# Kinetic theory of electrical conductivity in plasmas

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A recently developed quantum kinetic theory for time-correlation functions is applied to the calculation of the electrical conductivity in dense, strongly coupled plasmas. In the weak-collision limit the theory generalizes the Ziman expression to finite temperatures while, for strong collisions, it generalizes the result of Gould and of Williams and DeWitt to include strong ion coupling. Numerical results which compare the effects that strong ion coupling, bound (core) electrons, and strong collisions have upon the collision frequency are also presented.

## I. INTRODUCTION

This paper will discuss a kinetic theory approach to the calculation of electrical conductivities in nonideal plasmas. The starting point for this theory is the Kubo[1] expression for the electrical conductivity in terms of the equilibrium current-current time-correlation function so that a kinetic theory formalism may be applied to the problem. This formalism has been described in detail for a one-component system in the preceding paper[2] (hereafter referred to as I), and its generalization to a two-component system (electrons and ions) will be employed here. Use of this formalism allows the description of plasma electrical conductivity to be extended into regions of growing experimental interest[3] where the standard Debye-Huckel theory of plasma equilibrium properties and the Born approximation to the electron-ion scattering cross section fail simultaneously.

The correlation-function expression for the electrical conductivity in an isotropic medium may be written as[1]

$$\sigma = \frac{\beta}{3}\,\mathrm{Re}\int_0^\infty dt\,\frac{1}{\Omega}\langle\vec{\mathcal{J}}(0)\cdot\vec{\mathcal{J}}(t)\rangle_0\,, \tag{1.1}$$

where $\beta$ is the inverse temperature, and $\Omega$ is the volume of the system. The angular brackets indicate an average over an equilibrium ensemble of electrons and ions,

$$\langle\theta\rangle_0 \equiv \sum_{N_1\geq 0}\sum_{N_1\geq 0}\mathrm{Tr}^{(1)}_{1\cdots N_1}\mathrm{Tr}^{(2)}_{1\cdots N_2}\rho\theta\,, \tag{1.2}$$

$$\rho \equiv \frac{1}{\Xi}\,z_1^{N_1}z_2^{N_2}e^{-\beta H}\,. \tag{1.3}$$

The fugacities of the electrons and ions are $z_1$ and $z_2$; $\Xi$ is the partition function, and the superscripts on the trace operations indicate over which subspace, electron (1) or ion (2), the trace is to be performed. The Hamiltonian for the system is taken to be

$$H = \sum_{1\leq j\leq N_1}\frac{\vec{p}^{\,2}(j)}{2m} + \sum_{1\leq j<k\leq N_2}V_{11}(|\vec{r}(j)-\vec{r}(k)|)$$
$$+ \sum_{1\leq j\leq N_2}\frac{\vec{P}^{\,2}(j)}{2M} + \sum_{1\leq j<k\leq N_2}V_{22}(|\vec{R}(j)-\vec{R}(k)|)$$
$$+ \sum_{1\leq j\leq N_1}\sum_{1\leq k\leq N_2}V_{12}(|\vec{r}(j)-\vec{R}(k)|)\,. \tag{1.4}$$

Capital letters refer to ion operators while lowercase letters refer to electron operators. The electron-electron (1,1), ion-ion (2,2), and electron-ion (1,2) interactions are indicated by $V_{11}$, $V_{22}$, and $V_{12}$, respectively. The current operator appearing in Eq. (1.1) contains both electron and ion contributions,

$$\vec{\mathcal{J}}(0) \equiv -\frac{e}{m}\sum_{1\leq k\leq N_1}\vec{p}(k) + \frac{Ze}{M}\sum_{1\leq k\leq N_2}\vec{P}(k)$$
$$\equiv \sum_{1\leq k\leq N_1}\vec{j}(k) + \sum_{1\leq k\leq N_2}\vec{J}(k)\,, \tag{1.5}$$

where $(-e)$ is the charge on an electron and $Ze$ is the charge on an ion.

Rather than dealing with Eq. (1.1) directly, it is more convenient to represent $\sigma$ in terms of the transformed correlation function,

$$\tilde{C}(z) \equiv -i\int_0^\infty dt\,e^{izt}\,\frac{1}{\Omega}\langle\vec{\mathcal{J}}(0)\cdot\vec{\mathcal{J}}(t)\rangle_0 \quad \mathrm{Im}z>0\,. \tag{1.6}$$

Comparison of Eqs. (1.6) and (1.1) shows that

$$\sigma = -\frac{\beta}{3}\,\mathrm{Im}\lim_{\eta\to 0+}\tilde{C}(0+i\eta)\,. \tag{1.7}$$

The advantage in expressing the conductivity as in Eq. (1.7) is that the kinetic theory of I is designed to calculate correlation functions of precisely this form. Since this theory has already been described in detail for one component, only its major results, generalized for two components, will be presented here.

Because the current operator is initially a sum of single-particle operators, partial traces may be performed in Eq. (1.6) to write $\tilde{C}(z)$ as

$$\tilde{C}(z)=\frac{1}{\Omega}\,\mathrm{Tr}_1^{(1)}\vec{j}(1)\cdot\tilde{\psi}_1(1;z)+\frac{1}{\Omega}\,\mathrm{Tr}_1^{(2)}\vec{J}(1)\cdot\tilde{\psi}_2(1;z)\,, \tag{1.8}$$

where $\tilde{\psi}_1(1;z)$ and $\tilde{\psi}_1(1;z)$ are solutions to the exact, coupled kinetic equations, found by formally closing the BBGKY (Bogoliubov-Born-Green-Kirkwood-Yvon) hierarchy at the single-particle level,

$$[z-L_1(1)-\tilde{\Sigma}_{1;1}(1;z)]\tilde{\psi}_1(1;z)-\tilde{\Sigma}_{1;2}(1;z)\tilde{\psi}_2(1;z)=\psi_1(1;0)\,, \tag{1.9}$$

$$[z-L_2(1)-\tilde{\Sigma}_{2;2}(1;z)]\tilde{\psi}_2(1;z)-\tilde{\Sigma}_{2;1}(1;z)\tilde{\psi}_1(1;z)=\psi_2(1;0)\,. \tag{1.10}$$

Just as in the single-component case, the $\tilde{\Sigma}_{\alpha,\beta}$'s contain all of the many-body effects upon the time development of a single particle, and they are defined in an analogous manner to Eq. (2.8) in I,

$$\tilde{\Sigma}_{\alpha;\beta}(1;z)\equiv\mathrm{Tr}_2^{(\bar{\alpha})}L_{1\bar{\alpha}}(12)\tilde{U}_{1\bar{\alpha};\bar{\beta}}(12;z)\tilde{U}^{1}_{\bar{\beta};\beta}(1;z)\,, \tag{1.11}$$

where the barred indices are summed. The $\tilde{U}$ operators appearing in Eq. (1.11) relate the $\tilde{\psi}$'s to the initial data, $\vec{j}(1)$ and $\vec{J}(1)$. For example, one has

$$\tilde{\psi}_\alpha(1;z)=\tilde{U}_{\alpha;1}(1;z)\vec{j}(1)+\tilde{U}_{\alpha;2}(1;z)\vec{J}(1)\,, \tag{1.12}$$

$$\tilde{\psi}^{(2)}_{\alpha\beta}(12;z)=\tilde{U}_{\alpha\beta;1}(12;z)\vec{j}(1)+\tilde{U}_{\alpha\beta;2}(12;z)\vec{J}(1)\,. \tag{1.13}$$

The inverse of $\tilde{U}_{\alpha;\beta}(1;z)$ is defined by

$$\tilde{U}_{\alpha;\bar{\beta}}(1;z)\tilde{U}^{-1}_{\bar{\beta};\beta}(1;z)=\tilde{U}^{-1}_{\alpha;\bar{\beta}}(1;z)\tilde{U}_{\bar{\beta};\beta}(1;z)=\delta_{\alpha;\beta} \tag{1.14}$$

Finally, the single-particle Liouville operators in the above represent commutation with the kinetic energy of a single electron or ion,

$$L_1(1)\theta\equiv[\vec{p}^{\,2}(1)/2m,\theta]\,,\quad L_2(1)\theta\equiv[\vec{P}^2(1)/2M,\theta]\,, \tag{1.15}$$

while the two-particle Liouville operators represent commutation with the appropriate potential energy operator,

$$L_{11}\theta\equiv[V_{11},\theta]\,,\quad L_{22}\theta\equiv[V_{22},\theta]\,,\quad L_{12}\theta\equiv[V_{12},\theta]\,. \tag{1.16}$$

