is the renormalized rest mass and $\vec{K}$ is an externally applied force. $\vec{G}$ represents the radiation reaction force that we write as[47]

$$\vec{G}=\frac{2}{3}\frac{e^2}{c^3}\frac{d^2\vec{v}}{dt^2}+(\cdots)R+(\cdots)R^2+\cdots\ . \tag{47}$$

The coefficients in the powers of the classical radius of the particle $R=\Gamma c$ contain higher-order derivatives of $\vec{v}$. Their form is model dependent, and different internal charge distributions give different expressions for the various coefficients. The Lorentz-Dirac equation for relativistic radiation damping is obtained by considering a small particle radius R. Equation (47) gives for small R the standard form of the equation of Abraham Lorentz which only in the extreme $R=0$ case becomes an exact equation [Eq. (46) with $\vec{G}=m\Gamma\, d^2\vec{v}/dt^2$]. Next we write the resulting expression for the $R=0$ case in covariant form, assuming that $\vec{K}$ represents the applied Lorentz force due to the external electromagnetic field,[48]

$$mc\frac{du^i}{ds}=\frac{e}{c}F^{ik}u_k+g^i\ , \tag{48}$$

where F^{ik} is the field tensor and g^i is the Lorentz-Dirac damping four-force of Eq. (C1). g^i is obtained by writing the standard form of the Abraham Lorentz damping force in covariant manner, recalling that g^i should obey the usual condition for force four-vectors, namely, $g^iu_i=0$. The Lorentz-Dirac damping force is clearly valid only for classical charge monopoles of zero radius. Therefore, when considering particles of nonzero radius in the limit $v\rightarrow c$, we have to find a covariant form for the radiation reaction $\vec{G}$ represented by the infinite series in Eq. (47). Hence our expression of Eq. (C12) for the average relativistic damping force $\langle\vec{f}\rangle$ is strictly applicable only in the unphysical $R\rightarrow 0$ limit.

There are several well-known difficulties inherent in the classical theory of charged particles. Those difficulties propagate naturally into all re-

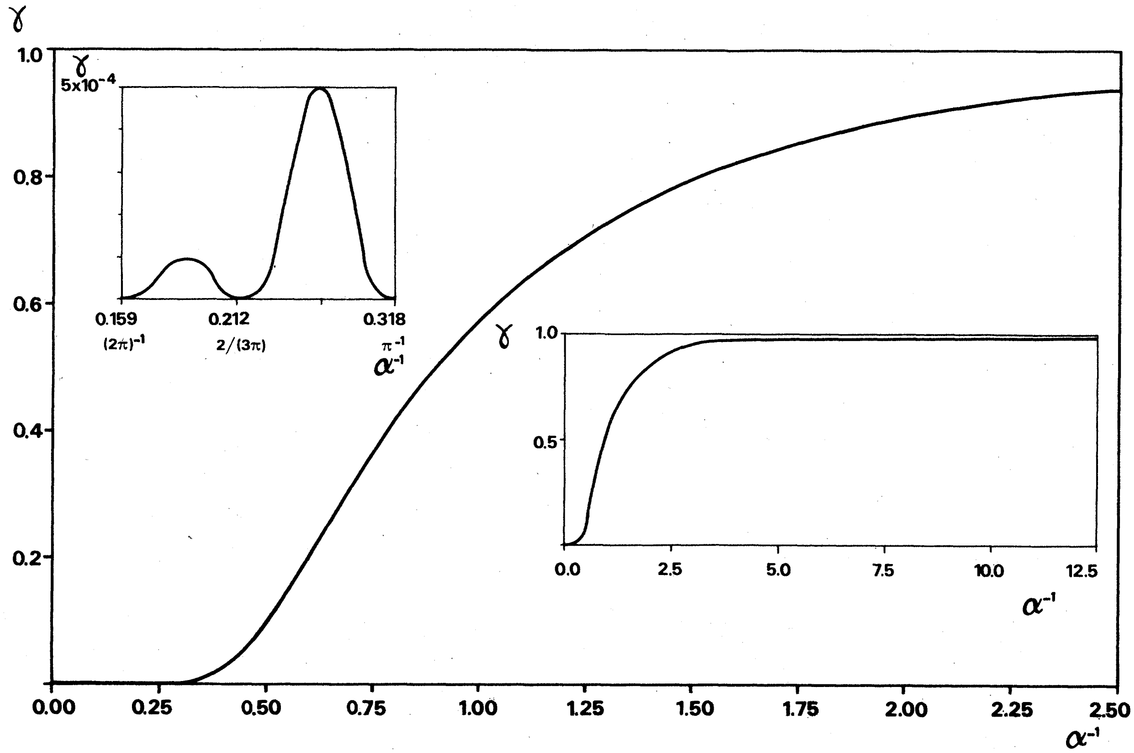

Fig. 1. Display of convergence form factor $\gamma(\alpha)$, $\alpha = \omega R/c$ versus wavelength, $\alpha^{-1} = \lambda/2\pi R$. The very small wiggles at high frequencies are displayed in the small chart in the upper left corner. Observe the very amplified scale for the vertical axis. The general behavior at long wavelengths is seen from the chart in the lower right corner, where the cutoff becomes evident at about $\lambda_c \simeq 2R$.

lated theories, hence we find them in quantum theory as well as in SED. However, if extreme care is exerted in the use of the classical equations of charged-particle motion in SED, the risk of uncritically accepting spurious results like that of Eq. (C12) is minimized. There is another likely source for erroneous predictions in SED, and that obviously is the introduction of the cemzpf of Eq. (2) with its associated divergent spectrum of Eq. (1). Clearly, the several points of rigor, here and elsewhere introduced, are enough to guarantee that the conspicuous behavior of classical polarizable particles (random walk in velocity space, $E^{-\text{const}}$, const ≥ 1, energy spectrum, etc.) is correctly predicted from the basic assumptions of SED, and hence that such a prediction springs forth from the conjunction of the classical theory of charge-particle motion with the cemzpf. It looks, hence, natural to propose that an observational search for such predicted behavior that is presented in SED may provide an immediate observational test, both to the SED theory itself as well as to the validity of the rather simple model of the vacuum contained in the cemzpf concept of Eq. (2). However, the proposed test would probably have to be carried out within an astrophysical context since, apparently, the low particle number densities ($n \simeq 10^{-6}$ cm^{-3}) required for the phenomenon to be detectable[26] may well lie orders of magnitude below those characteristic of the partial vacuum provided by present-day technology.