In the single-component case, it was shown that the static, $z$-independent, limit of $\tilde{\Sigma}$ could be isolated, and the remainder analyzed through a formally exact closure of the hierarchy at the two-particle level. This program can also be carried out here with the result that

$$\tilde{\Sigma}_{\alpha;\beta}(1;z)=B_{\alpha;\beta}(1)+\tilde{M}_{\alpha;\beta}(1;z)\,, \tag{1.17}$$

where

$$B_{\alpha;\beta}(1)\equiv\mathrm{Tr}_2^{(\bar{\alpha})}L_{\alpha\bar{\alpha}}(12)U_{\alpha\bar{\alpha};\bar{\beta}}(12)U^{-1}_{\bar{\beta};\beta}(1)\,, \tag{1.18}$$

and

$$\tilde{M}_{\alpha;\beta}(1;z)\equiv\mathrm{Tr}_2^{(\bar{\alpha})}L_{\alpha\bar{\alpha}}(12)\tilde{G}_{\alpha\bar{\alpha};\bar{\beta}\bar{\gamma}}(12;z)L_{\bar{\beta}\bar{\gamma}}(12)\times[U^{-1}_{\bar{\beta};\beta}(1)+U^{-1}_{\bar{\gamma};\beta}(2)\mathcal{P}_{12}]\,. \tag{1.19}$$

The permutation operator, $\mathcal{P}_{ij}$, interchanges the labels $i$ and $j$, and the $U$'s without tildes are the static limits,

$$U_{\alpha\beta\ldots\nu,\alpha'}(1\cdots n)\equiv\lim_{z\to\infty}z\tilde{U}_{\alpha\beta\ldots\nu;\alpha'}(1\cdots n;z)\,, \tag{1.20}$$

which can be written entirely in terms of the equilibrium correlations in the system. The transformations, $\tilde{G}_{\alpha\beta,\gamma\delta}(12;z)$, are the multicomponent generalization of the $\tilde{G}$ operator discussed in I. They are to be determined from the coupled set of equations

$$[z-L_\alpha(1)-L_\beta(2)-L_{\alpha\beta}(12)]\tilde{G}_{\alpha\beta;\gamma\delta}(12;z)-\tilde{\Sigma}^{(2)}_{\alpha\beta;\bar{\alpha}\bar{\beta}}(12;z)\tilde{G}_{\bar{\alpha}\bar{\beta};\gamma\delta}(12;z)=G_{\alpha\beta;\gamma\delta}(12)\,, \tag{1.21}$$

where

$$\tilde{\Sigma}_{\alpha\beta;\gamma\delta}(12;z)\equiv B^{(2)}_{\alpha\beta;\gamma\delta}(12)+\tilde{M}^{(2)}_{\alpha\beta;\gamma\delta}(12;z)\,, \tag{1.22}$$

$$B^{(2)}_{\alpha\beta;\gamma\delta}(12)\equiv\mathrm{Tr}_3^{(\bar{\alpha})}[L_{\alpha\bar{\gamma}}(13)+L_{\beta\bar{\gamma}}(23)]\times G_{\alpha\beta\bar{\gamma};\bar{\alpha}\bar{\beta}}(123)G^{-1}_{\bar{\alpha}\bar{\beta};\gamma\delta}(12)-U_{\alpha\beta;\bar{\gamma}}(12)U^{-1}_{\bar{\gamma};\gamma}(1)\,\mathrm{Tr}_3^{(\delta)}L_{\gamma\delta}(13)\mathcal{P}_{23}\,, \tag{1.23}$$

$$\tilde{M}^{(2)}_{\alpha\beta;\gamma\delta}(12;z)\equiv\mathrm{Tr}_3^{(\bar{\gamma})}[L_{\alpha\bar{\gamma}}(13)+L_{\beta\bar{\gamma}}(23)]\times[\tilde{G}_{\alpha\beta\bar{\gamma};\bar{\alpha}\bar{\beta}}(123;z)\tilde{G}^{-1}_{\bar{\alpha}\bar{\beta};\gamma\delta}(12;z)-G_{\alpha\beta\bar{\gamma},\bar{\alpha}\bar{\beta}}(123)G^{-1}_{\bar{\alpha}\bar{\beta};\gamma\delta}(12)]\,. \tag{1.24}$$

In the form Eq. (1.17), $\tilde{\Sigma}_{\alpha;\beta}(1;z)$ is the sum of a static, generalized mean-field operator, $B_{\alpha;\beta}(1)$, and a "collision" operator $\tilde{M}_{\alpha;\beta}(1;z)$. The mean field in Eq. (1.18) reflects the effects of static, many-body correlations upon the motion of a single particle. Equation (1.19) displays the collision operator in terms of the solution to the effective two-body problem defined by Eqs. (1.21)–(1.24). The following two sections (Secs. II and III) of this paper will discuss two related approximations to the two-body dynamics which are apppropriate to the calculation of the electrical conductivity in a dense plasma.

The simplest approximation, described in

Sec. II, is a multicomponent version of the disconnected approximation (DA) discussed by Gould and Mazenko[4] and Wallenborn and Baus.[5] This approximation leads to a result which is a generalization of the usual Ziman expression.[6] However, this description of the plasma does not include any of the strong collision ("ladder") terms which are required to replace the Born cross section with a $t$-matrix description. These types of terms are included in Sec. III, and it is shown that the collision operator may be approximated by a Boltzmann form which incorporates the effects of both long-range screening and close collisions. In this form, the collision operator is very similar to that first proposed by Gould[7] and used to describe conductivity by Williams and DeWitt.[8] In each of the results described above, the coupling of the ions is considered to be arbitrarily strong, so neither description is limited to a Debye plasma. The expressions derived in Secs. II and III are used in Sec. IV to obtain numerical estimates of the effects of strong ion coupling, bound electrons, and strong collisions.

## II. WEAK COLLISIONS AND THE ZIMAN LIMIT

The pair of equations (1.9) and (1.10) represent a coupled set of equations for $\tilde{\psi}_1(1;z)$ and $\tilde{\psi}_2(1,z)$ which are to be solved simultaneously. However, the very small electron-ion mass ratio may be exploited to gain a considerable simplification of this problem. The term in the electron equation which involves $\tilde{\psi}_2$ may be shown to be $m/M$ times as small as the term involving $\tilde{\psi}_1$ (Ref. 9). Hence, if the frequency of interest is in the range

$$(m/M)\omega_e \ll \omega \ll \omega_e , \tag{2.1}$$

where $\omega_e$ is the electron collision frequency then, as a first approximation, $\tilde{\psi}_1(1;z)$ can be found from the single equation,

$$[z - B_{1;1}(1) - \tilde{M}_{1;1}(1;z)]\tilde{\psi}_1(1;z) = \psi_1(1) . \tag{2.2}$$

The fact that $\tilde{\psi}_1(1;z)$ must be diagonal in the momentum representation has been used to drop the free streaming term. As discussed in the Appendix, Eq. (2.2) may be used to write the electron contribution to the electrical conductivity approximately as

$$\sigma = n_1 e^2/m\nu_e \tag{2.3}$$

where $n_\alpha$ is the density of species $\alpha$, and $\nu_e$ is the imaginary part of the electron collision frequency,

$$\omega_e \equiv \frac{-\beta}{3mn_1}\frac{1}{\Omega}\,\mathrm{Tr}_1^{(1)}\vec{p}(1)\cdot\tilde{M}_{1;1}(1;0+i\eta)\,U_{1;1}(1)\vec{p}(1) . \tag{2.4}$$

Hence, the conductivity is known as soon as an explicit expression for the collision operator is obtained.

In order to accomplish this, some reasonable approximation to Eq. (1.19) must be made. In this section, a multicomponent generalization of the disconnected approximation[4,5] will be discussed. Specifically, Eq. (3.21) of I is generalized to

$$\begin{aligned}\tilde{M}_{1;1}^{(\mathrm{DA})}(1;z) = -i\int_0^\infty dt\, e^{izt}\{&\mathrm{Tr}_2^{(1)}L_{11}(12)\,U_{1;\bar{\alpha}}(1;t)\,U_{1;\bar{\beta}}(2;t)\,U^{-1}_{\bar{\alpha};\bar{\gamma}}\,U^{-1}_{\bar{\beta};\bar{\delta}}(2)\\ &\times G_{\bar{\gamma}\bar{\delta};\bar{\bar{\alpha}}\bar{\bar{\beta}}}(12)\,L_{\bar{\bar{\alpha}}\bar{\bar{\beta}}}(12)[\,U^{-1}_{\bar{\bar{\alpha}};1}(1)+U^{-1}_{\bar{\bar{\beta}};1}(2)\mathcal{P}_{12}]\\ &+\mathrm{Tr}_2^{(2)}L_{12}(12)\,U_{1;\bar{\alpha}}(1;t)U_{2;\bar{\beta}}(2;t)\;U^{-1}_{\bar{\alpha};\bar{\gamma}}(1)\,U^{-1}_{\bar{\beta};\bar{\delta}}(1)G_{\bar{\gamma}\bar{\delta};\bar{\bar{\alpha}}\bar{\bar{\beta}}}(12)\\ &\times L_{\bar{\bar{\alpha}}\bar{\bar{\beta}}}(12)[U^{-1}_{\bar{\bar{\alpha}};1}(1)+U^{-1}_{\bar{\bar{\beta}};1}(2)\mathcal{P}_{12}]\}.\end{aligned} \tag{2.5}$$

This result may be interpreted just as in I. The two colliding particles propagate independently through the background plasma between interactions. However, the "propagators" in this case are a little more complicated in that they now form matrices.