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APPENDIX A: CONVERGENCE FORM FACTOR

Intuitively, one expects that for a monopolar particle of diameter $2R>0$ spectral components of random radiation whose wavelength λ is smaller than the size $2R$ of the particle cannot be effective in producing translational motion of the particle. Only spectral components of wavelengths $\lambda>2R$ can be responsible for translational motion of the particle as a whole. Spectral components of wavelengths smaller than the size of the particle, $\lambda<2R$, can only be effective in producing internal deformation of the particle. It is, however, interesting to give a specific example of this fact because despite the spectral divergence of the cemzpf and its associated infinite energy density, a natural cutoff should appear associated with the size of the particle, which means that one can do the electrodynamics of particles immersed in the cpmzpf without worrying unnecessarily much about the said divergence. This we proceed to check in detail by means of a specific example. This example is useful in Sec. II and in the next two appendices.

A convergence form factor is obtained by finding an upper bound to the energy available from the electromagnetic zero-point field for a charged particle of nonvanishing size. This is accomplished by a standard semiclassical method.[49] Consider the interaction of the zero-point field with a classical charged particle. Take the particle as an homogeneously charged sphere. Because of the Lorentz-invariant character of the spectrum of the zero-point radiation, one should not worry about velocity effects, and one can work in the frame of reference where the particle is instantaneously at rest. Let the particle have a small nonzero volume v, $v\ll V$, where V is the electromagnetic cavity volume, and let the time duration of the interaction be a short nonzero time interval $\tau>0$. This is the time required for a measurable effect to occur. Our immediate purpose is to study the translational effect of the field on the particle motion. For that purpose we need only know the average field. Consider the jth field component

$$(E_j)_{v,\tau}=(v\tau)^{-1}\int_v d\vec{x}\int_\tau dt\,E_j(\vec{x},t);\qquad j=1,2,3. \quad \text{(A1)}$$

The expectation value of E_j^2 is given by averaging over particle volume and time of measurement

$$\langle 0|(E_j)^2_{v,\tau}|0\rangle=(v\tau)^{-2}\int_{v,\tau}\int_{v,\tau}d\vec{x}\,d\vec{x}'\,dt\,dt'\langle 0|E_j(\vec{x},t)E_j(\vec{x}',t')|0\rangle\,. \quad \text{(A2)}$$

The matrix element in the integrand is

$$\langle 0|E_j(\vec{x},t)E_j(\vec{x}',t')|0\rangle=\left(\frac{2\pi}{V}\right)\sum_{sv}\hbar\omega_s\exp\{i[\vec{k}_s\cdot(\vec{x}-\vec{x}')-\omega_s(t-t')]\}\,(\hat{e}_{sv}\cdot\hat{e}_j), \quad \text{(A3)}$$

where the ket $\langle 0|$ represents the so-called vacuum state, i.e., the state where all cavity modes contain only the zero-point field. $2\pi/V$ is a normalization constant (see Eq. (2) in the text), and the summation is carried over an implicit plane-wave expansion. For each plane wave, indicated by s, there are two directions of polarization, $\nu=1,2$. $\hat{e}_{s\nu}$ is the unit vector in the direction of the electric field of the plane wave, and $\vec{k}_s$ is the associated wave vector with $\omega_s=c|\vec{k}_s|$. Substituting Eq. (A3) in Eq. (A2) and summing over the field components by means of $\sum_{j=1}^{3}(\hat{e}_{s\nu}\cdot\hat{e}_j)=1$, it follows that

$$\langle 0|(\vec{E})^2_{v\tau}|0\rangle=\left(\frac{2\pi}{V}\right)(v\tau)^{-2}\int_{v,\tau}\int_{v,\tau}d\vec{x}\,d\vec{x}'\,dt\,dt'\,2\sum_s\hbar\omega_s\exp\{i[\vec{k}_s\cdot(\vec{x}-\vec{x}')-\omega_s(t-t')]\}. \quad \text{(A4)}$$

The time integrations can be carried out. The summation over s can be replaced by an integration in the standard manner $V^{-1}\sum_s(\cdots)=(2\pi)^{-3}\int d\vec{k}(\cdots)$. As $\omega_s=c|\vec{k}|=ck$ and using the expression for the energy density $u=\langle\vec{E}^2\rangle/4\pi$ written in Gaussian units, and recalling that the total energy density is $u=V^{-1}\sum\frac{1}{2}\hbar\omega_{s\nu}$, we have the average available zpf energy density over the particle volume,

$$\langle u\rangle=\frac{\hbar c}{(2\pi)^3v^2}\int_v\int_v d\vec{x}\,d\vec{x}'\int_{\vec{k}}k\,d\vec{k}\exp[i\vec{k}\cdot(\vec{x}-\vec{x}')]\,\frac{\sin^2(\frac{1}{2}ck\tau)}{(\frac{1}{2}ck\tau)^2}\,. \quad \text{(A5)}$$