### A. The factorization approximation

To proceed further requires a detailed analysis of the equilibrium correlations in $G_{\alpha\beta;\gamma\delta}(12)$. However, a very interesting result may be obtained by simply writing

$$G_{\alpha\beta;\gamma\delta}(12) \to U_{\alpha;\gamma}(1)\,U_{\beta;\delta}(2) . \tag{2.6}$$

Using this and Eq. (1.14) allows Eq. (2.5) to be rewritten as

$$\begin{aligned}\tilde{M}_{1;1}^{(\mathrm{FA})}(1;z) \equiv -i\int_0^\infty dt\, e^{izt}\{&\mathrm{Tr}_2^{(1)}L_{11}(12)\,U_{1;\bar{\alpha}}(1;t)\,U_{1;\bar{\beta}}(2;t)\,L_{\bar{\alpha}\bar{\beta}}(12)[\,U^{-1}_{\bar{\alpha};1}(1)+U^{-1}_{\bar{\beta};1}(2)\mathcal{P}_{12}]\\ &+\mathrm{Tr}_2^{(2)}L_{12}(12)\,U_{1;\bar{\alpha}}(1;t)\;U_{2;\bar{\beta}}(2;t)\;L_{\bar{\alpha}\bar{\beta}}(12)[\,U^{-1}_{\bar{\alpha};1}(1)+U^{-1}_{\bar{\beta};1}(2)\mathcal{P}_{12}]\}.\end{aligned} \tag{2.7}$$

This approximation is considerably simpler than the disconnected approximation, and due to the form of Eq. (2.6) it will be called the factorization approximation (FA). The classical version of this approximation in a one-component system is what leads to the usual form of the Lenard-Balescu equation.[10]

Introducing the Weyl representation of the potential

$$V_{\alpha\beta}(12)=\int \frac{d^3k}{(2\pi)^3}\, v_{\alpha\beta}(k)e^{i\vec{k}\cdot[\vec{r}_\alpha(1)-\vec{r}_\beta(2)]}, \tag{2.8}$$

and using the identities

$$\frac{1}{\Omega}\,\mathrm{Tr}_1^{(\alpha)} e^{i\vec{k}\cdot\vec{r}_\alpha(1)}U_{\alpha;\beta}(1;t)e^{i\vec{k}'\cdot\vec{r}_\beta(2)}=\delta_{k,-k'}\,S_{\alpha\beta}(k;t), \tag{2.9}$$

$$[\vec{p}(1),e^{i\vec{k}\cdot\vec{r}(1)}]=-\vec{k}e^{i\vec{k}\cdot\vec{r}(1)}, \tag{2.10}$$

$$U^{-1}_{2;1}(1)U_{1;1}(1)\vec{p}(1)=0, \tag{2.11}$$

$$U^{-1}_{1;1}(1)U_{1;1}(1)\vec{p}(1)=\vec{p}(1), \tag{2.12}$$

yields the principal result of this section[11]

$$\omega_e\to\frac{i\beta}{3mn_1}\int_0^\infty dt\, e^{-\eta t}\int\frac{d^3k}{(2\pi)^3}\,v_{12}^2(k)k^2[\bar{S}_{11}(k;t)\bar{S}_{2,2}(k;t)-\bar{S}^2_{12}(k;t)]. \tag{2.13}$$

Owing to the conservation of momentum, the first term in Eq. (2.7) does not contribute to Eq. (2.13). The time-dependent structure factors in Eq. (2.13) are the density-density correlation functions in a true two-component system. However, to lowest order in the electron-ion interaction, they reduce to

$$\begin{aligned}&\bar{S}_{11}(k;t)\to\bar{S}_e(k;t),\\ &\bar{S}_{22}(k;t)\to\bar{S}_i(k;t),\\ &\bar{S}_{12}(k;t)\to 0,\end{aligned} \tag{2.14}$$

where $\bar{S}_e$ and $\bar{S}_i$ are the structure factors for an electron gas and a one-component plasma, respectively. Therefore, to second order in $v_{12}$, the imaginary part of $\omega_e$ may be expressed as

$$\mathrm{Im}\omega_e\equiv\nu_e\to\frac{\beta}{6mn_1}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}v_{12}^2(k)k^2S_e(k_1-\omega)S_i(k;\omega), \tag{2.15}$$

where

$$S(k;\omega)\equiv\int_{-\infty}^{\infty}dt\,e^{i\omega t}\bar{S}(k;t). \tag{2.16}$$

The result in Eq. (2.15) is fairly simple and can be understood in terms of heuristic arguments based upon Van Hove's theory of the scattering of a beam of particles by the density fluctuations in a gas.[12]

### B. Ziman's theory

The contact with Ziman's theory of electrical conductivity[6] occurs through the following substitutions for the structure factors,

$$S_i(k;\omega)=\frac{2\pi}{\omega}\int\frac{d^3P}{(2\pi)^3}\,\frac{\vec{k}\cdot\vec{P}}{M}f_2(P)\times\delta\left(\omega-\frac{\vec{k}\cdot\vec{P}}{M}\right)|\epsilon_i(k;\omega)|^{-2}, \tag{2.17}$$

$$\epsilon_i(k;\omega)=1+\beta^{-1}\bar{C}_i(k)X_i(k;\omega), \tag{2.18}$$

$$X_i(k;\omega)=\beta\int\frac{d^3P}{(2\pi)^3}\,\vec{k}\cdot\frac{\vec{P}}{M}\left(\frac{f_2(P)}{\omega+i\eta-\vec{k}\cdot\vec{P}/M}\right), \tag{2.19}$$

and

$$S_e(k;\omega)=\frac{4\pi}{(1-e^{-\beta\omega})}\int\frac{d^3p}{(2\pi)^3}f_1(p+k)\tilde{f}_1(p)(1-e^{-\beta\omega_k})\times\delta(\omega+\omega_k)|\epsilon_e(k;\omega)|^{-2}, \tag{2.20}$$

$$\epsilon_e(k;\omega)=1-\frac{4\pi e^2}{k^2}X_e(k;\omega), \tag{2.21}$$

$$X_e(k;\omega)=-2\int\frac{d^3p}{(2\pi)^3}\,\frac{f_1(p+k)\tilde{f}_1(p)}{\omega+i\eta+\omega_k}(1-e^{-\beta\omega_k}), \tag{2.22}$$

where $\omega_k=(-\vec{k}/m)\cdot(\vec{p}+\vec{k}/2)$ and $\bar{C}_i(k)$ is the direct correlation function for a one-component plasma. These approximations to the structure factors are exactly what would be obtained from the short-time equations discussed in Sec. III of paper I with $B(1)$ for the electrons given by Eq. (3.15) of I and $B(1)$ for the ions given by its exact classical limit. Substituting Eqs. (2.17) and (2.20) into Eq. (2.15) and performing the integrations over the delta functions yields

$$\nu_e=\frac{\beta}{6Z\pi^{3/2}}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{p}\,f_1(p)\int_{-A}^{A}dY\,e^{-Y^2}\int_0^{2p(1+Y/A)}dk\,k^3\left|v_S\left(k;\left(\frac{2}{\beta M}\right)^{1/2}kY\right)\right|^2\left|\epsilon_i\left(k;\left(\frac{2}{\beta M}\right)^{1/2}kY\right)\right|^{-2}\times\tilde{f}_1\left(p^2+2m\left(\frac{2}{\beta M}\right)^{1/2}kY\right)+\left[\int_A^\infty dY\,e^{-Y^2}\int_{-2p(1-Y/A)}^{2p(1+Y/A)}dk\,k^3\left|v_S\left(k;\left(\frac{2}{\beta M}\right)^{1/2}kY\right)\right|^2\left|\epsilon_i\left(k;\left(\frac{2}{\beta M}\right)^{1/2}kY\right)\right|^{-2}\times\tilde{f}_1\left(p^2+2m\left(\frac{2}{\beta M}\right)^{1/2}kY\right)\right], \tag{2.23}$$

with

$$Y \equiv \left(\frac{\beta M}{2k^2}\right)^{1/2}\left(\frac{\vec{k}\cdot\vec{p}}{m}+\frac{k^2}{2m}\right)=-\left(\frac{\beta M}{2k^2}\right)\omega_k\,, \tag{2.24}$$

$$A \equiv \frac{p}{m}\left(\frac{\beta M}{2}\right)^{1/2}=\frac{p}{p_T}\left(\frac{M}{m}\right)^{1/2}\gg 1\,, \tag{2.25}$$

$$p_T \equiv (2m/\beta)^{1/2}\,, \tag{2.26}$$

$$v_S(k;\omega)\equiv v_{12}(k)/\epsilon_e(k;\omega)\,. \tag{2.27}$$

For a typical electron-ion collision,

$$\left(\frac{2}{\beta M}\right)^{1/2} kY \sim \left(\frac{m}{M}\right)^{1/2}\bar{E}(\beta\bar{E})^{-1/2}\ll\bar{E}\,,$$

where $\bar{E}$ is an average electron energy. These factors can be set to zero in the electron terms, but they may not be ignored in the ion terms, since they are not negligible in comparison to the other terms appearing in Eq. (2.19). This approximation will be particularly accurate in the degenerate limit,[13] because in that limit $(\beta\bar{E})^{-1/2}\to 0$. Making this approximation and taking $A\to\infty$ leads to the considerably simplified expression,

$$\nu_e \to \frac{4\beta Ze^4}{3\pi}\int_0^\infty dp\, f_1(p)\bar{f}_1(p)p\Lambda(p)$$

$$= -\frac{4mZe^4}{3\pi}\int_0^\infty dp\,\frac{\partial f_1(p)}{\partial p}\,\Lambda(p)\,, \tag{2.28}$$

$$\Lambda(p)\equiv\left(\frac{1}{4\pi Ze^2}\right)^2\int_0^{2p} dk\,k^3\,|v_S(k;0)|^2 S_i(k)\,, \tag{2.29}$$

where $S_i(k)$ is the exact structure factor for the one-component plasma,

$$S_i(k)\equiv\frac{1}{n_2}\int_{-\infty}^{\infty} d\omega\, S_i(k;\omega)$$

$$=\frac{\pi^{-1/2}}{n_2}\int_{-\infty}^{\infty} dx\, e^{-x^2}\left|\epsilon_i\left(k;\left(\frac{2}{\beta M}\right)^{1/2}kx\right)\right|^{-2}. \tag{2.30}$$