We integrate first over $\vec{k}$, letting the k_3 axis be parallel to the vector $\vec{x}-\vec{x}'$,

$$\langle u\rangle=\frac{\hbar c}{(2\pi)^3v^2}\int_v d\vec{x}\int_v d\vec{x}'\int_{\vec{k}}k^3dk\,\sin\theta_k d\theta_k d\phi_k\,\frac{\sin^2(\frac{1}{2}ck\tau)}{(\frac{1}{2}ck\tau)^2}\exp(ik|\vec{x}-\vec{x}'|\cos\theta_k), \quad \text{(A6)}$$

where $\int_{\vec{k}}$ signifies the integral over the infinite sphere in k space. Writing $\mu = \cos\theta_k$ and using the fact that

$$\int_{-1}^{1} \exp(ik|\vec{x}-\vec{x}'|\mu)d\mu = 2\,\frac{\sin(k|\vec{x}-\vec{x}'|)}{(k|\vec{x}-\vec{x}'|)}\,, \tag{A7}$$

we obtain

$$\langle u\rangle = \frac{2\hbar c}{(2\pi v)^2}\int_v d\vec{x}\int_v d\vec{x}'\int_{k=0}^{\infty}\frac{\sin^2(\frac{1}{2}c\tau k)}{(\frac{1}{2}c\tau k)^2}\,\frac{\sin(k|\vec{x}-\vec{x}'|)}{(k|\vec{x}-\vec{x}'|)}\,k^3\,dk. \tag{A8}$$

This equation gives a divergent expression only if both $v=0$ and $\tau=0$.

This result is to be expected for a particle interacting with a field of infinite energy density. It follows that when the particle volume is infinitely small and also the time of interaction is infinitely short, there is no averaging of high-frequency components and the full effect of the field is manifested. We show next that if, on the contrary, both v and τ are finite nonzero quantities, the high-frequency components give no contribution to the energy available from the field. If either τ or v is zero, but not both, $\langle u\rangle$ is still convergent. A particular realization for the case of spherical particles is presented. In order to estimate the volume integrals, observe that

$$\frac{\exp(ik|\vec{x}-\vec{x}'|)}{2i|\vec{x}-\vec{x}'|} = 2\pi k\sum_{l=0}^{\infty} j_l(kr_<)h_l(kr_>)\sum_{m=-l}^{l} Y_{lm}(\theta',\phi')Y_{lm}(\theta,\phi), \tag{A9}$$

where j_l are the spherical Bessel functions, h_l are the spherical Hankel functions, Y_{lm} are the spherical harmonics, and

$$r_{\gtrless} = \begin{Bmatrix}\max\\ \min\end{Bmatrix}(|\vec{x}|,|\vec{x}'|). \tag{A10}$$

From the above we can write

$$\frac{\sin(k|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|} = 4\pi k\,\mathrm{Re}\left(\sum_{l=0}^{\infty} j_l(kr_<)j_l(kr_>)Y_{lm}(\theta',\phi')Y_{lm}(\theta,\phi)\right). \tag{A11}$$

We proceed to evaluate the integral

$$I = \int_v d\vec{x}\int_v d\vec{x}'\,\frac{\sin(k|\vec{x}-\vec{x}'|)}{|\vec{x}-\vec{x}'|}\,, \tag{A12}$$

assuming that v is a spherical volume of radius R, $v = 4\pi R^3/3$. As $Y_{lm} = \Theta_{lm}(\cos\theta)e^{im\phi}$, the integrations over ϕ and θ give zero unless $m=0$. Hence, writing $r \equiv |\vec{x}|$, $r' = |\vec{x}'|$, we have

$$I = (2\pi)^2 k\sum_{l=0}^{\infty}(2l+1)\int_0^R r^2 dr\int_0^R r'^2 dr' j_l(kr_<)j_l(kr_>)\left(\int_{-1}^{1} P_l(\delta)d\delta\right)^2, \tag{A13}$$

where $\delta = \cos\theta$, $\cos\theta'$. We obtain

$$I = k\left(\frac{4\pi}{k^3}\right)^2\left(\int_{\mu=0}^{kR} j_0(\mu)\mu^2 d\mu\right)^2. \tag{A14}$$

As $j_0(\mu) = \sin\mu/\mu$, the last integral can easily be evaluated:

$$\int_0^{kR}\mu\sin\mu\,d\mu = \sin(kR) - (kR)\cos(kR). \tag{A15}$$

Replacing in Eq. (A14), this gives

$$I = k\left(\frac{4\pi}{k^3}\right)^2[\sin(kR) - (kR)\cos(kR)]^2. \tag{A16}$$

The energy density of Eq. (A8) becomes then

$$\langle u\rangle = \frac{9c\hbar}{2\pi^2R^4}\int_0^{\infty}\frac{\sin^2(\frac{1}{2}c\tau k)}{(\frac{1}{2}c\tau k)^2}\left(\frac{\sin(kR)}{(kR)} - \cos(kR)\right)^2\frac{dk}{k}\,. \tag{A17}$$

A signal takes a maximum amount of time $2R/c$ in traversing the particle. It can be assumed that the maximum time of detection approximately corresponds to this amount. Setting then for simplicity $\tau = 2R/c$ in Eq. (A17) we obtain

$$\langle u \rangle = \frac{9c}{2\pi^2 R^4} \int_0^\infty \left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin\alpha}{\alpha} - \cos\alpha\right)^2 \frac{d\alpha}{\alpha}, \tag{A18}$$

where $\alpha = kR$ and where the last integral is a numerical constant that for $R > 0$ is positive and bounded. For the case $R > 0$, the energy available from the field, $U = v\langle u \rangle$, remains bounded:

$$U = v \int_0^\infty \gamma(\omega)\rho(\omega)d\omega, \tag{A19}$$

where

$$\gamma(\omega) = \gamma[\alpha] \equiv \frac{9}{\alpha^4}\left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin\alpha}{\alpha} - \cos\alpha\right)^2 \tag{A20}$$

and $\alpha = \omega R/c$. The function γ is displayed in Fig. 1 in terms of the wavelength λ. A cutoff is clearly seen around the critical wavelength $\lambda_c \equiv 2R$, giving substance to the intuitive statement that wavelengths smaller than the size of the particle produce internal deformation and do not directly contribute to the translational motion of the particle.[31]

Last, but not least, we would like to point out an important consequence of the sharpness of the cutoff given by Eq. (A20) and displayed in the lower right corner graph of Fig. 1. This sharp cutoff has remarkable relevance in the simplification of our treatment of the monopole in Sec. II and Appendix B. Furthermore, the energy growth rate for a charged monopolar particle under the cemzpf is a strongly dependent function of the cutoff frequency, i.e., most of the energy growth rate for a monopole is found to depend strongly on the highest frequencies of the random field available for monopolar translational motion[33] [see Eqs. (18) and (B16)].