Equations (2.28) and (2.29) combined are the Ziman expression for the collision frequency in a plasma, generalized to finite temperature. The precise Ziman form occurs in the degenerate limit where $\partial f_1/\partial p \to -\delta(p-p_F)$ and the conductivity, Eq. (2.3), becomes

$$\sigma \to \frac{3\pi n_1}{4Zm^2e^2\Lambda(p_F)}\,, \tag{2.31}$$

which is precisely the expression used by Minoo, Deutsch, and Hansen.[14] Similarly, in the nondegenerate limit, Eq. (2.3) reduces to

$$\sigma \to \frac{3}{16\pi^{1/2}Ze^2m^2\langle\Lambda\rangle}\left(\frac{2m}{\beta}\right)^{3/2}, \tag{2.32}$$

$$\langle\Lambda\rangle\equiv\frac{\beta}{m}\int_0^\infty dp\, p e^{-\beta p^2/2m}\Lambda(p)\,, \tag{2.33}$$

which is the single-polynomial approximation to the Lorentz gas conductivity[15] with Coulomb logarithm replaced by $\langle\Lambda\rangle$. This differs from the exact Loretnz gas result by a factor of $32/3\pi$.

The reason this approximation is so much more accurate in the degenerate limit is that, in that case, $\partial f_1/\partial p \to -\delta(p-p_F)$, and $\langle\Lambda^{-1}\rangle$ and $\langle\Lambda\rangle^{-1}$ become indistinguishable.[16] Significant improvement in the classical case can be obtained by going one step further to a two-polynomial approximation,[17] but that will not be pursued here.

### C. Initial correlations in the classical limit

Sections II A and II B discussed the factorization approximation and its connection with the Ziman expression for the electrical conductivity. The factorization approximation is very simple, but its simplicity comes at the expense of neglecting the initial static correlations contained in the $G_{\alpha\beta,\gamma\delta}$'s. Quantum mechanically the evaluation of these correlations requires the evaluation of the first four reduced distribution operators. However, in the classical limit only two-body correlations are required, since in that limit it can be shown that[18]

$$G_{\alpha\beta;\bar{\alpha}\bar{\beta}}(12)L_{\bar{\alpha}\bar{\beta}}(12)[y_{\bar{\alpha}}(p_1)+y_{\bar{\beta}}(p_2)]$$

$$\to -i\beta^{-1}f^{(2)}_{\alpha\beta}(12)\vec{\nabla}_1 \ln g_{\alpha\beta}(\vec{r}_1-\vec{r}_2)\left(\frac{\partial y_\alpha}{\partial\vec{p}_1}-\frac{\partial y_\beta}{\partial\vec{p}_2}\right), \tag{2.34}$$

where $g_{\alpha\beta}(r)$ is the radial distribution function for a particle of species $\alpha$ and another of species $\beta$. Substituting this into the classical limit of Eq. (2.5) and using the fact that

$$U^{-1}_{\alpha;\beta}(1)\to\delta_{\alpha;\beta}f^{-1}_\alpha(p_1)-\mathrm{Tr}_2 c_{\alpha\beta}(\vec{r}_1-\vec{r}_2)\Phi_{12}\,, \tag{2.35}$$

with the direct correlation functions, $c_{\alpha\beta}$, given by a multicomponent generalization of the Ornstein-Zernike equation

$$c_{\alpha\beta}(\vec{r}_1,\vec{r}_2)\equiv h_{\alpha\beta}(\vec{r}_1-\vec{r}_2)-n_{\bar{\beta}}\int d^3r_3\, c_{\alpha\bar{\beta}}(\vec{r}_1-\vec{r}_3)$$

$$\times h_{\bar{\beta}\beta}(\vec{r}_3-\vec{r}_2)\,, \tag{2.36}$$

$$h_{\alpha\beta}(\vec{r}_1-\vec{r}_2)\equiv g_{\alpha\beta}(\vec{r}_1-\vec{r}_2)-1\,, \tag{2.37}$$

leads to

$$\tilde{M}_{1;1}(1;z) \to -i\int_0^\infty dt\, e^{izt}\{\mathrm{Tr}_2^{(2)} L_{11}(12) U_{1;\bar{\alpha}}(1;t) U_{1;\bar{\beta}}(2;t)\mathcal{L}_{\bar{\alpha}\bar{\beta}}(12)[U^{-1}_{\bar{\alpha};1}(1) + U^{-1}_{\bar{\beta};1}(2)\Phi_{12}]$$

$$+ \mathrm{Tr}_2^{(2)} L_{12}(12) U_{1;\bar{\alpha}}(1;t) U_{\alpha;\bar{\beta}}(2;t)\mathcal{L}_{\bar{\alpha}\bar{\beta}}(12)[U^{-1}_{\bar{\alpha};1}(1) + U^{-1}_{\bar{\beta};1}(2)\Phi_{12}]\}\,. \tag{2.38}$$

In the above, the trace operation is now understood to represent an integration over the appropriate positions and momenta. Equation (2.38) is identical to Eq. (2.7) with $L_{\alpha\beta}(12)$ replaced by

$$\mathcal{L}_{\alpha\beta}(12) \equiv -i\beta^{-1}\nabla_1 c_{\alpha\beta}(\vec{r}_1 - \vec{r}_2)\left(\frac{\partial}{\partial \vec{p}_1} - \frac{\partial}{\partial \vec{p}_2}\right). \tag{2.39}$$

In other words, some of the potentials in Eq. (2.7) have been replaced by the corresponding direct correlation functions. As a result, the collision frequency found from Eq. (2.38) is

$$\omega_e = \frac{i}{3mn}\int_0^\infty dt\, e^{-\eta t}\int \frac{d^3k}{(2\pi)^3}\, v_{12}(k) c_{12}(k) k^2 \times [\bar{S}_{11}(k;t)\bar{S}_{22}(k;t) - \bar{S}_{12}^2(k;t)]\,, \tag{2.40}$$

which is identical to Eq. (2.13) with one of the potentials replaced by $c_{12}(k)$. Equation (2.40) is "asymmetric" in $c_{12}(k)$ because the collision operator was not made "symmetric" in the $G$'s. As a result, the high-frequency behavior of $\tilde{M}_{1;1}(1;z)$ has been preserved.[19]

## III. STRONG ELECTRON-ION COLLISIONS

### A. Approximate collision operator

In both its quantum and classical forms, the disconnected approximation discussed in Sec. II pictures the electron and ion as propagating independently between two interaction vertices, and the "ladder-type" terms which are required to adequately describe strong, close collisions are absent. The purpose of this section is to develop a more complete picture of electron-ion scattering which includes such terms.

To begin, it will be assumed that the electron-ion contribution to $\tilde{\Sigma}_{1;1}(1;z)$ is dominant.

$$\tilde{\Sigma}_{1;1}(1;z) \to \mathrm{Tr}_2^{(2)} L_{12}(12) U_{12;\bar{\alpha}}(12) U^{-1}_{\bar{\alpha};1}(1) + \mathrm{Tr}_2^{(2)} L_{12}(12)\tilde{G}_{12;\bar{\alpha}\bar{\beta}}(12;z) L_{\bar{\alpha}\bar{\beta}}(12)(U^{-1}_{\bar{\alpha};1}(1) + U^{-1}_{\bar{\beta};1}(2)\Phi_{12})\,. \tag{3.1}$$

This assumption is justifed, in part, by the fact that the first term in Eq. (2.5) made no contribution to $\omega_e$. Furthermore, it is known that electron-ion collisions dominate electron-electron collisions in a very degenerate plasma in which the electrons are "frozen" into a Fermi sea.[20] Electron-electron collisions appear to become more important as the electrons become less degenerate,[13] but even in a classical plasma they can be negligible if the ionization state of the plasma is high enough.[21] In any case they appear rarely to reduce the conductivity by more than about a factor of 2 (Refs. 3, 21). Therefore, even in regions where it is not strictly appropriate, neglecting electron-electron collisions should give a fairly good first estimate of the conductivity, especially in high-$Z$ plasmas.

In principle, $\tilde{G}_{12;\alpha\beta}(12;z)$, should be determined from the coupled set of equations (1.21). However, these equations will be separated just as those for $\tilde{\psi}_1^{(1)}$ and $\tilde{\psi}_2^{(1)}$. With this simplification, the collision part of Eq. (3.1) may be written as

$$\tilde{M}_{1,1}(1;z) \to \mathrm{Tr}_2^{(2)} L_{12}(12)[z - L_1(1) - L_2(2) - L_{12}(12) - B^{(2)}_{12;12}(12) - \tilde{M}_{12;12}(12;z)]^{-1} \times f_{12}^{(2)}(12)\mathcal{G}_{12;\bar{\alpha}\bar{\beta}}(12)[U^{-1}_{\bar{\alpha};1}(1) + U^{-1}_{\bar{\beta};1}(2)\Phi_{12}]\,, \tag{3.2}$$

where

$$\mathcal{G}_{12;\alpha\beta}(12) \equiv [f_{12}^{(2)}(12)]^{-1} G_{12;\alpha\beta}(12) L_{\alpha\beta}(12)\,, \tag{3.3}$$

is a "shielded interaction."[22]

As it stands, Eq. (3.2) is still very formidable, and in order to proceed the following set of approximations will be introduced.

(1) Treat the ions classically; in particular, neglect ion-ion exchange terms and allow the ion-ion correlation operators to factor into momentum and position-dependent parts.

(2) Ignore multiple collisions; i.e., set $\tilde{M}^{(2)}_{12;12}(12, z) = 0$.

(3) Evaluate $B^{(2)}_{12;12}(12)$ and $\mathcal{G}_{12;\bar{\alpha}\bar{\beta}}(12)[U^{-1}_{\bar{\alpha}}(1) + U^{-1}_{\bar{\beta};1}(2)\Phi_{12}]$ to lowest order in the electron-ion interaction; i.e., neglect the static correlations between the electron and ion systems in these terms.

Using these approximations allows the collision operator, Eq. (3.2), to be written as

$$\tilde{M}_{1;1}(1;z) \to \mathrm{Tr}_2^{(2)} L_{12}(12)[z - L_1'(1) - L_2'(2) - \mathcal{L}_{12}'(12) - \Delta_{12}(12)]^{-1} f_{12}^{(2)}(12)\mathcal{L}_{12}(12) U^{-1}_{1;1}(1)\,. \tag{3.4}$$

The new quantities appearing in the above are defined by,

$$L_1'(1) \equiv L_1(1) + \mathrm{Tr}_2^{(1)} L_{11}(12) U_{11;1}(12) U_{1;1}^{-1}(1)\,, \tag{3.5}$$

$$L_2'(2) \equiv L_2(2) + \mathrm{Tr}_3^{(2)} L_{22}(23) U_{22;2}(23) U_{2;2}^{-1}(2)\,, \tag{3.6}$$

$$\begin{aligned}\mathcal{L}_{12}'(12) y_{12}(12) \equiv{}& \tilde{f}_1(1) V_{12}(12) y_{12}(12) - y_{12}(12) V_{12}(12) \tilde{f}_1(1) + \mathrm{Tr}_3^{(2)} L_{12}(13) g_{22}^{(2)}(23) f_2^{-1}(2) y_{12}(12) \\ &+ \mathrm{Tr}_3^{(1)} L_{21}(23) g_{11}^{(2)}(13) f_1^{-1}(1) y_{12}(12)\,,\end{aligned} \tag{3.7}$$

$$\mathcal{L}_{12}(12) \equiv f_1^{-1}(1) f_2^{-1}(2) U_{1;1}(1) U_{2;2}(2) L_{12}(12)\,, \tag{3.8}$$

$$\begin{aligned}\Delta_{12}(12) y_{12}(12) \equiv{}& \mathrm{Tr}_3^{(2)} L_{12}(13) g_{22}^{(2)}(23) f_2^{-1}(3) y_{12}(13) \\ &+ \mathrm{Tr}_3^{(1)} [V_{21}(23) \hat{P}_{13} f_1(1) y_{12}(32) - f_1(1) g_{12}(32) \hat{P}_{13} V_{21}(23)] \\ &+ \mathrm{Tr}_3^{(1)} L_{21}(23) g_{11}^{(2)}(23) f_1^{-1}(3) g_{12}(32) + \mathrm{Tr}_{3,4}^{(2,2)} L_{12}(13) \lambda_{22;2}(23;4) y_{12}(14) \\ &+ \mathrm{Tr}_{3,4}^{(1;1)} L_{21}(23) \lambda_{11;1}(13;4) y_{12}(42)\,,\end{aligned} \tag{3.9}$$

where $y_{12}(12)$ is an arbitrary electron-ion operator, and the $\lambda$'s and $g$'s are multicomponent generalizations of Eqs. (B1) and (B14) in I. The single-particle operators, Eqs. (3.5) and (3.6), propagate the electron through the background of electrons and the ion through the background of ions. Retaining them alone in the "denominator" of Eq. (3.4) and using the ideal-gas limit of $f_{12}^{(2)}(12)$ leads to the Ziman expression discussed in Sec. II B. The effects of strong collisions are contained in $\mathcal{L}_{12}'(12)$. According to Eq. (3.7), $\mathcal{L}_{12}'(12)$ includes both the degeneracy modifications of the potential, which occur even for short-ranged forces,[23] and the long-range screening effects due to electron-electron and ion-ion correlations. From Eq. (3.9), all of the contributions to $\Delta_{12}$ involve traces over the electron-ion interaction and the operand. Therefore, $\Delta_{12}$ contains the dynamic correlations between the electron and ion subsystems. Since the essence of approximation (3) is to neglect the corresponding static correlations, it is consistent to make a fourth approximation;

(4) Neglect dynamic correlations between the electron and ion subsystems; i.e., set $\Delta_{12}(12) = 0$.

Approximations (1)–(4) then lead to the primary result of this section

$$\begin{aligned}M_{1;1}(1;z) \to{}& \mathrm{Tr}_2^{(2)} L_{12}(12) [z - L_1'(1) - L_2'(2) - \mathcal{L}_{12}'(12)]^{-1} \\ &\times f_{12}^{(2)}(12) \mathcal{L}_{12}(12) U_{1;1}^{-1}(1)\,,\end{aligned} \tag{3.10}$$

where all of the operators are defined in Eqs. (3.5)–(3.8). The corresponding result for the mean-field operator is

$$B_{1,1}(1) y_1(1) \to \mathrm{Tr}_2^{(2)} L_{12}(12) [f_{12}^{(2)}(12) f_1^{-1}(1) y_1(1) - f_1(1) U_{1;1}^{-1}(1) y_1^{(1)} g_{12}^{(2)}(12)]\,, \tag{3.11}$$

where $y_1(1)$ is an arbitrary electron operator. The second term in Eq. (3.11) results from considering electron exchange. These expressions must be supplemented by an appropriate expression for the equilibrium correlation function $f_{12}^{(2)}(12)$. Applying approximations (1)–(3) to the equilibrium hierarchy shows that a self-consistent $f_{12}^{(2)}(12)$ should be the solution to

$$[L(1) + L(2) + \mathcal{L}_{12}'(12)] f_{12}^{(2)}(12) = 0\,. \tag{3.12}$$

### B. Further simplifications and a $t$-matrix representation

In order to further explore the physical content of Eq. (3.10), the electrons will be modeled as a non-degenerate ideal gas. With this model

$$L_1'(1) \to L_1(1)\,, \tag{3.13}$$

and $L_2'(2)$ will be taken to be[24]

$$\begin{aligned}L_2'(2) y_2(2) &= L_2(2) y_2(2) \\ &\quad + \beta^{-1} \mathrm{Tr}_3^{(2)} [c_i(\vec{R}_2 - \vec{R}_3), f_2(2) y_2(3)] \\ &= [L_2(2) + B_2(2)] y_2(2)\,,\end{aligned} \tag{3.14}$$

where $c_i(\vec{R}_2 - \vec{R}_3)$ is the direct correlation function for a one-component plasma. Also, one finds that

$$\mathcal{L}_{12}'(12) \to \mathcal{L}_{12}(12) \to \mathcal{L}_S(12)\,, \tag{3.15}$$

where

$$\begin{aligned}\mathcal{L}_S(12) y_{12}(12) = \int \frac{d^3k}{(2\pi)^3} v_{12}(k) \{&[e^{i\vec{k}\cdot(\vec{r}_1 - \vec{R}_2)}, y_{12}(12)] + [S_i(k) - 1] e^{-i\vec{k}\cdot\vec{R}_2} [e^{i\vec{k}\cdot\vec{r}_1}, y_{12}(12)]\} \\ &- [V_S(12), y_{12}(12)]\,,\end{aligned} \tag{3.16}$$

and

$$V_S(12) \equiv \int \frac{d^3k}{(2\pi)^3} v_{12}(k) S_i(k) e^{i\vec{k}\cdot(\vec{r}_1 - \vec{R}_2)}\,, \tag{3.17}$$

is the electron-ion interaction screened by the static correlations among the ions. The arrow in Eq. (3.16) may be replaced by an equality whenever $y_{12}(12)$ does not contain the ion momentum.