APPENDIX B: MONOPOLAR MOTION UNDER THE CEMZPF

Consider the monopolar particle of Eq. (6) under the electric and magnetic fields of Eqs. (2)–(4). If again we assume that the effect of the field on the particles is not too strong, the resulting initial particle motion is slow enough, and then in a first approximation we can use, if a sufficiently short time interval τ is considered, the dipole approximation [$\vec{k} \cdot \vec{x} = 0$ in Eq. (2)] and neglect in a first approximation the effect of the magnetic field. The steady-state solution is then

$$\vec{x}(t) = \left(\frac{V}{(2\pi)^3}\right)^{1/2} \sum_{\lambda=1}^{2} \int d^3k \vec{x}(\vec{k},\lambda) \cos[\omega_{\vec{k}} t - \Theta(\vec{k},\lambda) + \delta(\omega_{\vec{k}})], \tag{B1}$$

where

$$\vec{x}(\vec{k},\lambda) = -\frac{3c^3}{2e\omega_{\vec{k}}^3} \mu(\omega_{\vec{k}}) \sin[\delta(\omega_{\vec{k}})] \hat{\epsilon}(\vec{k},\lambda), \tag{B2}$$

and

$$\tan[\delta(\omega_{\vec{k}})] = \Gamma \omega_{\vec{k}}, \tag{B3}$$

where Γ was defined by Eq. (7). We can now include the particle velocity $\dot{\vec{x}}$ in the Lorentz-force expression of the right-hand side of Eq. (6). In the time interval τ, the net impulse obtained by the particle is then

$$\vec{I}_\tau = \int_0^\tau \left(e\vec{E} + \frac{e}{c}\vec{v} \times \vec{B}\right) dt. \tag{B4}$$

It is easy to see that for a time interval τ sufficiently long, on the average the integral over the electric field may be neglected in comparison with the integral involving the magnetic field. The electric field gives only the jiggling motion of Eq. (13). It is the magnetic field that is responsible for the main contribution to the impulse of Eq. (B4). This is so because the cross product of the velocity and the magnetic field gives an impulse that on the average grows with $\tau^{1/2}$, as shown in what follows. We have

$$\frac{e}{c}\int_0^\tau(\dot{\vec{x}}\times\vec{B})\,dt=\left(\frac{V}{(2\pi)^3}\right)\frac{e}{c}\int_0^\tau\sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\int d^3k_1\int d^3k_2\,\frac{3}{2}\,\frac{c^3}{e\omega^2_{\vec{k}}}\,\mu(\omega_{\vec{k}_1})\mu(\omega_{\vec{k}_2})\sin[\delta(\omega_{\vec{k}_1})]\hat{\epsilon}(\vec{k}_1,\lambda_1)$$
$$\times[\hat{k}_2\times\hat{\epsilon}(\vec{k}_2,\lambda_2)]\sin[\omega_{\vec{k}_1}t+\delta(\omega_{\vec{k}_1})-\Theta(\vec{k}_1,\lambda_1)]$$
$$\times\cos[\omega_{\vec{k}_2}t-\Theta(\vec{k}_2,\lambda_2)]\,dt\,, \tag{B5}$$

where $\hat{k}=\vec{k}/|\vec{k}|$ and the calculation is analogous to a previous one.[26] The time integration is carried out first. We then square and take averages over the random phases using[26]

$$\langle\sin(\tfrac{1}{2}(\omega_1\pm\omega_2)\tau\mp\theta_2-\theta_1+\delta)\sin(\tfrac{1}{2}(\omega_1'\pm\omega_2')\tau\mp\theta_2'-\theta_1'+\delta)\rangle=\frac{(2\pi)^6}{V^2}\,\tfrac{1}{2}\,\delta(\vec{k}_1-\vec{k}_1')\,\delta(\vec{k}_2-\vec{k}_2')\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}\,. \tag{B6}$$

This yields

$$\langle\vec{\Delta}_\tau^2\rangle=\left(\frac{e}{c}\right)^2\int d^3k_1\int d^3k_2\,\mu^2(\omega_1)\mu^2(\omega_2)\left(\frac{3c^3}{2e}\right)^2\left(\frac{\sin\delta_1}{\omega_1}\right)^2$$
$$\times\sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\{\hat{\epsilon}(\vec{k}_1,\lambda_1)\times[\hat{k}_2\times\hat{\epsilon}(\vec{k}_2,\lambda_2)]\}^2\,\frac{1}{2}\left(\frac{\tau}{2}\right)^2\left(\frac{1}{[\frac{1}{2}(\omega_1+\omega_2)\tau]^2}\sin^2[\tfrac{1}{2}(\omega_1+\omega_2)\tau]\right.$$
$$\left.+\frac{1}{[\frac{1}{2}(\omega_1-\omega_2)\tau]^2}\sin^2[\tfrac{1}{2}(\omega_1-\omega_2)\tau]\right), \tag{B7}$$

where we have already written $\langle\vec{\Delta}_\tau^2\rangle$ instead of $\langle\vec{I}_\tau^2\rangle$ because, for the case of motion under the zpf, there is no net drag force and the friction term $\vec{\Phi}=-P\tau\vec{v}_t$, as shown below, is equal to zero to all orders in v/c. Observe furthermore that $d^3k=(\omega^2/c^3)\,d\omega\,d\Omega_{\hat{k}}$. For long enough τ it can easily be seen that the second term in the curvilinear parenthesis behaves as a delta function. In comparison, the first term can be neglected. We use the relation

$$\int_{-\infty}^{\infty}\frac{\sin^2\beta}{\beta^2}=\pi\,. \tag{B8}$$

Furthermore, we observe that

$$\int d\Omega_1\int d\Omega_2\sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}\{\hat{\epsilon}(\vec{k}_1,\lambda_1)\times[\hat{k}_2\times\hat{\epsilon}(\vec{k}_2,\lambda_2)]\}^2=\int d\Omega_1\int d\Omega_2\sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}[\hat{\epsilon}(\vec{k}_1,\lambda_1)\times\hat{\epsilon}(\vec{k}_2,\lambda_2)]^2\,. \tag{B9}$$

Recalling the vector identity

$$(\vec{A}\times\vec{B})^2=\vec{A}^2\vec{B}^2-(\vec{A}\cdot\vec{B})^2\,, \tag{B10}$$

with the observation that if $|\vec{A}|=|\vec{B}|=1$, then

$$(\vec{A}\times\vec{B})^2=1-\cos^2\Theta(\vec{A},\vec{B})\,, \tag{B11}$$

we can, instead of Eq. (B9), write

$$\int d\Omega_1\int d\Omega_2\left(4-\sum_{\lambda_1=1}^{2}\sum_{\lambda_2=1}^{2}[\hat{\epsilon}(\vec{k}_1,\lambda_1)\cdot\hat{\epsilon}(\vec{k}_2,\lambda_2)]^2\right)=4(4\pi)^2-4(4\pi)(\tfrac{4}{3}\pi)=\tfrac{8}{3}(4\pi)^2\,. \tag{B12}$$

From Eqs. (B8), (B9), and (B12), and after neglecting the first term inside the curvilinear parenthesis of Eq. (B7) we obtain

$$\langle\vec{\Delta}_\tau^2\rangle=\left(\frac{3}{2}\right)^2\frac{8}{3}(4\pi)^2\,\frac{\tau}{2}\,\frac{1}{2}\pi\,\frac{1}{c^2}\int\left(\frac{\hbar\omega}{2\pi^2}\right)^2\frac{(\Gamma\omega)^2}{1+(\Gamma\omega)^2}\,d\omega. \tag{B13}$$

The rate of translational energy growth is then given by

$$\Omega\equiv\frac{dE}{dt}=\frac{\langle\vec{\Delta}_\tau^2\rangle}{2m\tau}\,. \tag{B14}$$

In the above derivation we have considered a point particle. For a particle of nonzero radius, $R>0$, we know from Appendix A that this translational energy growth should be finite even if the cemzpf energy density spectrum is itself divergent. Introducing then the convergence form factor of Eq. (A20) we finally obtain

$$\Omega = \frac{3}{\pi}\frac{\hbar^2}{mc^2}\int_{\omega=0}^{\infty}\gamma(\omega)\frac{(\Gamma\omega)^2}{1+(\Gamma\omega)^2}\,\omega^2 d\omega. \tag{B15}$$

As the classical particle radius is $R=\Gamma c$, we can approximate the integration of Eq. (B15) by the introduction of a cutoff at $\omega_m \cong \pi/\Gamma = \pi c/R$. This gives

$$\Omega \cong \frac{3}{5\pi}(\Gamma\omega_m)^2\left(\frac{\hbar\omega_m}{mc^2}\right)(\hbar\omega_m)\,\omega_m. \tag{B16}$$

For a more precise evaluation of the integral of Eq. (B15) structural information on the specific particle is required. For elementary particles found in nature, i.e., electrons, that information is not always available. However, for the idealized homogeneously charged spherical particles of Appendix A, it can be seen that Eq. (B16) is a fairly reasonable approximation. It is interesting furthermore to realize that the rate of energy growth is strongly sensitive to the highest frequency ranges available to the particle.[33] It remains to show that in the cemzpf case the average impulse of Eq. (17) transmitted to the particle is equal to zero.

Consider a new frame of reference S' moving with velocity $\vec{v}$ with respect to the previous reference frame that we call S. We can make this velocity $\vec{v}$ coincide with the instantaneous velocity that the particle has in the frame S at a specific time, say, the time $t=0$, i.e., $\vec{v}=\dot{\vec{x}}(0)$. The field transformations give

$$\vec{E}'(\vec{x}',t') = \left(\frac{V}{(2\pi)^3}\right)^{1/2}\sum_{\lambda=1}^{2}\int d^3k\,\mu(\omega_{\vec{k}})\left[\epsilon_x\hat{i}_x + \gamma\left(\epsilon_y - \frac{v}{c}\frac{(\vec{k}\times\hat{\epsilon})_z}{|\vec{k}|}\right)\hat{i}_y + \gamma\left(\epsilon_z + \frac{v}{c}\frac{(\vec{k}\times\hat{\epsilon})_y}{|\vec{k}|}\right)\hat{i}_z\right] \times\cos[\omega't' - \vec{k}'\cdot\vec{x}' - \Theta(\vec{k},\lambda)], \tag{B17}$$

where for simplicity we have chosen the x and x' axes to coincide with the axis of the relative velocity $\vec{v}$ of S and S'. Beware that $\gamma=(1-v^2/c^2)^{-1/2}$ is different from the previous $\gamma(\omega)$. The wave vectors transform as

$$k_x=\gamma(k_x'+v\omega'/c^2),\quad k_y=k_y',\quad k_z=k_z',\quad d^3k=d^3k'\gamma(1+vk_x'/\omega'),\quad \omega=\gamma(\omega'+vk_x'),\quad \omega t-\vec{k}\cdot\vec{x}=\omega't'-\vec{k}'\cdot\vec{x}'. \tag{B18}$$