However, due to the smallness of the electron-ion mass ratio, the change in the ion momentum is almost always a very small fraction of the ion momentum itself, so the simpler commutator representation of $\mathcal{L}_S$ should be very accurate even when $y_{12}(12)$ does contain ion momentum dependence.

Using Eqs. (3.13)–(3.15) in the collision operator, Eq. (3.10), yields

$$M_{1;1}(1;z) \to \mathrm{Tr}_2^{(2)} L_{12}(12)[z - L_1(1) - L_2(2) - \mathcal{L}_S(12) - B_2(2)]^{-1} f_{12}^{(2)}(12)\mathcal{L}_S(12) f_1^{-1}(1). \tag{3.18}$$

The role of $B_2(2)$ is the dynamic screening of the remaining bare interaction. This may be made explicit through the use of the identity

$$\mathcal{R}(12;z) \equiv [z - L_1(1) - L_2(2) - \mathcal{L}_S(12) - B_2(2)]^{-1} = \mathcal{R}_0(12;z) + \mathcal{R}_0(12;z) B_2(2) \mathcal{R}(12;z), \tag{3.19}$$

$$\mathcal{R}_0(12;z) \equiv [z - L_1(1) - L_2(2) - \mathcal{L}_S(12)]^{-1}, \tag{3.20}$$

to obtain the form

$$\tilde{M}_{1;1}(1;z) \to \mathrm{Tr}_2^{(2)} \mathcal{L}_D(12;\Delta z)\mathcal{R}_0(12;z) f_{12}^{(2)} \mathcal{L}_S(12) f_1^{-1}(1). \tag{3.21}$$

The dynamically screened interaction is defined by

$$\mathcal{L}_D(12;\Delta z) y_{12}(12) \equiv \int \frac{d^3k}{(2\pi)^3} v_{12}(k) \times [e^{ik\cdot(r_1 - R_2)} \epsilon_i^{-1}(k;\Delta z) y_{12}(12)], \tag{3.22}$$

where $\Delta z \equiv z - L_1(1)$ and the dielectric function is the same as that in Eq. (2.18). Consistent with approximation (4), the contribution of $V_{12}$ to the dielectric function has been neglected.[25]

Equation (3.21) simplifies even further in the low-frequency or Boltzmann limit, $z \to 0 + i\eta$. Using Eqs. (3.12) and (3.15) and noting that only the first term in Eq. (3.11) survives in the nondegenerate limit allows $\tilde{\Sigma}_{1;1}$ to be written as

$$\tilde{\Sigma}_{1;1}(1;0+i\eta)\tilde{\psi}_1(1;z) \to \mathrm{Tr}_2^{(2)}[L_{12}(12) - \mathcal{L}_D[12;0+i\eta - L_1(1)]f_2^{(2)}(12) f_1^{-1}(1)\tilde{\psi}_1(1;z) + \mathrm{Tr}_2^{(2)} \mathcal{L}_D(12;0+i\eta - L_1(1)) f_{12}^{(2)}(12)\, \hat{\Omega}_s f_1^{-1}(1)\, \tilde{\psi}_1(1;z) \hat{\Omega}_s^\dagger, \tag{3.23}$$

where

$$\hat{\Omega}_s \equiv \lim_{t\to\infty} e^{-itH_s} e^{itH_0}, \tag{3.24}$$

$$H_s \equiv H_0 + V_s, \quad H_0 \equiv p_1^2/2m + P_2^2/2M, \tag{3.25}$$

defines the Möller operator associated with the screened potential, $V_s$. The second-order approximation to the first term in Eq. (3.23) has been analyzed in the Appendix and it was found to be negligible compared to the second term. Hence, only the second term in Eq. (3.23) will be retained here. To further analyze this term, it is noted from Eqs. (3.12) and (3.16) that $f_{12}^{(2)}(12)$ must be a function of $H_s$,

$$f_{12}^{(2)}(12) = F(H_s),$$

such that

$$F(H_0) = f_1(1) f_2(2).$$

Using this and the so-called intertwining relation

$$H_s \hat{\Omega}_s = \hat{\Omega}_s H_0,$$

the momentum matrix element of Eq. (3.23) becomes

$$\langle p_1 | \tilde{\Sigma}_{1;1}(1;0+i\eta)\tilde{\psi}_1(1;z) | p_1 \rangle \to \sum_{p_2} [\langle p_1 p_2 | V_D(\Delta E_1) \hat{\Omega}_s f_2(2) \hat{\psi}_1(1;z) \hat{\Omega}_s^\dagger | p_1 p_2 \rangle - \langle p_1 p_2 | \hat{\Omega}_s f_2(2) \tilde{\psi}_1(1;z) \hat{\Omega}_s^\dagger V_D^\dagger(\Delta E_1) | p_1 p_2 \rangle], \tag{3.26}$$

where

$$\langle p_1 p_2 | V_D(\Delta E_1) | p_1' p_2' \rangle \equiv v_{12}(k)/\Omega\, \epsilon^{-1}(p_1 - p_1';\ (p_1^2 - p_1'^2)/2m)\, \delta_{p_1+p_2;p_1'+p_2'}. \tag{3.27}$$

The Born limit of Eq. (3.26) reproduces the Ziman expression Eq. (2.28) with electron screening ignored, but further analysis of Eq. (3.26) is hampered by the appearance of both the static and dynamically screened potentials. If $V_D(\Delta E)$ were to be replaced by $V_s$, then Eq. (3.26) would take the Boltzmann form,

$$\langle p_1|\tilde{\Sigma}_{1;1}(1;\,0+\eta)\tilde{\psi}_1(1;\,z)|p_1\rangle - \langle p_1|\Sigma_s(1)\tilde{\psi}_1(1;\,z)|p_1\rangle$$
$$\equiv 2\pi i \sum_{\bar{p}_2}\sum_{\bar{p}_1\bar{p}_2} |\langle p_1 p_2|T_s|\bar{p}_1\bar{p}_2\rangle|^2 \delta(E_1+E_2-\bar{E}_1-\bar{E}_2)[f_2(\bar{p}_2)\tilde{\psi}(\bar{p}_1,z)-f_2(p_2)\tilde{\psi}(p_1;\,z)], \quad (3.28)$$

where $T_s \equiv V_s\hat{\Omega}_s$ is the $t$-matrix associated with $V_s$. However, this expression is incorrect in the Born limit, so rather than writing the collision operator simply in the form, Eq. (3.28), the Born contribution to Eq. (3.26) will be explicitly isolated and the collision operator written as[7]

$$\langle p_1|\tilde{\Sigma}_{1;1}(1;\,0+i\eta)\tilde{\psi}_1(1;\,z)|p_1\rangle = \langle p_1|\tilde{\Sigma}_2(1)\tilde{\psi}_1(1;\,z)|p_1\rangle + \langle p_1|\tilde{\Sigma}_3(1)\tilde{\psi}_1(1;\,z)|p_1\rangle, \quad (3.29)$$

where $\tilde{\Sigma}_2$ is precisely the dynamic Born term analyzed in Sec. II B with electron screening effects ignored. If $V_D$ is now replaced by $V_s$ in $\tilde{\Sigma}_3$ only, then Eq. (3.26) becomes,

$$\langle p_1|\tilde{\Sigma}_{1;1}(1;\,0+i\eta)\tilde{\psi}_1(1;\,z)|p_1\rangle - \langle p_1|\tilde{\Sigma}_2(1)\tilde{\psi}_1(1;\,z)|p_1\rangle + \langle p_1|\tilde{\Sigma}_s(1)\tilde{\psi}_1(1;\,z)|p_1\rangle - \langle p_1|\tilde{\Sigma}_B(1)\tilde{\psi}_1(1;\,z)|p_1\rangle, \quad (3.30)$$

where $\tilde{\Sigma}_B$ is the Born approximation to $\tilde{\Sigma}_s$, defined in Eq. (3.28). This result generalizes the collision kernel used by Williams and DeWitt[8] to include strong ion coupling, but it neglects electron screening.

The collision frequency found from Eq. (3.30) is

$$\nu_e = \frac{4\beta Ze^4}{3}\int_0^\infty dp\, p f_1(p)\Lambda_T(p), \quad (3.31)$$

where $\Lambda_T(p)$ is defined by

$$\Lambda_T(p) \equiv \Lambda_s(p) + \Lambda_2(p) - \Lambda_B(p), \quad (3.32)$$

$$\Lambda_s(p) = \frac{p^2}{m^2Z^2e^4}\sum_{l=0}^{\infty}(l+1)\sin^2(\delta_{l+1}-\delta_l), \quad (3.33)$$

$$\Lambda_2(p) = \left(\frac{1}{4\pi Ze^2}\right)^2\int_0^{2p} dk\, k^3 v_{12}^2(k)S_i(k), \quad (3.34)$$

$$\Lambda_B(p) = \left(\frac{1}{4\pi Ze^2}\right)^2\int_0^{2p} dk\, k^3 v_{12}^2(k)S_i^2(k). \quad (3.35)$$

This result describes electron-ion scattering events in terms of the phase shifts, $\delta_l$, associated with the statically screened potential, $V_s$, plus a correction to account, at least partially, for dynamic screening effects. It was obtained through the application of approximations (1)–(4) listed above plus the neglect of electron-electron correlations. These approximations have not been rigorously justified in terms of a small-parameter expansion, but they are consistent with the intuitive idea that, at least for high-$Z$ plasmas, the ion-ion correlations should be dominant. Of course, the electron-electron and electron-ion correlations are contained in the exact expression for $\tilde{\Sigma}_{1;1}$, and they were included in the discussion of weak collisions presented in Sec. II. So, perhaps a different set of approximations can be found to include these extra effects in a tractable strong collision theory. However, this will not be pursued here; and instead, simple numerical results based on the discussion of this and Sec. II will be presented.

## IV. NUMERICAL RESULTS

The results of Secs. II and III, especially Eqs. (2.29) and (3.32), provide fairly simple mathematical expressions which can be used to make numerical comparisons of various physical effects on the electron collision frequency. Equation (2.29) may be used to study the effects of strong ion coupling, electron screening, and bound electrons, while Eq. (3.32) may be used to assess the validity of the weak-collision approximation. The types of plasmas for which most of these calculations are tailored are those described experimentally in Ref. 3. Typically, these aluminum plasmas achieve electron densities between $10^{20}$ to $10^{24}$ cm$^{-3}$ and temperatures between 10 to 500 eV.

### A. Strong ion coupling and electron screening

In a weakly coupled plasma, the ion structure factor is well represented by its Debye-Huckel (DH) approximation,

$$S_{DH}(k) = k^2/(k^2+k_D^2) \quad (4.1)$$

where $k_D^2 = 4\pi n_1\beta e^2$. This expression breaks down in strongly coupled plasmas and it must be replaced. Calculating exact structure factors is in general quite difficult,[26] however Baus and Hansen[27] (BH) have proposed a semianalytic expression which simulates Monte Carlo results for $S(k)$. This improved approximation has the form

$$S_{BH}(k) = k^2/[k^2+k_D^2\,h(kr_0)], \quad (4.2)$$

where, in this paper, $h(kr_0)$ is taken to be

$$h(kr_0) = 3j_1(kr_0)/kr_0. \quad (4.3)$$

The parameter, $r_0$, is an increasing function, determined numerically, of the ion coupling param-

eter, $\Gamma \equiv \beta Z^2 e^2 (\frac{4}{3}\pi n_2)^{1/3}$.

Owing to the appearance of the spherical Bessel function in Eq. (4.2), the Baus-Hansen structure factor approaches unity more rapidly than the Debye-Huckel result. Values for $\Lambda$, calculated from Eq. (2.29), using the Baus-Hansen structure factor with electron screening ignored and the corresponding values found using the Debye-Huckel expression are compared in Tables I and II. As expected, the results found with the improved $S(k)$ are consistently larger than the Debye-Huckel answers, implying that the effect of strong coupling is to decrease the conductivity of the plasma by as much as 10–20%.

The effects of electron screening in nondegenerate plasmas[28] may be estimated by taking $v_s$ in Eq. (2.29) to be

$$v_s(k) = -4\pi Ze^2/(k^2 + k_D^2/Z). \tag{4.4}$$

Tables III and IV list values of $\Lambda$ found using Eq. (4.4) in Eq. (2.29) for various ionization states. Comparison of these values with the corresponding values in Tables I and II show a significant decrease in $\Lambda$ due to electron screening, but this effect is less pronounced for higher $Z$ values and lower $\Gamma$ values.

### B. Bound electrons

The effect of the electrons bound to the ions on the electron-ion cross section may be estimated by assuming that the density distribution for the bound electrons remains unaltered during a collision, and that this distribution is given accurately by its Hartee-Fock (HF) approximation. If this is taken to be the case, and electron screening is neglected, then

$$v_{12}(k) = (Ze^2/k^2)R(k)\,, \tag{4.5}$$

$$R(k) = [A - \rho(k)]/Z\,, \tag{4.6}$$

TABLE I. Comparison of $\Lambda$ calculated using the Baus-Hansen estimate of $S(k)$ ($\Lambda_{BH}$) to the same result calculated using the Debye-Huckel $S(k)$ ($\Lambda_{DH}$). The value of $\Gamma$ is 1.96 and $P$ is measured in units of $a^{-1} \equiv (4\pi n_2/3)^{1/3}$. Electron screening effects have been ignored.

| $P$ | $\Lambda_{BH}$ | $\Lambda_{DH}$ |
|---|---|---|
| 2 | 0.833 | 0.657 |
| 3 | 1.24 | 0.982 |
| 4 | 1.53 | 1.24 |
| 5 | 1.75 | 1.45 |
| 6 | 1.94 | 1.62 |
| 7 | 2.09 | 1.77 |
| 8 | 2.22 | 1.90 |
| 9 | 2.34 | 2.02 |
| 10 | 2.45 | 2.12 |

TABLE II. Same comparison as in Table I except that $\Gamma$ is now 5.15.

| $P$ | $\Lambda_{BH}(p)$ | $\Lambda_{DH}(p)$ |
|---|---|---|
| 2 | 0.601 | 0.438 |
| 3 | 1.02 | 0.602 |
| 4 | 1.31 | 0.818 |
| 5 | 1.53 | 1.01 |
| 6 | 1.72 | 1.17 |
| 7 | 1.87 | 1.31 |
| 8 | 2.01 | 1.43 |
| 9 | 2.12 | 1.54 |
| 10 | 2.23 | 1.65 |

where $A$ is the atomic number of the ion, and $\rho(k)$ is the Fourier transform of the bound-electron density distribution. For small $k$, $R(k)$ approaches unity, while for large $k$ it approaches $A/Z$. The effective charge on the ion, therefore, appears to increase with the energy of the incoming electron. Substitution of Eq. (4.5) into Eq. (2.29) yields

$$\Lambda_{HF} = \int_0^{2p} \frac{dk}{k} S_i(k) R^2(k)\,. \tag{4.7}$$

From the behavior of $R(k)$ it is apparent that the bound electrons should decrease the conductivity and that for a fixed net charge this decrease should be more significant at higher energies. These expectations are borne out in Table V which shows about a ten percent increase in $\Lambda_{HF}$ over $\Lambda_{DH}$ at 1 eV and about a forty percent increase at 10 eV. Table VI compares values of $\Lambda_{HF}$ for various values of net ionization. The trend is for $\Lambda_{HF}$ to decrease with increasing ionization due to the smaller size of the bound electron cloud. This effect is particularly pronounced in Al between $Z=2$ and $Z=3$ where the last $3s$ electron is stripped.

TABLE III. Values of $\Lambda_{BH}$ including electron screening for various ionization states with $\Gamma$=1.96.

| $P$ | $Z=3$ | $Z=5$ | $Z=10$ |
|---|---|---|---|
| 2 | 0.420 | 0.513 | 0.619 |
| 3 | 0.770 | 0.885 | 1.01 |
| 4 | 1.04 | 1.16 | 1.29 |
| 5 | 1.25 | 1.38 | 1.51 |
| 6 | 1.42 | 1.55 | 1.69 |
| 7 | 1.57 | 1.70 | 1.84 |
| 8 | 1.70 | 1.84 | 1.97 |
| 9 | 1.82 | 1.95 | 2.09 |
| 10 | 1.92 | 2.06 | 2.20 |

TABLE IV. Same as Table III, but now $\Gamma=5.15$.

| $P$ | $Z=3$ | $Z=5$ | $Z=10$ |
|---|---|---|---|
| 2 | 0.200 | 0.276 | 0.375 |
| 3 | 0.488 | 0.609 | 0.749 |
| 4 | 0.722 | 0.862 | 1.02 |
| 5 | 0.918 | 1.07 | 1.23 |
| 6 | 1.08 | 1.24 | 1.41 |
| 7 | 1.23 | 1.39 | 1.56 |
| 8 | 1.36 | 1.52 | 1.69 |
| 9 | 1.47 | 1.63 | 1.81 |
| 10 | 1.57 | 1.74 | 1.91 |

### C. Validity of the weak-collision approximation

All of the results discussed so far were obtained from Eq. (2.29) which is based upon the Born approximation to the electron-ion cross section. For temperatures below about 30 eV this approximation is expected to be inaccurate, and an improved expression like Eq. (3.32) should be used. In order to accomplish this, the phase shifts for the statically screened Debye potential were calculated for $l \leq 145$ for various temperatures and densities in the ranges $10^{18} \leq n_1 \leq 10^{22}$ cm$^{-3}$ and $0.1 \leq T \leq 100$ eV. The method employed was to integrate the radial Schrödinger equation using the Numerov technique[29] and to compare the locations of the zeros of the radial wave functions to the locations of the zeroes of the corresponding spherical Bessel functions. The integration was terminated when two successive estimates of $\delta_l$ differed by less than $\pm 10^{-4}\ \pi$ or when the wave function had been through 100 oscillations. In no case was the inaccuracy of $\delta_l$ greater than $\pm 3\times 10^{-3}\ \pi$. Table VII compares $\Lambda$ calculated from Eq. (3.32) to the corresponding values of $\Lambda_{DH}$, and it is apparent that the Born approximation is overestimating the cross section and giving conductivities which are too small, by as much as a factor of 2.