However, the field $\vec{E}'$ can also be expressed directly in the particle frame of reference as

$$\vec{E}'(\vec{x}',t') = \left(\frac{V'}{(2\pi)^3}\right)^{1/2}\sum_{\lambda'=1}^{2}\int d^3k'\,\hat{\epsilon}'(\vec{k}',\lambda')\mu'[\vec{k}']\cos[\omega't'-\vec{k}'\cdot\vec{x}'-\Theta(\vec{k}',\lambda')], \tag{B19}$$

where $\mu[\vec{k}']$ denotes a function that in general is not necessarily isotropic in $\vec{k}'$. From Eqs. (B17) and (B19) after squaring and averaging it is shown that

$$\mu'^2[\vec{k}'] = \frac{\mu^2[\omega\gamma(1+\vec{v}\cdot\vec{k}'/\omega')]}{\gamma(1+\vec{v}\cdot\vec{k}'/\omega')}. \tag{B20}$$

If $v/c \ll 1$, Eq. (B20) reduces to

$$\mu'^2[\vec{k}'] \cong \mu^2(\omega') - \left(\mu^2(\omega') - \frac{\partial\mu^2(\omega')}{\partial\omega'}\omega'\right)\frac{\vec{v}\cdot\vec{k}'}{\omega'}. \tag{B21}$$

In general the radiation is not isotropic except for the S frame where $\vec{v}=0$. However, if the expression in large parentheses identically cancels, the random radiation may be isotropic in all inertial frames. This can be so if $\mu^2(\omega')\sim\omega'$, which from Eq. (4) we know is exclusively the case of the cemzpf of Eq. (1). In that case then,

$$\mu_c'^2[\vec{k}'] = \mu_c^2(\omega') \sim \omega'. \tag{B22}$$

The impulse in the frame S' can be written as

$$\vec{I}_\tau' = \int_0^\tau e\vec{E}'dt' + \frac{e}{c}\int_0^\tau(\dot{\vec{x}}'\times\vec{B}')dt'. \tag{B23}$$

For a long enough time interval we can neglect the first term of the Lorentz force in comparison with the second. Hence, we obtain that

$$\langle\vec{I}_\tau'\rangle = \frac{e}{c}\int_0^\tau\langle\dot{\vec{x}}'\times\vec{B}'\rangle\,dt' = \frac{1}{2}\frac{e}{c}\tau\sum_{\lambda'=1}^{2}\int d^3k'\frac{3}{2}\frac{c^3}{e\omega_{\vec{k}'}'^2}\mu'^2[\vec{k}']\sin^2\delta(\omega_{\vec{k}'}')\hat{\epsilon}'\times(\hat{k}'\times\hat{\epsilon}'), \tag{B24}$$

where Eq. (B5) has been used. The averaging has been performed first and the time integration has been performed next. As $\hat{k}\cdot\hat{\epsilon}=0$, Eq. (B24) can be simplified even more, yielding

$$\langle \vec{I}'_T\rangle = \tau\frac{3c^2}{2}\int d^3k'\,\sin^2\delta(\omega'_{\vec{k}'})\frac{\mu^2[\vec{k}']}{\omega'^2_{\vec{k}'}}\vec{k}'=0, \tag{B25}$$

where the last equality results from simple symmetry considerations. This last result is to be expected and was all we needed. However, for completeness of presentation, we can consider the case of general thermal plus zpf radiation ($T>0$) at low enough particle speeds. Replacing then Eq. (B21) in the second term of Eq. (B25) we obtain

$$\langle I'_T\rangle = -\vec{v}\,\frac{2\pi\tau}{c^3}\int\left(\mu^2(\omega')-\omega'\frac{\partial\mu^2(\omega')}{\partial\omega'}\right)\sin^2\delta(\omega')\omega'd\omega'. \tag{B26}$$

We see that we can then write that the drag force is proportional to the velocity

$$\vec{f}=-P\vec{v}, \tag{B27}$$

with $P\neq 0$ in general, except in the special case of the cemzpf where $P=0$ identically, as follows from Eqs. (B25) and (B26).

APPENDIX C: LORENTZ-DIRAC FRICTIONAL FORCE FOR MONOPOLES MOVING THROUGH THE ZPF—A SPURIOUS PREDICTION

Consider the Lorentz-Dirac damping force four-vector[48]

$$g^i=\frac{2e^2}{3c}\left(\frac{d^2u^i}{ds^2}-u^iu^k\frac{d^2u_k}{ds^2}\right), \tag{C1}$$

where u^i is the four velocity, c the speed of light, e the charge of a monopolar particle, and

$$ds=c\,dt(1-v^2/c^2)^{1/2} \tag{C2}$$

represents the proper time interval. The Lorentz-Dirac damping for a monopolar particle of Eq. (C1) can be written in terms of the field tensor F^{ij} of the external field acting on the particle,[48]

$$g^i=\frac{2e^3}{3mc^3}\frac{\partial F^{ik}}{\partial x^l}u_ku^l-\frac{2e^4}{3m^2c^5}F^{il}F_{kl}u^k+\frac{2e^4}{3m^2c^5}(F_{kl}u^l)(F^{km}u_m)u^i. \tag{C3}$$