## V. DISCUSSION

The theory presented in Secs. I–III represents an example of the practical application of the formally exact kinetic theory developed in I to the study of plasma electrical conductivity. Section II applies the disconnected approximation of Gould and Mazenko[4] to the collision operator of the kinetic theory. This approximation is appropriate when strong collisions are not important, and to second order in the electron-ion interaction, it reproduces the Ziman[6] expression for the electron collision frequency, including electron screening effects. As seen by the numerical results presented in Sec. IV A the electron-electron correlations can make a significant contribution to the collision frequency. But even with them included, the picture is imcomplete, since there are also correlations between the electron and ion subsystem which should be included. In the classical limit, such correlations are included exactly in Eq. (2.40). Numerical results based upon this expression have not yet been obtained, so no statement regarding the actual importance of electron-ion correlations may be made.

The neglect of electron-ion correlations is, in essence, equivalent to modeling the plasma as a Lorentz gas. This model is known to be fairly accurate for degenerate plasmas, but it is much less accurate for nondegenerate plasmas due to the increased importance of electon-electron collisions and electron correlations.[13] Actually,

TABLE V. Comparison $\Lambda_{HF}$ to $\Lambda_{DH}$. In each case the material is $Al^{3+}$ at $n_1=10^{20}$ cm$^{-3}$, and $p=(6m/\beta)^{1/2}$.

| $T$ (eV) | $\Lambda_{HF}$ | $\Lambda_{DH}$ |
|---|---|---|
| 1.00 | 2.24 | 2.04 |
| 1.65 | 2.86 | 2.53 |
| 2.72 | 3.57 | 3.03 |
| 4.48 | 4.42 | 3.53 |
| 7.39 | 5.52 | 4.03 |
| 12.2 | 6.99 | 4.53 |
| 20.1 | 8.99 | 5.03 |

TABLE VI. Comparison of $\Lambda_{HF}$ for $Al^{2+}$, $Al^{3+}$, and $Al^{4+}$ at a density of $10^{20}$ cm$^{-3}$. The upper integration limit is $p=(6m/\beta)^{1/2}$.

| $T$ (eV) | $\Lambda_{HF}$ ($Z=2$) | $\Lambda_{HF}$ ($Z=3$) | $\Lambda_{HF}$ ($Z=4$) |
|---|---|---|---|
| 1.00 | 2.92 | 2.24 | 2.03 |
| 1.65 | 3.88 | 2.86 | 2.59 |
| 2.72 | 5.09 | 3.57 | 3.21 |
| 4.48 | 6.69 | 4.42 | 3.90 |
| 7.39 | 8.86 | 5.52 | 4.73 |
| 12.2 | 11.9 | 6.99 | 5.74 |
| 20.1 | 16.0 | 8.99 | 7.03 |

TABLE VII. Comparison of $\Lambda_T$ calculated from Eq. (3.32) using the phase shifts for the Debye screened potential to $\Lambda_{DH}$. The material is $Al^{3+}$ at $n_1=10^{20}$ cm$^{-3}$, and $p=(6m/\beta)^{1/2}$.

| $T$ (eV) | $\Lambda_{DH}$ | $\Lambda_T$ |
|---|---|---|
| 1.00 | 2.04 | 0.962 |
| 1.58 | 2.49 | 1.30 |
| 2.51 | 2.95 | 1.71 |
| 3.98 | 3.41 | 2.18 |
| 6.31 | 3.88 | 2.73 |
| 10.0 | 4.33 | 3.25 |
| 15.8 | 4.79 | 3.83 |
| 25.1 | 5.25 | 4.37 |

in the case of a very degenerate plasma one may not only neglect electron collisions and correlations, but the electron-ion cross section may be modeled in the Born approximation. The primary thrust of Sec. III, is to deal with this latter approximation and to provide, within the context of the Lorentz gas model, an improved picture of electron-ion scattering. This is presented in the principal result to that section, Eq. (3.10) which combines both the long-range electron-electron and ion-ion screening effects and close collisions. When electron-electron correlations are neglected in addition to electon-ion correlations, this result may be written in the simpler, more numerically useful, $t$-matrix form of Eq. (3.30). It is significant to note that the effect of strong collisions is not simply to replace the Born cross section in the Ziman expression, Eq. (2.29), with the square of a $t$-matrix, so that the integrand would involve something like the product $|t|^2 S_i(k)$. Rather, the structure factor is contained inside the $t$-matrix itself. Hence, the Ziman expression appears to be rigorous only in the weak-collision limit.


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## APPENDIX

The purpose of this appendix is to justify Eq. (2.3) as an approximation to the electrical conductivity. From Eqs. (1.7), (1.8), and (2.2), the electron contribution to the conductivity is

$$\sigma=\frac{\beta e^2}{3\Omega m^2}\,\mathrm{Im}\mathrm{Tr}_1\vec{p}(1)\cdot\left[\tilde{\Sigma}_{1,1}(1;0+i\eta)\right]^{-1}U_{1,1}(1)\vec{p}(1)\,. \tag{A1}$$

An approximation to Eq. (A1) which is equivalent classically to a single-Sonine polynomial approximation[30] to the solution of the kinetic equation is

$$\sigma\to-\frac{ne^2}{m}\,\mathrm{Im}\left(\frac{1}{\omega'+\omega_e}\right)\,, \tag{A2}$$

where $\omega_e$ is defined by Eq. (2.4). The quantity, $\omega'$, is real and is defined by

$$\omega'=\frac{-\beta}{3mn_1}\,\frac{1}{\Omega}\,\mathrm{Tr}_1^{(1)}\vec{p}(1)\cdot B_{1;1}(1)U_{1;1}(1)\vec{p}(1). \tag{A3}$$

Provided $\mathrm{Re}(\omega'+\omega_e)\ll\mathrm{Im}\omega_e\equiv\nu_e$, Eq. (A2) is equivalent to

$$\sigma\to\sigma_1\equiv\frac{n_1e^2}{m\nu_e}\,, \tag{A4}$$

which is just Eq. (2.3).

In order to ascertain if the condition mentioned above is reasonable, the second-order approximation to $\omega'+\omega_e$ will be considered. To second order in $V_{12}$, the real part of $\omega_e$ is

$$\mathrm{Re}\omega_e\to\int\frac{d\omega}{2\pi}\int\frac{d\bar{\omega}}{2\pi}\int\frac{d^3k}{(2\pi)^3}\,v_{12}^2(k)S_i(k;\omega)S_e(k;\bar{\omega})PP\,\frac{1}{\omega+\bar{\omega}}$$

$$=\int\frac{d^3k}{(2\pi)^3}\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3P_2}{(2\pi)^3}\,v_{12}^2(k)\,\vec{k}\cdot\vec{p}_1\tilde{f}_1(p_1-k)f_1(p_1)f_2(P_2)\,|\epsilon_i(k;\Delta E_2)|^{-2}\,|\epsilon_e(k;\Delta E_1)|^{-2}\,\frac{(1-e^{-\beta(\Delta E_1+\Delta E_2)})}{\Delta E_1+\Delta E_2}\,, \tag{A5}$$

where $\Delta E_1\equiv-\vec{k}\cdot\vec{p}_1/m-k^2/2m$. Using the definition of $B_{1;1,}(1)$, Eq. (1.18), and the Bloch-DeDominicis[31] rules for evaluating equilibrium distribution functions, the second-order approximation to $\omega'$ is found to give

$$\mathrm{Re}(\omega'+\omega_e)\to-\int\frac{d^3k}{(2\pi)^3}\left(\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3P_2}{(2\pi)^3}\,v_{12}^2(k)[1-|\epsilon_i(k;\Delta E_2)|^{-2}\,|\epsilon_e(k;\Delta E_1)|^{-2}]\,\vec{k}\cdot\vec{p}_1\tilde{f}_1(p_1-k)f_1(p_1)f_2(P_2)\right.$$

$$\left.\times\frac{(1-e^{-\beta(\Delta E_1+\Delta E_2)})}{(\Delta E_1+\Delta E_2)}\right)\,. \tag{A6}$$

For small $k$, $\epsilon_e^{-1}\to\epsilon_i^{-1}\to0$ and the quantity in the large parentheses goes to

$$\beta n_2 v_{12}^2(k)\int\frac{d^3p_1}{(2\pi)^3}\,\vec{k}\cdot\vec{p}_1\tilde{f}_1(p_1)f_1(p_1)=0.$$

For large $k$, $\epsilon_e^{-1}\to\epsilon_i^{-1}\to1$, and once again the large parentheses enclose a small quantity. Therefore, it seems reasonable to expect $\mathrm{Re}(\omega'+\omega_e)\ll\nu_e$, as required above.

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