The associated three-dimensional expression for the damping force $\vec{f}$ in the relativistic case can easily be found in terms of the electric $\vec{E}$ and magnetic $\vec{H}$ fields measured in the laboratory frame of reference for a particle of charge e and rest mass m moving with respect to the laboratory with velocity $\vec{v}$,[48]

$$\vec{f}=\frac{2e^3}{3mc^3}\gamma\left[\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)\vec{E}+\frac{\vec{v}}{c}\times\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)\vec{H}\right]$$
$$+\frac{2e^4}{3m^2c^4}\left[\vec{E}\times\vec{H}+\vec{H}\times(\vec{H}\times\vec{v}/c)+\vec{E}\left(\frac{\vec{v}}{c}\cdot\vec{E}\right)\right]-\frac{2e^4}{3m^2c^4}\gamma^2\frac{\vec{v}}{c}\left[\left(\vec{E}+\frac{\vec{v}}{c}\times\vec{H}\right)^2-\left(\vec{E}\cdot\frac{\vec{v}}{c}\right)^2\right], \tag{C4}$$

where $\gamma\equiv(1-v^2/c^2)^{-1/2}$. When $\vec{E}$ and $\vec{H}$ refer to the zero-point field we can, because of the homogeneity and isotropy of the random field, assign the conditions specified by Bourret for random radiation, namely,[5]

$$\langle H_iH_j\rangle=\langle E_iE_j\rangle=\langle E_i^2\rangle\delta_{ij}=\tfrac{4}{3}\pi U\delta_{ij}, \tag{C5a}$$

$$\langle E_i\rangle=\langle H_j\rangle=0;\quad \langle\vec{S}\rangle=\langle\vec{E}\times\vec{H}\rangle=0, \tag{C5b}$$

where H_i and E_i represent the field components, the brackets indicate averaging over an ensemble of similarly prepared random fields, and U is the field volume energy density

$$U=\int_0^\infty\gamma(\omega)\rho(\omega)d\omega. \tag{C6}$$

The volume and spectral energy density ρ is given in Eq. (1), and the integration can be carried out introducing the convergence form factor $\gamma(\omega)$ of Appendix A. Hence we write

$$U=\int_0^{\omega_m}\rho(\omega)d\omega. \tag{C7}$$

Equations (C5) and (C6) define the instantaneous correlation properties of the field with itself. We also require the correlation properties of the field with the translational motion of the particle. From the random field at hand it can easily be seen that the instantaneous value of a field component is only weakly correlated with the present value of the particle velocity, since for the zero-point field the correlation time of the field with itself, because of the spectral divergence of the spectrum, if nonzero, should be extremely short. Hence we assume

$$\langle E_i^n H_j^m v_k^l \rangle = \langle E_i^n H_j^m \rangle \langle v_k^l \rangle + \epsilon \,, \tag{C8}$$

where ϵ represents a relatively small correction term and where n, m, and l represent integral powers of the field components and of the velocity, respectively. We use furthermore the vector relations

$$\vec{H} \times \left(\vec{H} \times \frac{\vec{v}}{c}\right) = \vec{H}\left(\vec{H} \cdot \frac{\vec{v}}{c}\right) - H^2 \frac{\vec{v}}{c} \,, \tag{C9a}$$

$$\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{H}\right)^2 = E^2 + \left(\frac{\vec{v}}{c} \times \vec{H}\right)^2 - 2\frac{\vec{v}}{c} \cdot (\vec{E} \times \vec{H}) \,. \tag{C9b}$$

From Eqs. (C4)–(C9) we then obtain that

$$\langle \vec{f} \rangle = \tfrac{3}{2}(\Gamma c)^2 \left[2\left\langle \vec{E}\left(\frac{\vec{v}}{c} \cdot \vec{E}\right)\right\rangle - \left\langle E^2 \frac{\vec{v}}{c}\right\rangle - \left\langle \gamma^2 E^2 \frac{\vec{v}}{c}\right\rangle - \left\langle \gamma^2 \left(\frac{\vec{v}}{c} \times \vec{H}\right)^2 \frac{\vec{v}}{c}\right\rangle + \left\langle \gamma^2 \left(\vec{E} \cdot \frac{\vec{v}}{c}\right)^2 \frac{\vec{v}}{c}\right\rangle \right] . \tag{C10}$$

From now on we neglect the small ϵ term of Eq. (C8), as carrying it unnecessarily complicates the algebra. Because of Eqs. (C8) and (C5) it easily follows that

$$\left\langle \left(\vec{E} \cdot \frac{\vec{v}}{c}\right)\vec{E}\right\rangle = \left\langle \vec{E}\left(\frac{v_x}{c} E_x + \frac{v_y}{c} E_y + \frac{v_z}{c} E_z\right)\right\rangle = \tfrac{1}{3}\left\langle E^2 \frac{\vec{v}}{c}\right\rangle , \tag{C11a}$$

$$\left\langle \gamma^2 \left(\frac{\vec{v}}{c} \times \vec{H}\right)\frac{\vec{v}}{c}\right\rangle = \left\langle \gamma^2 \left[\left(\frac{v_x}{c} H_y - \frac{v_y}{c} H_x\right)^2 \frac{\vec{v}}{c} \cdots\right]\right\rangle = \tfrac{2}{3}\left\langle \left(\frac{\gamma v}{c}\right)^2 E^2 \frac{\vec{v}}{c}\right\rangle , \tag{C11b}$$

$$\left\langle \gamma^2 \left(\vec{E} \cdot \frac{\vec{v}}{c}\right)\frac{\vec{v}}{c}\right\rangle = \left\langle \gamma^2 \left(E_x \frac{v_x}{c} + E_y \frac{v_y}{c} + E_z \frac{v_z}{c}\right)^2 \frac{\vec{v}}{c}\right\rangle = \tfrac{1}{3}\left\langle \left(\gamma \frac{v}{c}\right)^2 E^2 \frac{\vec{v}}{c}\right\rangle . \tag{C11c}$$

We thus obtain for the average force of Eq. (C10) that

$$\langle \vec{f} \rangle = -8\pi R^2 U \langle \gamma^2 \vec{v}/c \rangle \,, \tag{C12}$$

where we used Eq. (C5a) and the fact that $1 = \gamma^2 - (\gamma \vec{v}/c)^2$. $R = \Gamma c$ is the classical radius of the particle. Equation (C12) is a rather curious result that runs contrary to all our expectations. The drag force of Eq. (C12) can be zero for our ultrarelativistic monopolar particles only in the following two cases: (i) If the classical radius of the particle is zero. However, that is an inadmissible proposition. On the one hand, if we take into consideration Eq. (C7), we should have $\langle \vec{f} \rangle \to \infty$, $R \to 0$, a fact which strongly excludes the $\langle \vec{f} \rangle = 0$ possibility; on the other hand, if we do not consider Eq. (C7), we have to accept in the theory particles with infinite renormalized mass, as can be seen from Eq. (7). Furthermore, when $R \to 0$ the convergence form factor γ tends to 1 everywhere, and hence one would have to accept monopolar particles acted on by the whole zero-point-field divergent spectrum of Eq. (1). (ii) Another possible reason for claiming that Eq. (C12) gives $\langle \vec{f} \rangle = 0$ occurs if one assumes a null zero-point-field energy density spectrum.

In order to throw some light into the peculiar nature of the above displayed paradox, we consider the nonrelativistic form of Eq. (C12) and compare it with the corresponding expression of Appendix B, namely, with Eq. (B26). We can write Eq. (C12) in the nonrelativistic limit

$$\vec{f} = -8\pi R^2 U \vec{v}/c \,, \tag{C13}$$

where for simplicity the averaging, although not indicated, is understood. From Eqs. (4), (C7), and (B26) we have

$$\vec{f} = \langle I_\tau' \rangle / \tau = -\frac{\vec{v}}{c} 2\pi R^2 \left[\int_0^{\omega_m} \rho(\omega') d\omega' - \int_0^{\omega_m} \omega'^3 \frac{\partial}{\partial \omega'}\left(\frac{\rho(\omega')}{\omega'^2}\right) d\omega' \right] = -\frac{\vec{v}}{c} 8\pi R^2 \left(U - \tfrac{1}{4} \int_0^{\omega_m} \omega' \rho(\omega')\right) , \tag{C14}$$

where we introduced the cutoff of Appendix A, and hence could neglect the $(\Gamma\omega')^2$ term in the denominator of Eq. (B26), and used the fact that the classical radius of the particle is $R = \Gamma c$. Observe that Eqs. (C13) and (C14) differ only in the term that results from the integration by parts. Clearly, for thermal and other random radiation of a convergent spectrum such that $\rho \to 0$ sufficiently fast as $\omega \to \infty$, there is no problem with the integrated term and both Eqs. (C14) and (C13) give exactly the same result. It is for radiation fields of a divergent spectrum that such a term becomes relevant. In particular we can see that for the case of the cemzpf we have that the integrated term is exactly equal to U, and hence that Eq. (C14) gives $\vec{f}=0$, as on physical grounds it was obviously to be expected.

Therefore, in Eq. (C13) and hence in Eq. (C12), there is a missing term that gives the clue to the solution of the apparent paradox depicted in this appendix. The missing term, which is the integrated term derived in Eq. (C14) from the results of Appendix B, can be introduced in Eq. (C13) in an *ad hoc* manner as a "renormalization term" that corrects the divergence that ensues from the use of the LD equation in a random field of divergent spectrum. Hence we can see that the correct solution arises spontaneously from the derivation of Appendix B, that starts from the Abraham Lorentz Eq. (6), and not from the derivation of this appendix that starts from the LD Eq. (48) with the damping force Eq. (C3). It is this damping force that we should examine next.

Written in four-vector notation, the Abraham Lorentz Eq. (6) reads like Eq. (48) but with the damping force

$$g^i = \frac{2e^2}{3c}\frac{d^2u^i}{ds^2}. \tag{C15}$$

The additional term of Eq. (C1), as is well known, is required for proper covariance of the resulting equation and is chosen in such a manner that the orthogonality condition $g^i u_i = 0$ is obeyed. It is a simple matter to check that this last term does precisely lead to the last term of Eq. (C3). At large speeds it is also this last term that dominates the space components of the four force. In the ultrarelativistic regime we then approximately have[48]

$$\vec{f} = \frac{2e^4}{3m^2c^5}(F_{kl}u^l)(F^{km}u_m)\vec{v}. \tag{C16}$$

Furthermore, if we consider random radiation as in Eq. (C5), then Eq. (C16) leads exactly to Eq. (C12). Hence we have that the additional term of Eq. (C1) is exclusively responsible for the frictional force of Eq. (C12). If there were not this particular additional term, introduced exclusively from covariance requirements, there would not be the electromagnetic friction of Eq. (C12).

The physical interpretation of the additional term to Eq. (C1) can easily be obtained when examining the four momentum that the moving particle transmits to the field in a given time interval. We have

$$\Delta P^i = -\int g^i\,ds = \frac{2e^2}{3c}\int u^k \frac{d^2u_k}{ds^2}dx^i = -\frac{2e^2}{3c}\int\left(\frac{du^k}{ds}\right)\left(\frac{du_k}{ds}\right)dx^i = -\frac{2e^4}{3m^2c^5}\int (F_{kl}u^l)(F^{km}u_m)u^i ds\,,$$

and hence

$$\Delta\vec{P} = -\int \vec{f}\,ds. \tag{C17}$$

However, as seen from Eq. (C12) this transmitted impulse from the particle to the field goes to zero in the $R\to 0$ limit. The contradiction found comes from not considering particles in this limit of zero radius. For particles of finite size, as pointed out in the text, we have to use a more complex version of the Abraham Lorentz equation, namely, Eqs. (46) and (47), and then from it, by requiring the covariance of the expression, derive the corresponding generalized version of the LD equation. This last approach should not lead to the difficulties displayed in this appendix. Its actual feasibility is, however, greatly hindered by the mathematical complications created by the model dependent coefficients of Eq. (47).

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